1.1 Routine activities or problems

There are many things that we automatically do every day and would not consider to be 'problems'. For example, consider the action of making a cup of instant coffee – if you analyse the processes you need to go through, they are quite complicated. Just the list of items you need is quite long: a cup, a teaspoon, a jar of coffee, a kettle, water – and milk and sugar if you take them. Having found all these items, you fill the kettle and boil it; use the teaspoon to put coffee into the cup; pour the boiling water into the cup, just to the right level; stir; add milk and sugar; then put all the things you used away again. We could, if necessary, break this down even further, for example by explaining in detail what needs to be done to boil the kettle.

Although this is complicated, it is an everyday task that you probably do without thinking (assuming that you drink coffee that is, although the same principle applies for making tea or preparing your breakfast). To continue the coffee example, if over the years you have made many cups of coffee, you may have tried adding different quantities of coffee granules, milk and sugar to work out the strength of coffee that you prefer. Once you have established this, you simply make the coffee to your preferred specifications from that point onwards.

When you encounter something new, even a task which is no more complicated than others that you have previously completed, the processes required to complete the task may need considerably more thought and planning. Experience of similar problems is often helpful, however: when trying a new type of hot drink you may be able to more readily work out what the best combination of ingredients is for you.

Problem solving takes place when you are required to perform a task that you have not previously completed. These tasks may be very similar in nature to ones that you have done in the past or may be completely new. Experience of a wide range of different problems and their solutions is therefore very useful in developing problem-solving skills.

ACTIVITY 1.1

Imagine you are going to book tickets for a concert. List the pieces of information you need and the processes that you need to go through in order to book the tickets and get to the concert. In what order should you do them? First list the main things, then try to break each down into smaller parts.

1.2 What skills are needed to solve problems?

There are a range of different skills that can be required to solve problems, which can be divided into the following categories:

1 **Understanding the problem that needs to be solved**

Information might be presented in a range of different forms, using tables or diagrams in addition to text. Different pieces of information will be related to each other in many different ways, which might be explained through a simple mathematical model.

Being able to identify the appropriate pieces of information, and understand how they are related to each other, is necessary if the solution to a problem is going to be found.

2 **Processing the information that is available to solve the problem**
Reaching the solution to the problem may require some simple mathematics to be performed, based on the relationships that exist between the different pieces of information. There may also be certain criteria that need to be met – for example, a maximum weight that can be transported over a bridge in one journey.

Processing the information allows deductions to be made which lead to the solution to the problem.

3 **Analysing data and representing it in different ways**
Presenting information in a different way often allows different insights to be gained about the data that is available. Recognising patterns that exist in data can be very helpful for solving problems.

When considering problems that involve objects that can be moved into different positions and orientations, it can be very useful to be able to identify key features that confirm that the shape is unchanged.

4 **Developing a problem that has already been solved to consider more complex situations that might occur**
Many problems are solved by first considering much simpler versions of the problem. Once simplified versions of problems have been solved, it is important to be able to look at how that solution helps with the solutions to more complicated problems.

In addition to these problem-solving skills, there are a number of mathematical skills which are needed – many tasks will require some simple arithmetic and, as mentioned above, some familiarity with methods for representing data, such as bar charts and pie charts, is very useful.

EXAMPLE ACTIVITY 1.1

Imagine that you have to work out the best way to fit a number of rectangular packages into a box. Which of the four categories of skill identified above would be relevant to solving the problem?

Commentary

This problem involves mainly categories 1 and 2, but could involve some of category 4 as well:

1 Understand the problem that needs to be solved:

The dimensions of the box and the packages need to be known. Once these have been worked out you also need to think about the different orientations in which the packages could be placed into the box. You might make an initial assumption about the best orientation, which may turn out later to be wrong.

2 Process the information that is available:

Once you know the relevant dimensions, some appropriate calculations should enable you to work out how many boxes can be fitted in for each orientation.

4 Consider more complex situations:

If we know how to solve the problem in situations where all of the packages are the same shape and size then similar methods may help in situations where there is more than one shape of package.

If you had to solve the problem, you would need to be systematic and have some sort of strategy. In the case of problems such as this one it may also be difficult to be confident that you have the best solution – what if the best solution that you have found leaves a gap and the volume of the gap is larger than the volume of the box, how can you be sure that a different arrangement would not have managed to pack an extra box?

With some problems the method of finding an answer might be quite clear. With others there may be no systematic method and you might have to use trial and error from the start. Some problems will require a combination of both methods or can be solved in more than one way.

The example below is a simple problem; you can give either a simple answer or a more complicated one, depending on the degree of detail you consider necessary.

EXAMPLE ACTIVITY 1.2

Luke has a meeting, in a town that is 50 miles away, at 3 p.m. tomorrow. He is planning to travel by train from the town where he lives to the town where the meeting is, walking to and from the station at both ends.

List the pieces of information Luke needs in order to decide what time he must leave home.

Commentary

Let us start by thinking of everything that Luke does from leaving his house to arriving at the meeting:

1 He leaves his house.
2 He walks to the station.
3 He buys a train ticket.
4 He goes to the platform.
5 He boards the train when it arrives.
6 He sits on the train until it reaches its destination.
7 He leaves the train.
8 He walks to where his meeting is to be held.

We can construct the pieces of information that he needs from this list. They are:

1 The time taken to walk from his house to the station.
2 The time needed to buy a ticket (including the time waiting in a queue).
3 The time to walk to the platform.
4 The train timetable.
5 The time taken to walk from the station to where the meeting is being held.

EXAMPLE ACTIVITY 1.3

Suppose that we are given the following additional information:

It takes Luke 15 minutes to walk to the station and will take him 20 minutes to walk to the meeting. The trains leave the station every 30 minutes from 9 a.m. and the train journey takes a total of 50 minutes. Luke estimates that it will take him 10 minutes to buy his ticket and walk to the platform.

What time should Luke leave home in order to get to his meeting on time?

Commentary

We could choose a start time based on when we *think* Luke should leave home and then work out what time Luke would arrive, but a better strategy would be to work out when Luke should leave by working backwards from the meeting start time.

The sections of the journey can be considered one at a time. The start time of one activity is always the time by which the previous activity must have been completed.

Section of journey	Must be completed by	Time required	Latest start time
Walk to meeting	3:00 p.m.	20 minutes	2:40 p.m.
Train	2:40 p.m.	50 minutes	1:30 p.m.
The latest train that arrives by 2:40 p.m. must leave no later than 1:50 p.m., and the 1:30 p.m. train is the latest such train.			
Walk to station, buy ticket, walk to platform	1:30 p.m.	25 minutes	1:05 p.m.

Of course, you could do the whole thing by guesswork, but you might get it all wrong and, more to the point, you cannot be confident that you will have got it right.

1.3 Selecting a problem-solving strategy

In the sense we are using the word in this book, a 'problem' means a situation where we need to find a solution from a set of initial conditions. In some cases, as in the example above, the method for solving the problem is not difficult to work out once you have identified the information that is important.

In order to solve problems we must use the information that we are given in a certain way. The way in which we use it may be quite straightforward; it may for example be simply a matter of searching a table for a piece of data that matches given conditions. In other cases, instead of searching for a piece of data, we may have to search for a method of solution. The important thing in either case will be to have a strategy that will lead to the solution.

Imagine you are going out and can't find your house keys. Finding them is a problem in the sense meant by this section of the book.

- One method (and sometimes the quickest) is to run around all the *likely* places to see if they are there.
- After the likely places, you start looking at the *less likely* places, and so on until they turn up or you have to resort to more systematic methods.

There are two systematic ways of searching.

- The first (using experience) involves thinking carefully about when you last came into the house and what you did: this can be the quickest method.
- The other (which in mathematical terms is often known as the 'brute force' method) involves searching every room of the house thoroughly until they are found. This is often the most reliable method but can take a very long time and most people will use it as a last resort.

When people are solving problems similar to the one identified above, they may use any one or more of these methods. One of the prime skills that you need is to make a good judgement of which method is the most appropriate one to use in each particular situation.

In any problem, you will be presented with some initial pieces of information – these may be in the form of words, a table of numbers, a graph or a picture. You will also know what question it is that you need to answer. The first thing to do is to identify which pieces of information are most likely to be useful in proceeding to the solution and to try to work out how these pieces of information may be used. In many situations much more information than is needed is available and identifying which of it is redundant (not needed for the solution to the problem) is an important part of the problem-solving process.

The example below is relatively easy. It is not difficult to find a way of approaching the problem, and the necessary calculations are clear and simple.

EXAMPLE ACTIVITY 1.4

Julia has been staying in a hotel on a business trip. When she checks out, the hotel's computer isn't working, so the receptionist makes a bill by hand from the receipts, totalling $471. Julia thinks she has been overcharged, so she checks the itemised bill carefully.

Room:	4 nights at $76.00 per night
Breakfast:	4 at $10.00 each
Dinner:	3 at $18.00 each
Telephone:	10 units at $1.70 per unit
Bar:	Juices and soft drinks totalling $23.00
Laundry:	3 blouses at $5.00 each

It appears that the receptionist miscounted one of the items when adding up the total. Which item has Julia been charged too much for?

Commentary

Although this example is simple, it illustrates many of the methods used in solving problems:

- Look at the data provided. Identify which pieces are relevant and which are irrelevant. In this case all of the information is relevant to the solution as we will need to know how much Julia should have been charged.
- Make an intermediate calculation before you can reach the answer. In this case it was necessary to calculate the value that the bill should have been in order to identify where Julia had been overcharged. The sum of the charges on the itemised bill is $453, so Julia has been overcharged by $18.

- Search the data for a piece of information that helps to solve the problem. In this case, we need to find an item on the bill that could cost $18 (either singly or for some number of them). The only item on the itemised bill for which this is true is dinner, so Julia must have been charged for one extra dinner.

This is an example of using a systematic procedure to solve the problem.

The example below, whilst still being relatively simple, involves a slightly different type of problem where the method of solution is less obvious.

EXAMPLE ACTIVITY 1.5

The SuperSave supermarket sells Sudsy washing-up liquid for $1.20 a bottle. At this price they are charging 50% more than the price at which they buy the item from the manufacturers. Next week SuperSave is having a 'Buy two, get a third free' offer on this item. The supermarket does not want to lose money on this offer, so it expects the manufacturers to reduce their prices so SuperSave will make the same actual profit on every three bottles sold.

By how much will the manufacturers have to reduce their prices?

Commentary

There are several pieces of information that we will need in order to solve this problem.

1 How much do SuperSave normally buy a bottle of Sudsy washing up liquid for?

To make a 50% profit on the sale of each bottle, SuperSave must pay 80¢ for each bottle (two-thirds of the price for which they sell it).

2 How much profit do SuperSave normally make on the sale of three bottles of Sudsy washing up liquid?

The profit is 40¢ for each bottle, so the profit is normally $1.20 on three bottles.

3 How much would SuperSave need to pay for three bottles in order to make this profit under their new offer?

Under the offer, SuperSave would only be paid $2.40 for three bottles, so need to pay just $1.20 for three bottles.

4 By how much do the manufacturers need to reduce the price?

The price needs to be reduced to 40¢ per bottle, which is a reduction of 40¢ per bottle (or half of the normal price).

In fact, it is possible to deduce that the manufacturers would have to reduce the price to half the normal price without knowing that SuperSave's normal selling price is $1.20:

1 Since the amount that SuperSave must pay for the bottles is two-thirds of the price for which they sell it, the profit that SuperSave make on the sale of three bottles must be the price for which they sell one bottle. Additionally, SuperSave currently buy three bottles for the same price at which they sell two.

2 Once the offer is in place, customers will only be paying for two bottles when they buy three. To make the same profit on the sale of three bottles, SuperSave will therefore need to have bought the three bottles for the price at which they sell one.

3 The price that SuperSave pay for bottles must therefore have reduced to half of its previous value.

There are other ways in which the solution could be found. For example, it would be possible to choose a value for the reduction, see whether it would give the required result and then adjust it accordingly. The systematic approach is more efficient however (unless you are lucky enough to choose the correct value on your first guess).

ACTIVITY 1.2

Match the key terms (1–7) with the correct definition (A–G).

Note: you will not be asked to give definitions in the problem-solving exam, but it will help you develop your problem-solving skills if you understand these key terms.

Key terms

1 Data
2 Problem-solving
3 Information
4 Redundant
5 Systematic
6 Strategy
7 Trial and error

Definitions

A Using logic, mathematics or a systematic process in order to work out a way to do a task that you have not done before.
B Plan for how to achieve something.
C Involving a method, plan or step-by-step process to do something.
D Way of solving a problem, or finding the best way to achieve a desired result, by trying one thing or another until something succeeds.
E Knowledge, facts or news about a person, situation or event.
F Facts, numbers and statistics used to calculate, plan or analyse something.
G Information that is not useful or necessary.

Summary

Having read this chapter you should be able to:

- identify the information that is important in solving a problem
- identify an appropriate method to solve the problem.

End-of-chapter question

You want to buy a new mobile phone. You will use it for texting, making a few phone calls and keeping in touch with friends and family on social media. You have found two phones:

- a 'Mobes4U' phone that costs $36 if you sign a two-year contract with the company
- a 'Pay'N'Go' phone that costs $50 but you are not tied to a contract.

a What other pieces of information should you consider before you decide which phone to buy?

b Explain how you would use the information to make a decision.

Exam-style question

Nazim is planning to cook curries and sell them. He will take orders for parties. He will cook the curry and rice beforehand, then put the cooked food in serving dishes that will keep it hot. Nazim will pay his friend Raj, who has a van, to make deliveries and to fetch the serving dishes back after the party.

Nazim's first order is for a party of 27 people. He has a recipe using vegetables, chick peas, onions and tomatoes. He is now trying to work out the right quantities of spices. For 15 people he would normally use 100 g of spices, made up of ginger (20%), turmeric (25%), coriander (35%), and cumin seeds (20%).

a What weight of turmeric must Nazim put in a curry for 27 people?

b The food has been ordered to arrive at the party at 7:30 in the evening. Nazim knows he needs 30 minutes to prepare the vegetables. The curry takes 2 hours to cook and the rice takes 20 minutes. Putting the food in the dishes takes 10 minutes and putting the dishes in the van is another 10 minutes. Raj's journey to the party will take 25 minutes.

What is the latest time Nazim can start preparing the food?

c Nazim is buying rice. The supermarket sells 10 kg of basmati rice for $15.45. Today they have a 'buy 3, get the 4th free' offer. The wholesaler also offers 50 kg sacks of rice for $59.00.

Based on price, which rice should Nazim buy?

d Nazim wants to calculate the prices he should charge to customers to make sure he can make a profit. State three pieces of information that he needs.