

Contents

<i>Preface</i>	<i>page xi</i>
PART I PRELIMINARIES	1
1 Kummer theory	3
Some basics	3
Cyclic extensions	5
Some classical applications	7
Kummer extensions	8
The Kummer pairing	10
The fundamental theorem	14
Examples	16
2 Local number fields	23
Construction of $\mathbb{Q}_p$	24
Around Hensel’s lemma	31
Local number fields	37
Unramified extensions	43
Higher ramification groups	49
3 Tools from topology	54
Topological groups	54
Profinite groups	57
Infinite Galois extensions	62
Locally compact fields	67
4 The multiplicative structure of local number fields	72
Initial observations	72
Exponential and logarithm	74
Module structures	76

viii	CONTENTS	
	Summary and first applications	77
	The number of extensions (is finite)	80
	PART II BRAUER GROUPS	83
5	Skewfields, algebras, and modules	85
	Skewfields and algebras	85
	Modules and endomorphisms	89
	Semisimplicity	93
	The radical	98
	Matrix algebras	101
6	Central simple algebras	107
	The Brauer group revisited	107
	Tensor products	108
	The group law	113
	The fundamental theorems	115
	Splitting fields	118
	Separability	120
7	Combinatorial constructions	122
	Introduction	122
	Group extensions	124
	First applications: cyclic groups	130
	Crossed product algebras	134
	Compatibilities	136
	The cohomological Brauer group	141
	Naturality	143
8	The Brauer group of a local number field	146
	Preliminaries	146
	The Hasse invariant of a skewfield	149
	Naturality	153
	PART III GALOIS COHOMOLOGY	157
9	Ext and Tor	159
	Preliminaries	159
	Resolutions	162
	Complexes	165
	The Ext groups	171
	Long exact sequences	176
	The Tor groups	180

CONTENTS	ix
10 Group cohomology	187
Definition of group (co)homology	187
The standard resolution	191
Low degrees	198
Profinite groups and Galois cohomology	200
11 Hilbert 90	204
Hilbert's Theorem 90 in Galois cohomology	204
More Kummer theory	207
The Hilbert symbol	208
12 Finer structure	214
Shapiro's isomorphism	214
A few explicit formulae	217
The corestriction	221
The conjugation action	225
The five-term exact sequence	227
Cup-products	230
Milnor and Bloch–Kato	237
PART IV CLASS FIELD THEORY	241
13 Local class field theory	243
Statements	243
Cohomological triviality and equivalence	244
The reciprocity isomorphisms	248
Norm subgroups	250
Tate duality	252
The Existence theorem	254
The local Kronecker–Weber theorem	255
A concise reformulation	257
14 An introduction to number fields	261
Number fields and their completions	261
The discriminant	267
The (global) Kronecker–Weber theorem	272
The local and global terminology	275
Some statements from global class field theory	276
Appendix: background material	281
Norms and traces	281
Tensor products	282
<i>Notes and further reading</i>	286
<i>References</i>	290
<i>Index</i>	292