ZEROS OF POLYNOMIALS AND SOLVABLE NONLINEAR EVOLUTION EQUATIONS

Reporting a novel breakthrough in the identification and investigation of solvable and integrable systems of nonlinearly coupled evolution equations, this book includes many examples that illustrate this approach. Beginning with systems of ODEs, including second-order ODEs of Newtonian type, it then discusses systems of PDEs, and systems evolving in discrete time. It reports a novel, differential algorithm to evaluate all the zeros of a generic polynomial of arbitrary degree: a remarkable development of a fundamental mathematical problem with a long history. This book will be of interest to applied mathematicians and mathematical physicists working in the area of integrable and solvable nonlinear evolution equations; it can also be used as supplementary reading material for general Applied Mathematics or Mathematical Physics courses.

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CAMBRIDGE
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University Printing House, Cambridge CB2 8BS, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi – 110025, India

79 Anson Road, #06–04/06, Singapore 079906

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning, and research at the highest international levels of excellence.

www.cambridge.org
Information on this title: www.cambridge.org/9781108428590
DOI: 10.1017/9781108553124

First published 2018

Printed and bound in Great Britain by Clays Ltd, Elcograf S.p.A.

A catalogue record for this publication is available from the British Library.

Library of Congress Cataloging-in-Publication Data
Names: Calogero, Francesco, 1935– author.
Title: Zeros of polynomials and solvable nonlinear evolution equations / Francesco Calogero (Sapienza University of Rome, emeritus).
Description: Cambridge : Cambridge University Press, 2018. | Includes bibliographical references.
Identifiers: LCCN 2018017272 | ISBN 9781108428590 (hardback)
Subjects: LCSH: Evolution equations, Nonlinear. | Zero (The number) | Polynomials. | Differential equations, Partial. | Differential equations, Nonlinear.
Classification: LCC QC20.7.E88 C268 2018 | DDC 530.1/55353–dc23
LC record available at https://lccn.loc.gov/2018017272

ISBN 978-1-108-42859-0 Hardback

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Preface

This short book is based on the *algebraic* relationship among the N *zeros* and the N *coefficients* of a (monic) polynomial of *arbitrary* degree N. This is a core mathematical topic since time immemorial; indeed the important mathematical subdiscipline called "algebraic geometry" originated from investigations of this relationship and is focused on its ramifications, which are quite deep. The approach used in this book is instead quite elementary; it is essentially based on the simple idea to consider (monic) polynomials of arbitrary degree N in their argument—say, the *complex* variable z—which moreover also depend on a parameter—say the *real* variable t—generally conveniently interpreted as "time" (the dependence on a second parameter playing the role of additional "space" variable is also considered, see Chapter 5). This implies that both the N *coefficients* and the N *zeros* of such a time-dependent monic polynomial *depend on time*, and this simple notion allows a certain number of interesting developments relevant to the topics mentioned in the title of this book. Some of these developments emerged quite recently; indeed this book is a compilation of results obtained and published by its author—alone or with collaborators—over the last three years. This is reflected in its bibliography and in the fact that occasionally the text below is drawn—even *verbatim*—from these publications. But unpublished findings are also included.

The author is very grateful to his co-authors—Oksana Bihun, Mario Bruschi, and François Leyvraz—who have been instrumental in obtaining some of the findings reported in this book (as specified below), and also to his co-authors of previous papers on related topics—several of which are identified in the bibliographic notes at the end of each chapter, and/or are referred to in the papers quoted there—including Fabio Briscese, Antonio Degasperis, Silvana De Lillo, Luca Di Cerbo, Riccardo Droghei, Marianna Euler, Norbert Euler, Jean Pierre Françoise, David Gómez-Ullate, Sandro Graffi, Vladimir I. Inozemtsev, Sabino Iona, Edwin Langmann, Mauro Mariani, Paolo Maria Santini, Matteo Sommacal, Ji Xiaoda, and Ge Yi. Interactions with these colleagues and with others I met in Rome and at

international meetings over the years have been quite useful, in particular those facilitated by the series of international Gatherings of Scientists hosted—for three to five weeks in November-December every second year since 2000—by the Centro Internacional de Ciencias (CIC) in Cuernavaca, Mexico, as well as the series of Nonlinear Evolution Equations and Dynamical Systems meetings held since 1980 and the series of Physics and Mathematics of Nonlinear Phenomena meetings held since 1979 (the names of these meetings have evolved over time).

This book is written in plain language: it is addressed to a wide readership, including mathematicians, mathematical and theoretical physicists, and, more generally, practitioners interested in getting acquainted with mathematical tools useful to model complex phenomena via *systems* of *nonlinear* evolution equations—including *ordinary differential* equations, *partial differential* equations, and equations evolving via *discrete* time-steps. The phenomena for which the tools provided within this book might prove useful include many-body problems in classical mechanics, chemical reactions, ecology, population dynamics, economics, you name it. The readership of this book might moreover include students, indeed the results reported offer a fertile ground for master and PhD level dissertations exploiting the simple technique described herein to identify and investigate *new* systems of *nonlinear* evolution equations amenable to *exact* treatments. This book might be used—at the advanced undergraduate or at the graduate level—as basic text for a topical course or as background reading material for a pure and applied mathematics course where evolution equations play a significant role. It has been accordingly written so as to facilitate a *non-systematic* reading of it from beginning to end—at the cost of some repetitions (my teaching experience over more than half a century taught me that *repetita iuvant*). It is, however, advisable to read first Chapter 1 and especially Chapter 2; while the material in Chapter 3 should mainly appeal to *numerical* analysts, who might then hopefully become interested in investigating the comparative disadvantages/advantages—relative to traditional methods—of the *differential algorithm* to compute *all* the zeros of a *generic* polynomial reported there.

Finally, let us mention that throughout this book we do not hesitate from using *italics* whenever we consider this typographical device possibly useful to help the reader identify the *more significant* aspect of the text; our apologies to those readers who dislike this trick.