
Contents

	<i>Preface</i>	<i>page ix</i>
1	Introduction and Constructions	1
	1.1 Basic Definitions	4
	1.2 Cayley Digraphs	11
	1.3 Double Coset Digraphs	19
	1.4 Orbital Digraphs	26
	1.5 Metacirculant Digraphs	31
	1.6 Exercises	38
2	The Petersen Graph, Blocks, and Actions of A_5	45
	2.1 The Automorphism Group of the Petersen Graph	46
	2.2 Blocks and Imprimitive Groups	48
	2.3 More Ways of Finding Blocks	54
	2.4 Primitive and Quasiprimitive Groups	59
	2.5 Actions of A_5	66
	2.6 Orbital Digraphs of A_5	74
	2.7 Exercises	82
3	Some Motivating Problems	87
	3.1 Cubic s -Arc-transitive Graphs	88
	3.2 Arc-, Edge-, and Half-arc-transitive Graphs	92
	3.3 Introduction to the Hamiltonicity Problem	102
	3.4 Vertex-transitive Graphs That Are Not Hamiltonian	103
	3.5 Connected Cayley Graphs of Abelian Groups Are Hamiltonian	110
	3.6 Block Quotient Digraphs	112
	3.7 The Spiral Path Argument and Frucht Notation	119
	3.8 Semiregular Elements	123
	3.9 Exercises	128

vi	<i>Contents</i>	
4	Graphs with Imprimitive Automorphism Group	132
4.1	Permutation Isomorphism and Equivalent Representations	133
4.2	Wreath Products of Digraphs and Groups	141
4.3	The Embedding Theorem	149
4.4	Groups of Prime Degree and Digraphs of Prime Order	152
4.5	The Automorphism Group of the Heawood Graph	158
4.6	Exercises	163
5	The End of the Beginning	167
5.1	The Automorphism Group of Wreath Product Graphs	168
5.2	2-Closed Permutation Groups	173
5.3	Generalized Wreath Products	179
5.4	Primitive Groups and the O’Nan–Scott Theorem	188
5.5	Covers	197
5.6	Exercises	208
6	Other Classes of Graphs	214
6.1	The Coxeter Graph	214
6.2	Kneser Graphs and Odd Graphs	223
6.3	The Johnson Graphs	225
6.4	Levi Graphs	237
6.5	The Generalized Petersen Graphs	248
6.6	Fermat Graphs	262
6.7	Exercises	268
7	The Cayley Isomorphism Problem	272
7.1	Graph Isomorphisms in Quasipolynomial Time	273
7.2	Introduction and Basic Definitions	274
7.3	Basic Tools	277
7.4	Non-CI-groups With Respect to Graphs	283
7.5	Groups of Order a Product of Two Primes	286
7.6	Pálffy’s Theorem	296
7.7	Isomorphisms of Circulant Digraphs	303
7.8	Exercises	317
8	Automorphism Groups of Vertex-Transitive Graphs	322
8.1	Normal Cayley Digraphs	322
8.2	The Transfer	328
8.3	More Tools and Automorphism Groups of Circulants of Order p^2	333
8.4	Automorphism Groups of Cayley Digraphs of \mathbb{Z}_p^2	341
8.5	Automorphism Groups of Circulant Digraphs	346
8.6	Exercises	353

<i>Contents</i>		vii
9	Classifying Vertex-Transitive Graphs	357
	9.1 Digraphs of Prime Power Order	357
	9.2 Graphs of Order a Product of Two Distinct Primes	365
	9.3 Circulant Digraphs of Order n with $\gcd(n, \varphi(n)) = 1$	372
	9.4 Exercises	376
10	Symmetric Graphs	378
	10.1 The 17 Families of Cubic Symmetric Graphs	379
	10.2 Cubic Symmetric Graphs of Small Girth	381
	10.3 Rigid Cells	385
	10.4 Valency Preserving Quotienting	391
	10.5 Symmetric Graphs of Larger Valencies	395
	10.6 Exercises	399
11	Hamiltonicity	401
	11.1 Historic Perspectives	401
	11.2 The Lifting Cycle Technique	406
	11.3 Vertex-Transitive Graphs of Particular Orders	415
	11.4 Specific Approaches for Cayley Graphs	421
	11.5 Embeddings on Surfaces	425
	11.6 A Final Observation	435
	11.7 Exercises	435
12	Semiregularity	437
	12.1 Historic Perspectives	437
	12.2 Orders of (Di)graphs	443
	12.3 Valency of Graphs	444
	12.4 Types of Group Actions	448
	12.5 Further Directions	449
	12.6 Exercises	450
13	Graphs with Other Types of Symmetry:	
	Half-arc-transitive Graphs and Semisymmetric Graphs	451
	13.1 Historic Perspectives	451
	13.2 Half-arc-transitive Graphs: A Geometric Approach	453
	13.3 Quartic Half-arc-transitive Graphs	458
	13.4 Semisymmetric Graphs	462
	13.5 Exercises	465

14	Fare You Well	468
14.1	Algebraic Graph Theory: Towards CFSG-Free Arguments	468
14.2	Even/Odd Automorphisms Problem	472
	<i>References</i>	474
	<i>Index of Graphs</i>	497
	<i>Index of Symbols</i>	500
	<i>Select Author Index</i>	503
	<i>Index of Terms</i>	506