Part I

Models

1

How to Teach and Think About Spontaneous Wave Function Collapse Theories: Not Like Before

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A simple and natural introduction to the concept and formalism of spontaneous wave function collapse can and should be based on textbook knowledge of standard quantum state collapse and monitoring. This approach explains the origin of noise driving the paradigmatic stochastic Schrödinger equations of spontaneous localization of the wave function Ψ. It reveals, on the other hand, that these equations are empirically redundant and the master equations of the noise-averaged state $\hat{\rho}$ are the only empirically testable dynamics in current spontaneous collapse theories.

1.1 Introduction

> "We are being captured in the old castle of standard quantum mechanics. Sometimes we think that we have walked into a new wing. It belongs to the old one, however." [1]

The year 1986 marked the birth of two theories, prototypes of what we call the theory of spontaneous wave function collapse. Both the GRW paper published in *Physical Review D* [2] followed by Bell's insightful work [3] and the author's thesis [4] constructed strict stochastic jump equations to explain unconditional emergence of classical behavior in large quantum systems. Subsequently, both theories obtained their time-continuous versions, driven by white noise rather than by stochastic jumps. The corresponding refinement of the GRW proposal [5, 6, 7] is the Continuous Spontaneous Localization (CSL) theory. The author's gravity-related spontaneous collapse theory [4, 8, 9] used to be called DP theory after Penrose concluded to the same equation for the characteristic time of spontaneous collapse in large bodies [10]. These theories modified the standard theory of quantum mechanics in order to describe the irreversible process of wave function collapse. The mathematical structure of modification surprised the proponents themselves, and it looked strange and original for many of the interested as well. In fact, these theories were considered new physics with new mathematical structures to replace standard equations like Schrödinger's. The predicted effects of spontaneous collapses are extremely small and have thus remained untestable for the lack of experimental technique. After three decades, fortunately, tests on nanomasses are now becoming gradually available. Theories like GRW, CSL, and DP have not changed over the decades apart from their parameter ambiguities; see reviews by Bassi *et al.* [11, 12]. But our understanding and teaching of spontaneous collapse should be revised radically.

Personally, I knew that GRW's random jumps looked like unsharp measurements, but, in the late 1980s, I believed that unsharp measurements were phenomenological modifications of von Neumann standard ones. My belief extended also for the time-continuous limit of unsharp measurements [13] that DP collapse equations [9] were based on. Finally in the 1990s I got rid of my ignorance and learned that unsharp measurements and my time-continuous measurement (monitoring) could have equally been derived from standard quantum theory [14, 15].

That was disappointing [1]. Excitement about the radical novelty of our modified quantum mechanics evaporated. Novelty got reduced to the concept that tiny collapses which get amplified for bulk degrees of freedom happen everywhere and without measurement devices. That's why we call them spontaneous. But they are standard collapses otherwise. I have accordingly stressed their revised interpretation recently [16], and the present work is arguing further toward such demand.

1.2 How to Teach GRW Spontaneous Collapse?

We should build as much as possible on standard knowledge, using standard concepts, equations, and terminology. The key notion is unsharp generalized measurement, which has been standard ever since von Neumann showed how inserting an ancilla between object and measuring device will control measurement unsharpness [17]. Hence we are in the best pedagogical position to explain GRW theory to educated physicists. No doubt, for old generations measurement means the projective (sharp) one, but this has changed recently due to the boom in quantum information science. For younger scientists, generalized measurements are the standard ones, while projective measurements are the specific case [18, 19]. For the new generation, there is a natural way to get acquainted with spontaneous collapse. The correct and efficient teaching goes like this.

GRW theory assumes that independent position measurements of unsharpness (precision) $r_C/\sqrt{2}$, with GRW choice $r_C = 10^{-5}$ cm, are happening randomly at average frequency $\lambda = 10^{-19}$ Hz on each (non-relativistic) particle in the Universe. The two parameters r_C, λ are considered new universal constants of Nature. The mathematical model of unsharp measurements is exactly the same as for independent von Neumann detectors [17] where the Gaussian ancilla wave function has the width $\sigma = r_C/\sqrt{2}$ [1]. The difference from standard von Neumann detection is the concept of being spontaneous: GRW are measurements supposed to happen *without* the presence of detectors.

The merit of GRW is wave function *localization* in bulk degrees of freedom, e.g., the center of mass (c.o.m.) of large objects. Quantum theory allows for arbitrary large quantum fluctuations of macroscopic degrees of freedom in large quantized systems. The extreme example is a Schrödinger cat state in which two macroscopically different wave functions would be superposed. In GRW theory such macroscopic superpositions or fluctuations become suppressed by GRW spontaneous measurements but the superpositions of microscopic degrees of freedom will invariably survive. These complementary features are guaranteed by the chosen values of parameters σ and λ. Due to the extreme low rate of measurements, individual particles are almost never measured. But among an Avogadro number (A) of constituents some $N = A\lambda \sim 10^4$ become spontaneously measured in each second, meaning that their collective variables, e.g. center of mass, are measured each second with a precision of $\sigma/\sqrt{N} \sim 10^{-7}$ cm, leading to extreme sharp c.o.m. localization on the long run. That's what we expect of spontaneous localization theories.

The *mathematical model* is the following. We model the Universe or part of it by a quantized N-body system satisfying the Schrödinger equation

$$\frac{d|\Psi\rangle}{dt} = -\frac{i}{\hbar}\hat{H}|\Psi\rangle \tag{1.1}$$

apart from instances of spontaneous position measurements that happen randomly and independently at rate λ on every constituent. Spontaneous position measurements are standard generalized measurements. Accordingly, when the kth coordinate $\hat{\mathbf{x}}_k$ endures a measurement, the quantum state undergoes the following collapse:

$$|\Psi\rangle \Longrightarrow \frac{\sqrt{G(\mathbf{x}_k - \hat{\mathbf{x}}_k)}|\Psi\rangle}{\|\sqrt{G(\mathbf{x}_k - \hat{\mathbf{x}}_k)}|\Psi\rangle\|}. \tag{1.2}$$

The effects of unsharp position measurement take the Gaussian form:

$$G(\mathbf{x}_k - \hat{\mathbf{x}}_k) = \frac{1}{(2\pi\sigma^2)^{3/2}} \exp\left(-\frac{(\mathbf{x}_k - \hat{\mathbf{x}}_k)^2}{2\sigma^2}\right), \tag{1.3}$$

where $\mathbf{x}_k$ is the random outcome of the unsharp position measurement on $\hat{\mathbf{x}}_k$, and σ sets the scale of unsharpness (precision). The probability of the outcomes $\mathbf{x}_k$ is defined by the standard rule:

$$p(\mathbf{x}_k) = \|\sqrt{G(\mathbf{x}_k - \hat{\mathbf{x}}_k)}|\Psi\rangle\|^2. \tag{1.4}$$

We have thus specified the mathematical model of GRW in terms of standard unsharp position measurements targeting every constituent at rate λ and precision σ. These measurements are *selective* measurements if we assume that the measurement outcomes $\mathbf{x}_k$ are accessible. If they are not, we talk about *non-selective* measurements, and the jump equation (1.2) should be averaged over the outcomes, according to the probability distribution (1.4). The mathematical model of the GRW theory reduces to the following master equation for the density matrix $\hat{\rho}$:

$$\begin{aligned}\frac{d\hat{\rho}}{dt} &= -\frac{i}{\hbar}[\hat{H}, \hat{\rho}] + \lambda \sum_k \left(\int d\mathbf{x}_k \sqrt{G(\mathbf{x}_k - \hat{\mathbf{x}}_k)}\hat{\rho}\sqrt{G(\mathbf{x}_k - \hat{\mathbf{x}}_k)} \right) - \lambda\hat{\rho} \\ &= -\frac{i}{\hbar}[\hat{H}, \hat{\rho}] + \lambda \sum_k \mathcal{D}[\hat{\mathbf{x}}_k]\hat{\rho}. \end{aligned} \tag{1.5}$$

The decoherence superoperator is defined by

$$\mathcal{D}[\hat{\mathbf{x}}]\hat{\rho} = \int d\mathbf{x}\sqrt{G(\mathbf{x} - \hat{\mathbf{x}})}\hat{\rho}\sqrt{G(\mathbf{x} - \hat{\mathbf{x}})} - \hat{\rho}. \tag{1.6}$$

We can analytically calculate it in coordinate representation $\rho(\mathbf{x}, \mathbf{x}')$ of the density matrix. Its contribution on the rhs of the master equation (1.5) shows spatial decoherence, saturating for large separations:

$$\frac{d\rho(\mathbf{x}, \mathbf{x}')}{dt} = \cdots - \lambda \sum_k \left(1 - \exp\left(-\frac{(\mathbf{x}_k - \mathbf{x}'_k)^2}{8\sigma^2}\right)\right) \rho(\mathbf{x}, \mathbf{x}'), \tag{1.7}$$

where ellipsis stands for the Hamiltonian part.

The *amplification mechanism* is best illustrated in c.o.m. dynamics. As we said, for the individual particles the decoherence term remains negligible, whereas for bulk degrees of freedom, e.g. the c.o.m., it becomes crucial to damp Schrödinger cats, as we desired. Assume, for simplicity, free spatial motion of a many-body object. Then the non-selective GRW equation (1.5) yields the following autonomous equation for the reduced c.o.m. density matrix $\hat{\rho}_{\text{cm}}$:

$$\frac{d\hat{\rho}_{\text{cm}}}{dt} = -\frac{i}{\hbar}[\hat{H}_{\text{cm}}, \hat{\rho}_{\text{cm}}] + N\lambda\mathcal{D}[\hat{\mathbf{x}}_{\text{cm}}]\hat{\rho}_{\text{cm}}. \tag{1.8}$$

As we see, the decoherence term concerning the c.o.m. coordinate has been amplified by the number N of the constituents [2], ensuring the desired fast decay of macroscopic superpositions:

$$\frac{d\rho_{\text{cm}}(\mathbf{x}_{\text{cm}}, \mathbf{x}'_{\text{cm}})}{dt} = \cdots - N\lambda \left(1 - \exp\left(-\frac{(\mathbf{x}_{\text{cm}} - \mathbf{x}'_{\text{cm}})^2}{8\sigma^2}\right)\right) \rho(\mathbf{x}_{\text{cm}}, \mathbf{x}'_{\text{cm}}). \tag{1.9}$$

In the selective evolution, the individual GRW measurements (1.2) entangle the c.o.m., rotation and internal degrees of freedom; hence $|\Psi_{\rm cm}\rangle$ does not exist in general. It does exist in a limiting case of rigid many-body motion when the unitary evolution of $|\Psi_{\rm cm}\rangle$ is interrupted by spontaneous σ-precision measurements of the c.o.m. coordinate $\hat{x}_{\rm cm}$ similar to (1.2), just the average rate of the measurements becomes $N\lambda$ [20] instead of λ.

1.3 Localization Is Not Testable, but Decoherence Is

The standard concept of selective measurement implies that we have access to the measurement outcomes, which are the values $\mathbf{x}_k$ in GRW. If they are accessible variables, then the stochastic jump process of the GRW state vector $|\Psi\rangle$ is testable; otherwise it is not. If not, then the same spontaneous measurement is called non-selective, and what is testable is the density operator $\hat{\rho}$. The stochastic jump process (1.1–1.4) becomes *illusory*, and the master equation (1.5) contains the whole GRW physics.

This latter sentence holds in GRW where, as a matter of fact, the $\mathbf{x}_k$s remain inaccessible. Consider the conservative preparation-detection scenario. Assume we prepare a well-defined pure initial state $\hat{\rho}_0 = |\Psi_0\rangle\langle\Psi_0|$ and after time t we desire to test it for the presence of GRW collapses (1.2), but we perform no test prior to this one. As a matter of fact, the relevant state is $\hat{\rho}_t$, being the solution of the master equation (1.5), which does not know about GRW collapses but about GRW decoherence. This is equally valid in the particular case of the macroscopic Schrödinger cat initial state, i.e., a superposition of c.o.m. at two distant locations. The c.o.m. GRW master equation (1.8) will exhaustively predict the results of all subsequent tests on the c.o.m. (including the results and statistics of possible naked-eye observations).

Obviously, inference on stochastic collapse assumes our access to the measurement outcomes. In real laboratory quantum measurements it is the detector design and operation that determine if we have full (or partial) access to the measurement outcomes or we have no access at all. In the case of GRW collapse, accessibility of outcomes is not a matter of postulation. It is useless to postulate that $\mathbf{x}_k$s are accessible without a prescription of how to access them.

1.4 Digression: Random Unitary Process Indistinguishable From GRW

Let us consider an alternative to GRW random process in which the stochastic non-linear GRW jumps (1.2) are replaced by the following stochastic unitary jumps:

$$|\Psi\rangle \Longrightarrow e^{i\mathbf{k}\hat{\mathbf{x}}_k}|\Psi\rangle, \tag{1.10}$$

corresponding to the transfer of momentum $\hbar\mathbf{k}$ to the kth constituent. The probability distribution of momentum transfer is universal, independent of the particle and of the state:

$$p(\mathbf{k}) = \frac{1}{(2\pi\sigma^{-2})^{3/2}} \exp\left(-\frac{\mathbf{k}^2}{2\sigma^{-2}}\right). \tag{1.11}$$

The decoherence superoperator acts as

$$\mathcal{D}[\hat{\mathbf{x}}]\hat{\rho} = \int d\mathbf{k}\, p(\mathbf{k}) e^{i\mathbf{k}\hat{\mathbf{x}}} \hat{\rho} e^{-i\mathbf{k}\hat{\mathbf{x}}} - \hat{\rho}, \tag{1.12}$$

which looks completely different from the GRW structure (1.6) but coincides with it! Hence the master equation for the Schrödinger (1.1) dynamics with the averaged unitary jumps (1.10) will be the the master equation (1.5) derived earlier for the GRW theory. As we argued in Sec. 1.3, the GRW theory can only be tested at the level of the density operator; no experiment could tell us whether the underlying stochastic process of $|\Psi\rangle$ was the GRW stochastic localizing process (1.1–1.4) or the stochastic unitary process.

1.5 How to Think About CSL?

We could repeat what we said concerning correct and efficient teaching of GRW in Section 1.2. This time the standard discipline of modern physics, relevant to CSL, is time-continuous quantum measurement (monitoring), which is just the time-continuous limit of unsharp sequential measurements similar to those underlying GRW in Section 1.2. Quantum monitoring theory was not yet conceived in 1986 (GRW). It was born in 1988, and it became widely known in the 1990s as the standard theory of quantum monitoring in the laboratory [14, 15]. It played an instrumental role for semiclassical gravity's consistent introduction to spontaneous collapse theories [21, 22]. In what follows, I utilize the summary of standard Markovian quantum monitoring theory from [21].

So, how should we interpret CSL? It derives from GRW. The discrete sequence of spontaneous unsharp position measurements is replaced by spontaneous monitoring of the spatial number distribution of particles [6] (or, in a later version, of the spatial mass distribution of particles [7]). Accordingly, CSL introduces the smeared mass distribution

$$\hat{n}(\mathbf{x}) = \sum_k G(\mathbf{x} - \hat{\mathbf{x}}_k), \tag{1.13}$$

where, this time, the width of the Gaussian is r_C. Monitoring yields the measured signal in the form

$$n_t(\mathbf{x}) = \langle\Psi_t|\hat{n}(\mathbf{x})|\Psi_t\rangle + \delta n_t(\mathbf{x}), \tag{1.14}$$

where $\delta n_t(\mathbf{x})$ is the signal white noise, still depending on the spatial resolution/correlation of monitoring. The CSL signal noise is a spatially uncorrelated white noise:

$$\mathbb{E}\delta n_t(\mathbf{x})\delta n_s(\mathbf{y}) = \frac{1}{4\gamma}\delta(\mathbf{x}-\mathbf{y})\delta(t-s). \tag{1.15}$$

Just like in the case of GRW sequential spontaneous measurements, the conditional quantum state evolves stochastically, this time according to the following stochastic Schrödinger equation, *driven by the signal noise* in the Ito-sense:

$$\frac{d|\Psi\rangle}{dt} = \left\{-\frac{i}{\hbar}\hat{H} - \frac{\gamma}{2}\int d\mathbf{x}\left(\hat{n}(\mathbf{x}) - \langle\hat{n}(\mathbf{x})\rangle\right)^2 + 4\gamma\int d\mathbf{x}\left(\hat{n}(\mathbf{x}) - \langle\hat{n}(\mathbf{x})\rangle\right)\delta n(\mathbf{x})\right\}|\Psi\rangle. \tag{1.16}$$

So far we have introduced the equations of selective spontaneous monitoring, assuming that the signal (1.14) is accessible, which won't be the case, similar to GRW. In non-selective monitoring, the CSL physics reduces to the signal-averaged evolution of the conditional state, i.e., to the CSL master equation:

$$\frac{d\hat{\rho}}{dt} = -\frac{i}{\hbar}[\hat{H},\hat{\rho}] - \frac{\gamma}{2}\int d\mathbf{x}[\hat{n}(\mathbf{x}),[\hat{n}(\mathbf{x}),\hat{\rho}]]. \tag{1.17}$$

(Note $\gamma = (4\pi r_C^2)^{3/2}\lambda$ would ensure the coincidence with GRW's spatial decoherence rate at the single-particle level, although CSL defined a slightly different γ [6]).

The traditional CSL teaching differs in a single major point: it does not mention the theory of monitoring. Hence it does not use the notion of signal $n_t(\mathbf{x})$, and the equation (1.14) is not part of it. Instead, CSL's traditional definition postulates the stochastic Schrödinger equation:

$$\frac{d|\Psi\rangle}{dt} = \left\{-\frac{i}{\hbar}\hat{H} - \frac{\gamma}{2}\int d\mathbf{x}\left(\hat{n}(\mathbf{x}) - \langle\hat{n}(\mathbf{x})\rangle\right)^2 + \sqrt{\gamma}\int d\mathbf{x}\left(\hat{n}(\mathbf{x}) - \langle\hat{n}(\mathbf{x})\rangle\right)w(\mathbf{x})\right\}|\Psi\rangle, \tag{1.18}$$

which would correspond to the replacement $\delta n_t(\mathbf{x}) = 2\sqrt{\gamma}w_t(\mathbf{x})$ had CSL derived it from our (1.16). The traditional CSL dynamics is driven by the spatially uncorrelated standard white noise, satisfying

$$\mathbb{E}\delta w_t(\mathbf{x})\delta w_s(\mathbf{y}) = \delta(\mathbf{x}-\mathbf{y})\delta(t-s). \tag{1.19}$$

In CSL narrative (e.g. [12]) the origin of the noise field as well its anti-Hermitian coupling to density $\hat{n}(\mathbf{x})$ are mentioned among theory elements yet to be justified, still without reference to the spontaneous monitoring interpretation available already for a long enough time.

From arguments of Section 1.3, it follows that all testable predictions follow from the CSL master equation (1.17), the stochastic Schrödinger equation (1.18) is *empirically redundant*, collapse in the claimed quantitative sense is an illusion.

1.6 Final Remarks

Disregarding that spontaneous collapse theories are rooted in standard quantum mechanical collapse theories with hidden detectors has had too many drawbacks.

The principle one is the illusion that the quantitative models of spontaneous collapse (localization) in their current forms are relevant empirically like master equations of spontaneous decoherence are, which have already been under empiric tests due to recent breakthroughs in technology. This illusion is surviving despite no proposals having been ever made for a future experiment to test underlying localization effects of $|\Psi\rangle$ beyond decoherence of $\hat{\rho}$; all proposals have so far concerned the dynamical features (e.g. spontaneous decoherence) of the averaged state $\hat{\rho}$.

Secondary drawbacks concern illusions that teaching and interpretation of spontaneous collapse necessitate radical departure from standard quantum theory both conceptually and mathematically. This may have kept philosophers excited and may have prevented students from learning the subject faster and physicists from going deeper into their fundational investigations.

Physics research will gradually adapt itself to the option that spontaneous collapse fits better to standard quantum knowledge than we thought of it before. Monitoring theory roots were revealed for DP spontaneous collapse from the beginning and have been detailed and exploited for CSL, too, recently in [21, 22].

References

[1] L. Diósi, Emergence of classicality: From collapse phenomenologies to hybrid dynamics, *Lect. Notes Phys.* **538**, 243–250 (2000).
[2] G. C. Ghirardi, A. Rimini, and T. Weber, Unified dynamics for microscopic and macroscopic systems, *Phys. Rev.* **D 34**, 470–491 (1986).
[3] J. S. Bell, Are there quantum jumps? In *Schrödinger Centenary of a polymath* (Cambridge, Cambridge University Press, 1987).
[4] L. Diósi, A quantum-stochastic gravitation model and the reduction of the wavefunction. Thesis, 1986. Available from: http://wigner.mta.hu/~diosi/prints/thesis1986.pdf.
[5] N. Gisin, Stochastic quantum dynamics and relativity, *Helv. Phys. Acta* **62**, 363–371 (1989).
[6] G. C. Ghirardi, P. Pearle, and A. Rimini, Markov processes in Hilbert space and continuous spontaneous localization of systems of identical particles, *Phys. Rev.* **A 42**, 78–89 (1990).
[7] G. C. Ghirardi, R. Grassi, and F. Benatti, Describing the macroscopic world: Closing the circle within the dynamical reduction program, *Found. Phys.* **25**, 5–38 (1995).

[8] L. Diósi, A universal master equation for the gravitational violation of the quantum mechanics, *Phys. Lett.* **120A**, 377–381 (1987).
[9] L. Diósi, Models for universal reduction of macroscopic quantum fluctuations, *Phys. Rev.* **A 40**, 1165–1174 (1989).
[10] R. Penrose, On gravity's role in quantum state reduction. *Gen. Rel. Grav.* **28**, 581 (1996).
[11] A. Bassi and G. C. Ghirardi, Dynamical reduction models, *Phys. Rep.* **379**, 257–426 (2003).
[12] A. Bassi, K. Lochan, S. Satin, T. P. Singh, and H. Ulbricht, Models of wave-function collapse, underlying theories, and experimental tests, *Rev. Mod. Phys.* **85**, 471–527 (2013).
[13] L. Diósi, Continuous quantum measurement and Ito-formalism, *Phys. Lett.* **129A**, 419–423 (1988).
[14] H. Carmichael, *An open system approach to quantum optics* (Berlin, Springer, 1993).
[15] H. M. Wiseman and G. J. Milburn, *Quantum measurement and control* (Cambridge, Cambridge University Press, 2010).
[16] L. Diósi, Is spontaneous wave function collapse testable at all? *J. Phys. Conf. Ser.* **626**, 012008-(5) (2015).
[17] J. von Neumann, *Matematische Grundlagen der Quantenmechanik* (Berlin, Julius Springer, 1932).
[18] M. A. Nielsen and I. L. Chuang, *Quantum computing and quantum information* (Cambridge, Cambridge University Press, 2000).
[19] L. Diósi: *Short course in quantum information theory* (Berlin, Springer, 2011).
[20] L. Diósi, On the motion of solids in modified quantum mechanics, *Eurohys. Lett.* **8**, 285–290 (1988).
[21] A. Tilloy and L. Diósi, Sourcing semiclassical gravity from spontaneously localized quantum matter, *Phys. Rev.* **D 93**, 024026-(12) (2016).
[22] A. Tilloy and L. Diósi, Principle of least decoherence for Newtonian semi-classical gravity, *Phys.Rev.* **D96**, 104045-(6) (2017).