

Contents

	<i>Preface</i>	<i>page xi</i>
1	Introduction	1
	1.1 Orthogonal polynomials on the real line	1
	1.1.1 Pearson equation and semi-classical orthogonal polynomials	4
	1.2 Painlevé equations	8
	1.2.1 The six Painlevé differential equations	8
	1.2.2 Discrete Painlevé equations	9
2	Freud weights and discrete Painlevé I	13
	2.1 The Freud weight $w(x) = e^{-x^4+tx^2}$	13
	2.2 Asymptotic behavior of the recurrence coefficients	16
	2.3 Unicity of the positive solution of d-P _I with $x_0 = 0$	17
	2.4 The Langmuir lattice	21
	2.5 Painlevé IV	23
	2.6 Orthogonal polynomials on a cross	24
3	Discrete Painlevé II	27
	3.1 Orthogonal polynomials on the unit circle	27
	3.1.1 The weight $w(\theta) = e^{t \cos \theta}$	28
	3.1.2 The Ablowitz–Ladik lattice	31
	3.1.3 Painlevé V and III	32
	3.2 Discrete orthogonal polynomials	34
	3.2.1 Generalized Charlier polynomials	36
	3.2.2 The Toda lattice	43
	3.2.3 Painlevé V and III	44
	3.3 Unicity of solutions for d-P _{II}	46
4	Ladder operators	50
	4.1 Orthogonal polynomials with exponential weights	50

viii	<i>Contents</i>	
	4.2 Riemann–Hilbert problem for orthogonal polynomials	53
	4.3 Proof of the ladder operators	55
	4.4 A modification of the Laguerre polynomials	57
	4.5 Ladder operators for orthogonal polynomials on the linear lattice	60
	4.6 Ladder operators for orthogonal polynomials on a q -lattice	61
5	Other semi-classical orthogonal polynomials	64
	5.1 Semi-classical extensions of Laguerre polynomials	64
	5.2 Semi-classical extensions of Jacobi polynomials	65
	5.3 Semi-classical extensions of Meixner polynomials	66
	5.4 Semi-classical extensions of Stieltjes–Wigert and q -Laguerre polynomials	70
	5.5 Semi-classical bi-orthogonal polynomials on the unit circle	74
	5.6 Semi-classical extensions of Askey–Wilson polynomials	79
6	Special solutions of Painlevé equations	83
	6.1 Rational solutions	83
	6.1.1 Painlevé II	83
	6.1.2 Painlevé III	88
	6.1.3 Painlevé IV	91
	6.1.4 Painlevé V	98
	6.1.5 Painlevé VI	102
	6.2 Special function solutions	103
	6.2.1 Painlevé II	103
	6.2.2 Painlevé III	106
	6.2.3 Painlevé IV	108
	6.2.4 Painlevé V	109
	6.2.5 Painlevé VI	112
7	Asymptotic behavior of orthogonal polynomials near critical points	115
	7.1 Painlevé I	119
	7.2 Painlevé II	129
	7.3 Painlevé III	136
	7.4 Painlevé IV	137
	7.5 Painlevé V	141
	7.6 Painlevé VI	146

	<i>Contents</i>	ix
<i>Appendix</i>	Solutions to the exercises	147
	<i>References</i>	167
	<i>Index</i>	177