

Introduction

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We talk and think about our beliefs both in qualitative terms – as when we say that we believe A, or disbelieve A, or are agnostic about A – and in quantitative terms, as when we say that we believe A to a certain degree, or are more strongly convinced of A than of B. Traditionally, analytic philosophers, especially epistemologists, have focused on categorical (all-or-nothing) beliefs, to the almost complete neglect of graded beliefs. On the other hand, the Bayesian boom that started in the late 1980s has led many philosophers to concentrate fully on graded beliefs; these philosophers have sometimes rejected talk about categorical beliefs as being unscientific and as therefore having no place in a serious epistemology.

By now, many regard both approaches as misguided for being entirely one-sided. Both outright beliefs and graded beliefs occupy important places in the phenomenology of belief and they also both occur in much theoretically significant work. Once this is acknowledged, however, the question arises of how the two sorts of beliefs are connected. If anyone were to claim to believe A categorically while at the same time claiming that she deems not-A more likely than A, we would reject this as utterly unreasonable. This is enough to suggest that there must be a close connection between categorical and graded belief.

However, exactly *how* the two are connected is a matter of ongoing controversy. At first the solution may seem easy and to be given by what has been called “the Lockean Thesis” (LT). According to LT, it is rational to believe A categorically if, and only if, it is rational to believe A to a degree above a certain threshold value ϑ , where ϑ is close, but unequal, to 1. While *prima facie* entirely innocuous, LT is known to conflict with two seemingly equally uncontestable principles concerning outright belief – to wit, the Conjunction Principle (CP), according to which it is rational to believe the conjunction of any two propositions that are each rationally believable, and the No Contradictions Principle (NCP),

according to which it is never rational to believe an explicit contradiction. The problem – known as the “Lottery Paradox” – is that, jointly, LT, CP, and NCP lead to the contradictory conclusion that all tickets in a fair, large lottery that is guaranteed to have a winner will lose.

In the face of this paradox, some philosophers have argued that LT must be abandoned, or at least replaced by some paradox-free variant, while others have instead argued that CP be replaced by some weaker closure principle. (Very few philosophers have proposed to give up NCP.) However, we are nowhere near a consensus on how best to respond to the Lottery Paradox.

This volume contains eleven new chapters on the Lottery Paradox or relating to it. Questions that are being addressed are: What exactly are the options for responding to the Lottery Paradox? Specifically, which alternatives to LT are on offer and how reasonable are they? How does the Lottery Paradox connect to broader issues in epistemology and beyond, such as the belief–action connection, the Preface Paradox, and the connection between knowledge and rational belief? Some contributions are also devoted to investigating the assumptions – concerning the psychology of belief and the unity of the mind – that underlie the whole debate about how categorical and graded beliefs are related. In short, this volume aims to examine the nature of the Lottery Paradox, its ramifications and applications, and the challenges that it has encountered.

Overview of the Contributions

Dana Nelkin makes a case for the thesis that we are never rationally entitled to believe a proposition just because that proposition has a high statistical probability. This thesis would explain why we are reluctant to categorically believe that our ticket will lose, even if its winning chances are slim. The thesis has ramifications far beyond lotteries, however. Nelkin argues that while the thesis faces certain pressures, for instance from considerations having to do with the accuracy of our beliefs, it is to be maintained due to the important explanatory work it can do, for instance in the legal and moral realms but also in accounting for certain biases in our belief-forming practices.

Authors working on the Lottery Paradox and related issues tend to start from claims that are alleged to be pretheoretically uncontroversial, or intuitive, or natural to say, such as when we deny knowledge of our ticket being a loser. But experimental philosophers have shown that such claims

of what it is intuitive to suppose (which abound in analytic philosophy) are often not shared by the general public and hence can hardly be claimed to be pretheoretically obvious. John Turri presents new experimental findings concerning what people actually are and are not willing to say about knowledge of lottery propositions (i.e., propositions to the effect that a given ticket is a loser), showing how, in this context, the difference between two types of information affects people's knowledge judgments. This has important implications for views of how people see the connection between knowledge and justification.

Jennifer Nagel discusses the psychology underlying the intuitions that are generally taken to be involved in the Lottery Paradox. Specifically, she looks at empirical work on how probabilities are represented, arguing that this work shows that a range of interestingly different intuitive and reflective processes are deployed when we think about possible outcomes in different contexts. She further argues that if we pay attention to how our thinking can shift naturally, it is no longer clear that the Lottery Paradox brings to light deeply problematic aspects of the concept of knowledge. As she also points out, however, these shifts in our thinking raise new questions about how we ought to represent possible outcomes to begin with.

Julia Staffel discusses the Lottery Paradox in relation to two other problems: the statistical-evidence problem (also central to Nelkin's chapter) and the Harman–Vogel paradox. The former problem starts with the observation that we tend not to categorically believe things strictly on the basis of statistical grounds. But why is that? And is there any justification for this tendency? On the other hand, the Harman–Vogel paradox concerns the fact that we ordinarily claim to know certain things which imply some lottery proposition, which, however, we consider ourselves *not* to know. That goes against what would otherwise seem a perfectly innocuous assumption: that we know the (obvious) consequences of our knowledge. Staffel investigates whether the proposed solutions to the Lottery Paradox can help us in solving both the statistical-evidence problem and the Harman–Vogel paradox.

Historically, the first response to the discovery of the Lottery Paradox consisted in the proposal to abandon CP. Later, however, most philosophers came to see this solution as being too costly, and focused instead on replacing LT. This approach often comes with the claim that it is unintuitive to say that we could be justified in believing of a given ticket that it will lose. Going beyond appeals to intuition, Martin Smith in his contribution gives four arguments for holding that we cannot justifiedly believe

lottery propositions. As he shows, this puts pressure on some widely held views about how justification relates to knowledge, risk, and surprise.

Virtually all authors writing about the Lottery Paradox have assumed that there must be some way to describe formally the connection between categorical and graded beliefs. This broadly shared assumption may be based on what is perhaps the most fundamental, if typically unstated, tenet underlying the debate about the Lottery Paradox – that the mind is one and undivided. If beliefs and degrees of belief live in the same “space,” then it is difficult to see how they could not be related in some way that we can formally characterize. Igor Douven and Shira Elqayam argue that while the image of beliefs and degrees of belief all being in one and the same mind is captivating, it is also one that has come under pressure from recent work in psychology. They draw attention to a version of Dual Process Theory according to which different mental tasks may be carried out by different “ad hoc committees.” Their suggestion is that the epistemology of belief and that of degrees of belief may be governed by different such committees.

Gerhard Schurz generalizes implications from the Lottery and Preface Paradoxes to arrive at a number of impossibility results for rational belief. He shows that no minimally adequate epistemic system can at the same time satisfy LT and CP while also being fallibilist. On the positive side, he develops two new ways to escape the paradoxes and, more generally, his impossibility results. One, to be applied in practical contexts, places probabilistic constraints on CP, and the other, to be applied in epistemic contexts, gives pride of place to the notion of belief-as-approximately-true.

Hannes Leitgeb has recently proposed a new solution to the Lottery Paradox. In this proposal, LT gets replaced by the thesis that for a categorical belief in a proposition to be rationally held, one ought to have a *stably* high degree of belief in the proposition, meaning that the degree of belief should be high not only on one’s current evidence but should stay high also if one were to learn things one now deems epistemic possibilities. While this proposal solves the Lottery Paradox and has a number of other virtues as well, it comes at the cost of making rational belief sensitive to context. In his contribution to this volume, Leitgeb considers the question of how much of a concern this context-sensitivity is and also looks at ways for making his original proposal less context-sensitive.

Christoph Kelp and Francesco Praolini look at possible solutions to the Lottery Paradox from a rule-consequentialist perspective, specifically by asking how a candidate solution fares in light of our epistemic goal.

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They criticize a proposed solution that was explicitly meant to be rule-consequentialist, and argue that it can be salvaged by giving up the idea that our epistemic goal is truth and adopting instead the idea that that goal is constituted by knowledge. They also bring their argument to bear on the Preface Paradox.

Kevin T. Kelly and Hanti Lin offer a broad perspective on possible bridge principles between beliefs and degrees of belief. Most authors concerned with the Lottery Paradox have only looked at principles that might guarantee that beliefs and degrees of belief cohere synchronically in some way. As they argue, however, we should also consider the matter from a diachronic perspective. Coherent revisions of our beliefs should match coherent updates of our degrees of beliefs and vice versa. They show that various well-known proposals in the line of LT have no plausible extension that could also take into account diachronic constraints. On the other hand, a version of the odds threshold account, proposed by the authors in previous work, does satisfy the constraints and can still be regarded as being in the Lockean spirit.

Franz Dietrich and Christian List, finally, are also concerned with the question of how to connect categorical beliefs and degrees of belief with each other. As they note, all traditional proposals are *local* in the sense that they make a given proposition's rational believability a matter of the degree of belief assigned to just that proposition. They discuss a previous result of theirs showing that, given very minimal conditions on rationality, this locality cannot be maintained. They strengthen their result by proving that locality cannot be maintained even if the idea that categorical belief is a function of degrees of belief is abandoned.

CHAPTER I

Rational Belief and Statistical Evidence
Blame, Bias, and the Law

Dana Kay Nelkin

Introduction: The Lottery Paradoxes and Statistical Evidence

It is rational to believe all sorts of things, even things about which we are not absolutely certain, and about which we admit that there is a chance that we are wrong. And we know that it is possible to have excellent evidence for propositions that are false. For example, I believe – rationally – that I will be in Munich next month, based on my intention to be there, having taken all requisite steps to enact my plan, such as making a flight reservation, and yet I admit that there is a small chance that I will not in fact be in Munich. A small percentage of transatlantic flights are cancelled, there is a small chance of a natural disaster such as a volcano eruption that grounds all flights across Europe, and so on. Yet when it comes to some propositions, we can have apparently very strong evidence that makes their truth extremely likely – even more likely than that I will be in Munich next month – without its being rational to believe them. For example, there is a good argument that it is not rational for me to believe that my single ticket in a million-ticket lottery will lose, based on the evidence that 999,999 tickets will lose.¹ Is this correct? If so, how can we explain it?

One argument for this is based on the Rationality Version of the Lottery Paradox. In a nutshell: If it were rational to believe that my ticket will lose, it would, by parity of reasoning, be rational to believe that every other ticket in the lottery will lose, and yet it is also rational for me to believe that not every ticket in the lottery will lose. Thus, it would appear that it is

An earlier version of this chapter was presented at the Current Themes in Practical Philosophy Workshop at the Ludwig-Maximilians-Universität München, and I am very grateful for excellent feedback from the participants. Special thanks to Monika Betzler, Kathleen Connelly, Sam Rickless, and Anna Wehofsits for their very helpful comments on written versions of the paper. Finally, I am grateful to Tyler Burge for first bringing the Lottery Paradox to my attention as a graduate student working on free will and rational agency, and to Igor Douven for the opportunity to again bring together a variety of my interests in returning to it in the writing of this paper.

¹ See, for example, Harman (1968), Kaplan (1996), Nelkin (2000).

rational for me to believe a set of things that I can see are inconsistent (for each ticket, that it will lose, and that not every ticket will lose), and this is clearly a *reductio ad absurdum* of our assumptions. This set of claims together forms what I call the Rationality Version of the Lottery Paradox.²

The absurd conclusion can be avoided by rejecting the initial claim that it is rational for me to believe that my ticket will lose. If we accept this solution to the paradox, then we must also accept that it can be rational for me to believe things, such as that I will be in Munich next month, that are less likely than things that it is not rational for me to believe, such as that my ticket will lose in a million-ticket lottery. The same applies to beliefs about the past as well as those about the future. For example, it is rational for me to believe that the vote originally scheduled for December on Theresa May's Brexit plan was postponed, even though newspapers sometimes make mistakes and I sometimes misread, even if the chance of error is higher than that my lottery ticket will lose.

This solution to the paradox is controversial, although lately it appears to be gaining somewhat in popularity.³ To this point, philosophers have been much more likely to accept a parallel solution to the *Knowledge* Version of the Lottery Paradox. According to that version, if I know that my ticket will lose, it would, by parity of reasoning, be the case that I know that every other ticket in the lottery will lose, and yet I also know that not every ticket in the lottery will lose. Thus, it would appear that I can know a set of things that are inconsistent (for each ticket, that it will lose, and that

² More formally and completely:

- (1*) It is rational for Jim to believe that his ticket (t_1) will lose.
- (2*) If it is rational for Jim to believe that t_1 will lose, then it is rational for Jim to believe that t_2 will lose, it is rational for Jim to believe that t_3 will lose, . . . and it is rational for Jim to believe that $t_1,000,000$ will lose.
- (3*) It is rational for Jim to believe that t_1 will lose and . . . and it is rational for Jim to believe that $t_1,000,000$ will lose. (1*, 2*)
- (4*) It is rational for Jim to believe that t_1 will not lose or t_2 will not lose or . . . or $t_1,000,000$ will not lose.
- (5*) Propositions of the following form comprise an inconsistent set: (a) $p_1 \dots (n) p_n, (n+1) \text{ not-}p_1 \text{ or } \dots \text{ or not-}p_n$.
- (6*) Jim recognizes that the following propositions comprise an inconsistent set: (i) t_1 will lose. . . (n) $t_1,000,000$ will lose, t_1 will not lose or . . . or $t_1,000,000$ will not lose.
- (7*) It is rational for Jim to believe inconsistent things that he recognizes are inconsistent. (3*, 4*, 5*, 6*)
- (8*) It cannot be rational to believe inconsistent things that one recognizes are inconsistent. Therefore,
- (9*) (1*), (2*), (4*), (5*), (6*), or (8*) is false. (Adapted from Nelkin (2000).)

³ Admittedly, "gaining in popularity" should be understood relative to its starting point. Compare Redmayne (2008) to Buchak (2014), for example.

not every ticket will lose), and this is clearly a *reductio ad absurdum* of our assumptions.⁴ A widely accepted solution to this Knowledge Version of the Paradox is to reject the assumption that I know that my ticket will lose. I simply do not *know* that my ticket will lose despite its likelihood of doing so, and this does not sound counterintuitive. But when it comes to the Rationality Version, people have been much more reluctant to reject the parallel assumption that it is *rational* for me to believe that my ticket will lose. They might say, “I admit that I don’t *know* that my ticket will lose, but surely it’s rational for me to believe it, given the high likelihood.”

In reply, I have argued that there is a principled distinction to be made between, on the one hand, my beliefs that I will be in Munich and that the Brexit vote was postponed, and, on the other hand, the ticket-buyer’s belief that her lottery ticket will lose. The difference is that the evidential support for the belief that her ticket will lose is exhausted by statistical evidence, whereas this is not the case when it comes to the other fallible beliefs, such as those about Munich and Brexit. My evidence in the Munich case is not merely statistical: It includes my awareness of my intentions and plans, my visual confirmation of a receipt for my plane ticket, and so on. My evidence in the Brexit vote postponement case includes my having read about it in several trusted online news sources and my having discussed it with others who have additional sources. Because this proposed solution to the Lottery Paradox draws a distinction between beliefs supported solely by statistical evidence and beliefs supported by nonstatistical evidence, I call it the “Statistical Support Solution.” It relies on this key Statistical Thesis (ST):

(ST) For any proposition P, it is not rational to believe P solely on the basis of a high statistical probability of P.

In other words, it is not rational to form beliefs on the basis of what I will call “P-inferences,” namely, inferences of the form “p has a statistical probability of n [where n is a very high number] → p.”

⁴ More formally and completely:

- (1) Jim knows that his ticket (t₁) will lose.
- (2) If Jim knows that his ticket (t₁) will lose, then he knows that t₂ will lose, he knows that t₃ will lose, . . . and he knows that t_{1,000,000} will lose.
- (3) Jim knows that t₁ will lose and . . . and Jim knows that t_{1,000,000} will lose. (1, 2)
- (4) Jim knows that t₁ will not lose or t₂ will not lose or . . . or t_{1,000,000} will not lose.
- (5) Propositions of the following form comprise an inconsistent set: (a) p₁ . . . (n) p_n, (n+1) not-p₁ or . . . or not-p_n.
- (6) Jim knows propositions that form an inconsistent set. (3, 4, 5)
- (7) It is not possible to know propositions that form an inconsistent set. Therefore,
- (8) (1), (2), (4), (5), or (7) is false.

Thus, (ST) nicely sorts the cases in question, while offering a solution to the Rationality Version of the Lottery Paradox. But is there independent reason for thinking that it is true? I believe that there is, and it ultimately rests on the nature of rationality. Even among those who offer very different solutions to the Lottery Paradox, many agree that rationality is a guide to the truth, and this is the starting point to an explanation for the fact that P-inferences are not rational ones. For believing solely on the basis of statistical evidence is inconsistent with being able to posit a causal or explanatory connection between one's belief and the fact that makes it true. In the lottery case, what explains the fact that it is not rational to believe that one's ticket will lose is that one cannot see a causal or explanatory connection between one's belief and the fact that makes it true. In fact, given the nature of the evidence, one can see that there is no such connection to be found. The role of rationality in one's belief-forming activities is to guide one's beliefs toward the truth, via reasons. There is a way in which this role is fulfilled in inferences to beliefs such as that I will arrive in Munich next month, but not in the case of P-inferences. In the everyday case like that of my belief that I will be in Munich or my belief that there was a postponement of the vote on Theresa May's Brexit deal in the UK Parliament yesterday, the rationality of one's belief depends on one's being (rationally) committed to the proposition that there is an explanatory connection between one's belief and what the belief is about.

We can see the implications of this by supposing that I come to learn that my belief is false. Suppose that I find out that there was in fact a vote on the Brexit deal as originally scheduled after all. In that case, I would now reject something to which I was previously committed: that the newspaper reported accurately, or that there was a causal connection between the reporting and events in Parliament, or something else in my total evidential package. In recognizing the falsity of my belief, I realize that my evidence (although perhaps quite reliable) failed to connect to the truth in some way. Either the evidence was inaccurate or it was not connected with the truth in the right way.

Importantly, I do not need to see myself as having been irrational or at fault in any way. But once I realize that my belief is false, I must reject part of what I took to give me reason, where this can include considerations in the foreground such as that I viewed the headline on my computer screen, or in the background, such as that I assumed that the reporter witnessed the official act of postponement or had some other relevant explanatory connection to the event. In other words, in seeing that my reasons failed to

guide me to the truth, I must reject something in my total “package” of reasons. Notice, in contrast, that this is not the case for P-inferences. If I believe that p (say, my ticket will lose) on the basis of a high statistical probability for p , and I find out that not- p (I won!), then there is nothing at all in my reasons to reject. I still believe the same odds were in effect, and I still believe that they made my losing extremely probable. I have no reason to think that my evidence failed to bear a connection to my conclusion that I previously thought it did. Learning that my belief is false puts no pressure on me to find some problem in my reasons. Thus, there is a way in which they are not “sensitive” to the truth, or at least to what I conceive of as the truth. Given the role of rationality as a guide toward truth, this lack of sensitivity to the truth in the case of P-inferences helps to explain why such inferences are not rational.⁵

So far, then, we have a solution that sorts cases in an intuitive way, and a rationale in terms of the connection between rationality and truth. Notice that this solution also helps explain subtle differences, as well as similarities, in the behavior that probabilistic beliefs and outright beliefs rationalize. On the one hand, the behavior rationalized by the merely probabilistic belief that one’s ticket will win overlaps substantially with the behavior that the outright belief does. If I believe my ticket will very probably lose, then I should not plan on quitting my job or altering my life in any substantial way. This is also true if I believe my ticket will lose, but there is a difference, as well. For example, if I believe my ticket *will* lose (as I would if I thought the stub would be torn up immediately), then I wouldn’t be in a position to rationally buy it in the first place. If, however, I believe that it will *very probably* lose, it could still be rational for me to buy it. This point helps support the idea that lottery ticket buyers typically believe that their tickets *will probably* lose, but not that they typically believe that they *will* lose. For subtly different sets of behavior seem to be rationalized by the two beliefs. (In what follows, I will sometimes refer to beliefs that have probabilistic contents, such as that “there is a 1/1000 chance that my ticket will lose,” as “probabilistic beliefs” and beliefs without probabilistic contents, such as that “my ticket will lose,” as “outright beliefs.”)

Despite these advantages, the solution faces serious challenges. One source of resistance is precisely the kind of case with which I began: If it

⁵ It might be thought that what undermines the rationality of a P-inference in this case is the fact that the lottery belief in question is about the future.

We can, however, imagine that the lottery has already been conducted and the result kept secret for a time, and in that case, as well, the same explanation is effective. Similarly, as we will see, statistical evidence about past crimes poses a particular challenge in trial law.