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CHAPTER

1

Electro Magnetic Circuits

Chapter Objectives

After the completion of this unit, students/readers will be able to understand:

- ✓ What is magnetic field and its significance?
- ✓ What is a magnetic circuit?
- ✓ What are the important terms related to magnetism and magnetic circuits?
- ✓ What are the similarities and dissimilarities between magnetic and electric circuits?
- ✓ How series and parallel magnetic circuits are treated?
- ✓ What is leakage flux and how it affects magnetic circuits?
- ✓ What is magnetic hysteresis and hysteresis loss?
- ✓ What is electromagnetic induction phenomenon?
- ✓ What are Faraday’s laws of electromagnetic induction?
- ✓ What are self- and mutual inductances and what is their significance?
- ✓ What is the effective value of inductances when these are connected in series–parallel combination?
- ✓ What are electromechanical energy conversion devices?
- ✓ How does torque develop by the alignment of two fields?
- ✓ What are the factors on which torque depends?
- ✓ How to determine the direction of torque or induced emf in rotating machines?

Introduction

It is always advantageous to utilise electrical energy since it is cheaper, can be easily transmitted, easy to control and more efficient. The electrical energy is generally generated from natural resources such as water, coal, diesel, wind, atomic energy, etc. From these sources, first mechanical energy is produced by one way or the other and then that mechanical energy is converted into electrical energy by suitable machines. For the utilisation of electrical energy, it is again converted into other forms of energy such as mechanical, heat, light etc. It is a well-known fact that the electric drives have been universally adopted by the industry due to their inherent advantages. The energy conversion devices are always required at both ends of a typical electrical system. The devices or machines which convert mechanical energy into electrical energy and vice-versa are called **electro–mechanical energy conversion devices.**

The operation of all the electrical machines such as *DC* machines, transformers, synchronous machines, induction motors, etc., rely upon their magnetic circuits. The closed path followed by the magnetic lines of force is called a *magnetic circuit*. The operation of all the electrical devices (e.g., transformers, generators, motors, etc.) depends upon the magnetism produced by their magnetic circuits. Therefore, to obtain the required characteristics of these devices, their magnetic circuits have to be designed carefully.

In this chapter, we shall focus our attention on the basic fundamentals of magnetic circuits and their applications as electromechanical energy conversion devices.

1.1 Magnetic Field and its Significance

The region around a magnet where its poles exhibit a force of attraction or repulsion is called **magnetic field.**

The existence of the magnetic field at a point around the magnet can also be determined by placing a magnetic needle at that point as shown in Fig. 1.1. Although magnetic lines of force have no real existence and are purely imaginary, yet their concept is very useful to understand various magnetic effects. It is assumed (because of their effects) that the magnetic lines of force possess the following important properties:

(*i*) The direction of magnetic lines of force is from N-pole to the S-pole outside the magnet. But inside the magnet their direction is from S-pole to N-pole.
(*ii*) They form a closed loop.
(*iii*) Their tendency is to follow the least reluctance path.
(*iv*) They act like stretched cords, always trying to shorten themselves.
(*v*) They never intersect each other.
(*vi*) They repel each other when they are parallel and are in the same direction.
(*vii*) They remain unaffected by non-magnetic materials.

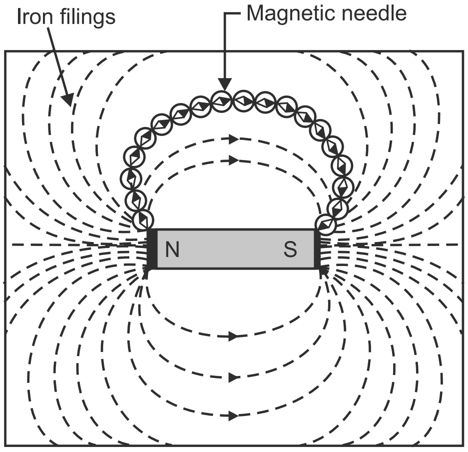


Fig. 1.1 Magnetic field around a bar magnet

1.2 Magnetic Circuit and its Analysis

The closed path followed by magnetic flux is called a **magnetic circuit.**

A magnetic circuit usually consists of magnetic materials having high permeability (e.g., iron, soft steel, etc.). In this circuit, magnetic flux starts from a point and finishes at the same point after completing its path.

Figure 1.2 shows a solenoid having N turns wound on an iron core (ring). When current I ampere is passed through the solenoid, magnetic flux ϕ Wb is set-up in the core.

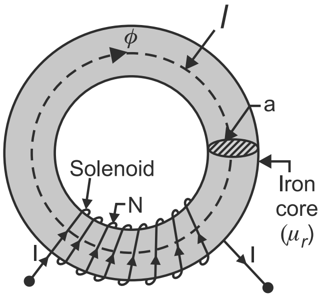


Fig. 1.2 Magnetic circuit

Let l = mean length of magnetic circuit in m;
a = area of cross-section of core in m^2;
μ_r = relative permeability of core material.

Flux density in the core material, $B = \dfrac{\phi}{a}$ Wb/m^2

Magnetising force in the core material.

$$H = \frac{B}{\mu_0 \, \mu_r} = \frac{\phi}{a \, \mu_0 \, \mu_r} \text{ AT/m}$$

According to work law, the work done in moving a unit pole once round the magnetic circuit (or path) is equal to the ampere-turns enclosed by the magnetic circuit.

i.e., $$Hl = NI \quad \text{or} \quad \frac{\phi}{a \, \mu_0 \, \mu_r} \times l = NI \quad \text{or} \quad \phi = \frac{NI}{\left(l \,/\, a \, \mu_0 \, \mu_r \right)} \text{Wb}$$

The above expression reveals that the amount of flux set-up in the core is

(*i*) directly proportional to N and I i.e., NI, called *magnetomotive force* (*mmf*). It shows that the flux increases if either of the two increases and *vice-versa*.

(*ii*) inversely proportional to $l/a \, \mu_0 \, \mu_r$ called *reluctance* of the magnetic path. In fact, reluctance is the opposition offered to the magnetic flux by the magnetic path. The lower is the reluctance, the higher will be the flux and *vice-versa*.

Thus, $$\text{Flux} = \frac{\text{m.m.f}}{\text{reluctance}}$$

It may be noted that the above expression has a strong resemblance to Ohm's law for electric current (I = emf/resistance). The mmf is analogous to emf in electric circuit, reluctance is analogous to resistance and flux is analogous to current. Because of this similarity, the above expression is sometimes referred to as *Ohm's law of magnetic circuits*.

1.3 Important Terms

While studying magnetic circuits, generally, we come across the following terms:

1. **Magnetic field:** The region around a magnet where its poles exhibit a force of attraction or repulsion is called *magnetic field*.

2. **Magnetic flux (ϕ):** The amount of magnetic lines of force set-up in a magnetic circuit is called *magnetic flux*. Its unit is weber (Wb). It is analogous to *electric current I* in electric circuit.
3. *The* **magnetic flux density** *at a point is the flux per unit area at right angles to the flux at that point.*

   It is, generally, represented by letter '*B*'. Its unit is Wb/m^2 or Tesla, i.e.,

$$B = \frac{\phi}{A}\text{Wb / m}^2 \quad \text{or} \quad \text{T } (1 \text{ Wb/m}^2 = 1 \times 10^4 \text{ Wb/cm}^2)$$

4. **Permeability:** *The ability of a material to conduct magnetic lines of force through it is called the* **permeability** *of that material.*

   It is generally represented by μ (*mu*, a Greek letter). The greater the permeability of a material, the greater is its conductivity for the magnetic lines of force and *vice-versa*. The permeability of air or vacuum is the poorest and is represented as μ_0 (where $\mu_0 = 4\pi \times 10^{-7}$ H/m).

   Relative permeability: The absolute (or actual) permeability μ of a magnetic material is much greater than absolute permeability of air μ_0. The relative permeability of a magnetic material is given in comparison with air or vacuum.

   Hence, the ratio of the permeability of material μ to the permeability of air or vacuum μ_0 is called the relative permeability μ_r of the material.

   i.e., $$\mu_r = \frac{\mu}{\mu_0} \quad \text{or} \quad \mu = \mu_0 \mu_r$$

   Obviously, the relative permeability of air would be $\mu_0/\mu_0 = 1$. The value of relative permeability of all the non-magnetic materials is also 1. However, its value is as high as 8000 for soft iron, whereas, its value for mumetal (iron 22% and nickel 78%) is as high as 1,20,000.
5. **Magnetic field intensity:** The force acting on a unit north pole (1 Wb) when placed at a point in the magnetic field is called the magnetic intensity of the field at that point. It is denoted by *H*. In magnetic circuits, it is defined as mmf per unit length of the magnetic path. It is denoted by *H*, mathematically,

$$H = \frac{\text{m.m.f}}{\text{length of magnetic path}} = \frac{NI}{l}\text{AT / m}$$

6. **Magnetomotive force (mmf):** The magnetic pressure which sets-up or tends to set-up magnetic flux in a magnetic circuit is called *magnetomotive force*. As per work law it may be defined as under:

   The work done in moving a unit magnetic pole (1 Wb) once round the magnetic circuit is called *magnetomotive force*. In general

$$\text{mmf} = NI \text{ ampere-turns (or AT)}$$

   It is analogous to *emf* in an electric circuit.
7. **Reluctance (S):** The opposition offered to the magnetic flux by a magnetic circuit is called its *reluctance*.

   It depends upon length (l), area of cross-section (a) and permeability ($\mu = \mu_0 \mu_r$) of the material that makes up the magnetic circuit. It is measured in AT/Wb.

$$\text{Reluctance, } S = \frac{l}{a\ \mu_0\ \mu_r}$$

It is analogous to *resistance* in an electric circuit.

8. **Permeance:** It is a measure of the ease with which flux can be set-up in the material. It is just reciprocal of reluctance of the material and is measured in *Wb/AT* or *henry*.

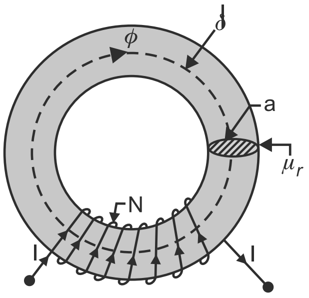
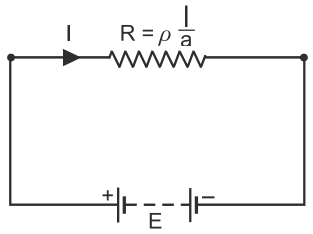
$$\text{Permeance} = \frac{1}{\text{reluctance}} = \frac{a\ \mu_0\ \mu_r}{l}\ \text{Wb/AT} \quad \text{or} \quad \text{H}$$

It is analogous to *conductance* in an electric circuit.

9. **Reluctivity:** It is specific reluctance and analogous to *resistivity* in electric circuit.

1.4 Comparison between Magnetic and Electric Circuits

Although magnetic and electric circuits have many points of similarity but still they are not analogous in all respects. A comparison of the two circuits is given below:

Magnetic Circuits	***Electrical Circuits***
 **Fig. 1.3** Magnetic circuit	 **Fig. 1.4** Electric circuit
Similarities	
1. The closed path for magnetic flux is called magnetic circuit.	1. The closed path for electric current is called electric circuit.
2. Flux = mmf/reluctance	2. Current = emf/resistance
3. Flux, ϕ in Wb	3. Current, ***I*** in ampere
4. mmf in AT	4. emf in V
5. Reluctance, $S = \frac{l}{a\mu} = \frac{l}{a\mu_0\ \mu_r}$ AT/Wb	5. Resistance, $\boldsymbol{R} = \rho\frac{l}{a}\Omega$ or $\boldsymbol{R} = \frac{1}{\sigma}\frac{l}{a}\ \Omega$
6. Permeance = 1/reluctance	6. Conductance = 1/resistance
7. Permeability, μ	7. Conductivity, $\sigma = 1/\rho$
8. Reluctivity	8. Resistivity
9. Flux density, $\boldsymbol{B} = \frac{\phi}{a}$ Wb/m^2	9. Current density, $\boldsymbol{J} = \frac{I}{a}$ A/m^2
10. Magnetic intensity, $\boldsymbol{H} = \boldsymbol{NI/l}$	10. Electric intensity, $\boldsymbol{E} = \boldsymbol{V/d}$

Dissimilarities	
1. In fact, the magnetic flux does not flow but it sets-up in the magnetic circuit (basically molecular poles are aligned).	1. The electric current (electrons) actually flows in an electric circuit.
2. For magnetic flux, there is no perfect insulator. It can be set-up even in the non-magnetic materials like air, rubber, glass etc. with reasonable mmf	2. For electric current, there are large number of perfect insulators like glass, air, rubber, etc., which do not allow it to follow through them under normal conditions.
3. The reluctance (***S***) of a magnetic circuit is not constant rather it varies with the value of ***B***. It is because the value of μ_r changes considerably with the change in ***B***.	3. The resistance (***R***) of an electric circuit is almost constant as its value depends upon the value of ρ which is almost constant. However, the value of ρ and ***R*** may vary slightly if temperature changes.
4. Once the magnetic flux is set-up in a magnetic circuit, no energy is expanded. However, a small amount of energy is required at the start to create flux in the circuit.	4. Energy is expanded continuously, so long as the current flows through an electric circuit. This energy is dissipated in the form of heat.

1.5 Ampere-turns Calculations

In a magnetic circuit, flux produced,

$$\phi = \frac{\text{m.m.f.}}{\text{reluctance}} = \frac{NI}{l/a\,\mu_0\,\mu_r}$$

or AT required, $$NI = \frac{\phi l}{a\mu_0\mu_r} = \frac{B}{\mu_0\mu_r}l = Hl$$

1.6 Series Magnetic Circuits

A magnetic circuit that has a number of parts of different dimensions and materials carrying the same magnetic field is called a *series magnetic circuit*. Such as series magnetic circuit (composite circuit) is shown in Fig. 1.5.

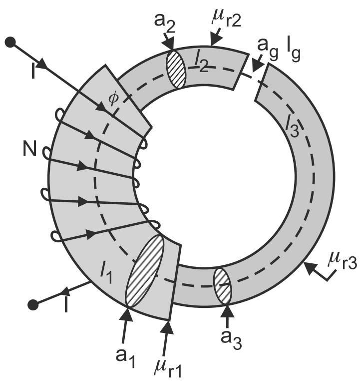


Fig. 1.5 Series magnetic circuit

Total reluctance of the magnetic circuit,

$$S = S_1 + S_2 + S_3 + S_g$$

$$= \frac{l_1}{a_1 \mu_0 \mu_{r1}} + \frac{l_2}{a_2 \mu_0 \mu_{r2}} + \frac{l_3}{a_3 \mu_0 \mu_{r3}} + \frac{l_g}{a_g \mu_0}$$

$$\text{Total mmf} = \phi S$$

$$= \phi\left(\frac{l_1}{a_1 \mu_0 \mu_{r1}} + \frac{l_2}{a_2 \mu_0 \mu_{r2}} + \frac{l_3}{a_3 \mu_0 \mu_{r3}} + \frac{l_g}{a_g \mu_0}\right)$$

$$= \frac{B_1 l_1}{\mu_0 \mu_{r1}} + \frac{B_2 l_2}{\mu_0 \mu_{r2}} + \frac{B_3 l_3}{\mu_0 \mu_{r3}} + \frac{B_g l_g}{\mu_0}$$

$$= H_1 l_1 + H_2 l_2 + H_3 l_3 + H_g l_g$$

1.7 Parallel Magnetic Circuits

A magnetic circuit which has two or more than two paths for the magnetic flux is called a *parallel magnetic circuit*. Its behaviour can be just compared to a parallel electric circuit.

Figure 1.6 shows a parallel magnetic circuit. A current carrying coil is wound on the central limb *AB*. This coil sets-up a magnetic flux ϕ_1 in the central limb which is further divided into two paths i.e., (*i*) path *ADCB* which carries flux ϕ_2 and (*ii*) path *AFEB* which carries flux ϕ_3.

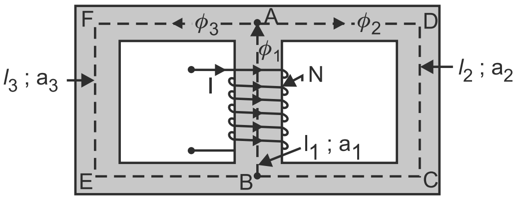


Fig. 1.6 Parallel magnetic circuit

It is clear that $\phi_1 = \phi_2 + \phi_3$

The two magnetic paths *ADCB* and *AFEB* are in parallel. The ATs required for this parallel circuit is equal to the ATs required for any one of the paths.

If S_1 = reluctance of path *BA* i.e., $l_1/a_1 \mu_0 \mu_{r1}$

S_2 = reluctance of path *ADCB* i.e., $l_2/a_2 \mu_0 \mu_{r2}$

S_3 = reluctance of path *AFEB* i.e., $l_3/a_3 \mu_0 \mu_{r3}$

∴ Total mmf required = mmf required for path *BA* + mmf required path *ADCB* or path *AFEB*.

i.e., Total mmf or $AT = \phi_1 S_1 + \phi_2 S_2 = \phi_1 S_1 + \phi_3 S_3$

1.8 Leakage Flux

The magnetic flux which does not follow the intended path in a magnetic circuit is called *leakage flux*.

When some current is passed through a solenoid, as shown in Fig. 1.7, magnetic flux is produced by it. Most of this flux is set-up in the magnetic core and passes through the air gap (an intended path). This flux is known as *useful flux* ϕ_u. However, some of the flux is just set-up around the coil and is not utilised for any work. This flux is called *leakage flux* ϕ_l.

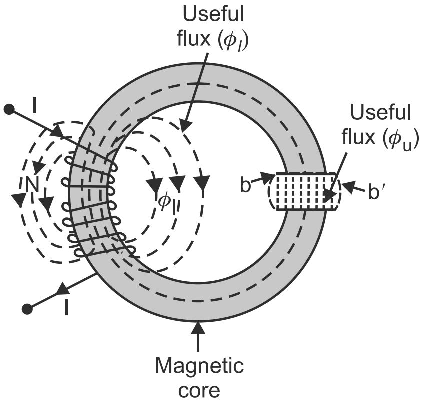


Fig. 1.7 Leakage flux

Total flux produced by the solenoid.

$$\phi = \phi_u + \phi_l$$

Leakage co-efficient or leakage factor: The ratio of total flux (ϕ) produced by the solenoid to the useful flux (ϕ_u) set-up in the air gap is known as *leakage co-efficient*. It is generally represented by letter ‘λ’.

$$\therefore \quad \text{Leakage co-efficient, } \lambda = \frac{\phi}{\phi_u}$$

Fringing: It may be seen in Fig. 1.7 that the useful flux when sets-up in the air gap, it tends to bulge outwards at *b* and *b′* since the magnetic lines set-up in the same direction repel each other. This increases the effective area in the air gap and decreases the flux density. This effect is known as *fringing*. The fringing is directly proportional to the length of the air gap.

Example 1.1

An iron ring of 400 cm mean circumference is made from round iron of cross-section 20 cm^2. Its permeability is 500. If it is wound with 400 turns, what current would be required to produce a flux of 0·001 Wb?

Solution:

The magnetic circuit is shown in Fig. 1.8.

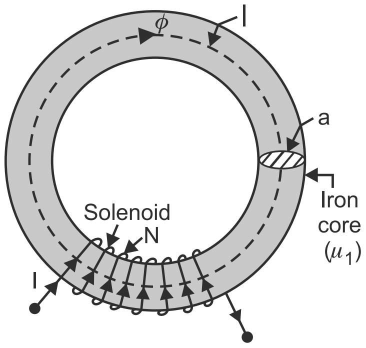


Fig. 1.8 Magnetic circuit

Mean length of magnetic path, $l_m = 400 \text{ cm} = 4 \text{ m}$

Area of X-section of iron ring, $a = 20 \times 10^{-4} \text{ m}^2$

Absolute permeability, $\mu_0 = 4\pi \times 10^{-7}$

Now mmf = flux × reluctance

$$NI = \phi \times \frac{l_m}{a\mu_0\mu_r}$$

$$400 \times I = 0{\cdot}001 \times \frac{4}{20 \times 10^{-4} \times 4\pi \times 10^{-7} \times 500}$$

$$\therefore \quad \text{Current, } I = \frac{0.001 \times 4}{20 \times 10^{-4} \times 4\pi \times 10^{-7} \times 500 \times 400} = \mathbf{7{\cdot}958} \ (\textit{Ans.})$$

Example 1.2

An electromagnet has an air gap of 4 *mm and flux density in the gap is* 1·3 Wb/m². *Determine the ampere-turns for the gap.*

Solution:

Here, $l_g = 4\text{mm} = 0{\cdot}4\text{ cm} = 4 \times 10^{-3}\text{ m};\ B_g = 1{\cdot}3\text{ Wb/m}^2$

Ampere-turns required for the gap

$$= H_g \times l_g = \frac{Bg}{\mu_0} \times l_g = \frac{1 \cdot 3}{4\pi \times 10^{-7}} \times 4 \times 10^{-3} = \mathbf{4136{\cdot}83\ AT}\ (\mathit{Ans.})$$

Example 1.3

A coil of insulated wire of 500 *turns and of resistance* 4 Ω *is closely wound on iron ring. The ring has a mean diameter of* 0·25 m *and a uniform cross-sectional area of* 700 *mm*². *Calculate the total flux in the ring when a DC supply of* 6*V is applied to the ends of the winding. Assume a relative permeability of* 550.

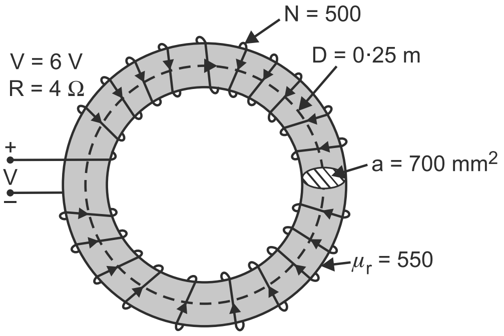


Fig. 1.9 Magnetic circuit

Solution:

Mean length of iron ring, $l = \pi D = \pi \times 0{\cdot}25 = 0{\cdot}25\ \pi\text{ m}$

Area of cross-section, $\alpha = 700\text{ mm}^2 = 700 \times 10^{-6}\text{ m}^2$

Current flowing through the coil,

$$I = \frac{\text{Voltage applied across coil}}{\text{Resistance of coil}}$$

$$= \frac{6}{5} = 1 \cdot 5\text{ A}$$

Total flux in the ring, $$\phi = \frac{N\,I}{l\,/\,a\,\mu_0\mu_r} = \frac{N\,I \times a\,\mu_0\mu_r}{l}$$

$$= \frac{500 \times 1 \cdot 5 \times 700 \times 10^{-6} \times 4\pi \times 10^{-7} \times 550}{0 \cdot 25\ \pi} = \mathbf{0{\cdot}462\ mWb}\ (\mathit{Ans.})$$

Example 1.4

What are the similarities between electrical circuits and magnetic circuits? An iron ring of mean length 50 *cm and relative permeability* 300 *has an air gap of* 1 *mm. If the ring is provided with winding of* 200 *turns and a current of* 1 *A is allowed to flow through, find the flux density across the airgap.*

Solution:

Here, $l_i = 50\text{ cm} = 0{\cdot}5\text{ m};\ \mu_r = 300;\ l_g = 1\text{mm} = 0{\cdot}001\text{ m};\ N = 200\text{ turns};\ I = 1\text{ A}$

Ampere-turns required for air gap $= \dfrac{B}{\mu_0}\, l_g$

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$$\text{Ampere-turns required for iron ring} = \frac{B}{\mu_0\ \mu_r} l_i$$

or $$\text{Total ampere-turns required} = \frac{B}{\mu_0} l_g + \frac{B}{\mu_0\ \mu_r} l_i \qquad ...(i)$$

$$\text{Ampere-turns provided by the coil} = N\,I = 200 \times 1 = 200 \qquad ...(ii)$$

Equating eqn. (*i*) and (*ii*), we get,

or $$200 = \frac{B}{\mu_0}\left(l_g + \frac{l_i}{\mu_r}\right) = \frac{B}{\mu_0}\left(0.01 + \frac{0.5}{300}\right)$$

$$= \frac{B}{\mu_0}(0 \cdot 001 + 0 \cdot 00167) = \frac{B}{\mu_0} \times 0 \cdot 00267$$

or $$\text{Flux density, } B = \frac{200 \times \mu_0}{0 \cdot 00267} = \frac{200 \times 4\pi \times 10^{-7}}{0 \cdot 00267} = \mathbf{0{\cdot}09425\ T}\ (Ans.)$$

Example 1.5

A coil of 1000 *turns is wound on a laminated core of steel having a cross-section of* 5 *cm*^{2}. *The core has an air gap of* 2 *mm cut at right angle. What value of current is required to have an air gap flux density of* 0·5 *T? Permeability of steel may be taken as infinity. Determine the coil inductance.*

Solution:

Here, $N = 1000$ turns; $a = 5\text{ cm}^2 = 5 \times 10^{-4}\text{ m}^2$;

$$l_g = 2\text{ mm} = 2 \times 10^{-3}\text{ m};\ B = 0{\cdot}5\text{ T};\ \mu_r = \infty$$

Total ampere-turns required,

$$AT = \frac{B}{\mu_0} l_g + \frac{B}{\mu_0\ \mu_r} l_i = \frac{0 \cdot 5}{4\pi \times 10^{-7}} \times 2 \times 10^{-3} + 0 = 796$$

$$\left(\text{As } \mu_r = \infty; \frac{B}{\mu_0 \mu_r} \times l_i = 0\right)$$

Current required, $$I = \frac{AT}{I} = \frac{796}{1000} = \mathbf{0{\cdot}796\ A}\ (Ans.)$$

Inductance of coil, $$L = \frac{N\phi}{I} = \frac{N \times B \times a}{I} = \frac{1000 \times 0 \cdot 5 \times 5 \times 10^{-4}}{0.796}$$

$$= \mathbf{0{\cdot}314\ H}\ (Ans.)$$

Example 1.6

A flux density of 1·2 Wb/m^2 *is required in 2 mm air gap of an electro-magnet having an iron path 1 metre long. Calculate the magnetising force and current required if the electro magnet has 1273 turns. Assume relative permeability of iron to be 1500.*

Solution:

$$\text{Flux density, } B = 1{\cdot}2\text{ Wb/m}^2$$