Superconductivity

Superconductivity is among the most exciting of quantum phenomena in condensed matter physics, and has important applications across science and technology, from fusion reactors to particle accelerators. This self-contained text provides a comprehensive account of the physical foundations of superconductivity and related recent developments in the field. Beginning with a detailed description of the BCS theory of superconductivity, the book then describes the subsequent successes of this landmark theory and proceeds to more advanced topics such as Josephson effects and vortices. The strong coupling theory of superconductivity is introduced in later chapters, providing a springboard to important current research on hydride superconductors, which have displayed very high critical temperatures. Recent manifestations of superfluidity in ultracold atoms physics are also described. This book will give readers a solid grounding in the theory and applications of superconductivity, and an appreciation of its broader importance in the field of modern condensed matter physics.

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An Introduction

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Preface

As is often the case, this book is an outgrowth of lecture notes for a course on superconductivity I had the pleasure to teach for several years. Teaching implies naturally severe limitations of time for the course duration, which leads to strong constraints on the subject matter that is taught. These boundary conditions disappear in principle for a book, and I have taken this opportunity to include a number of points I could not address at all during my lectures, for lack of time.

However, this book has not been written with the a priori intention of extending the scope of what I taught. My teaching was an introduction to superconductivity, and my aim in this book has been to stay at the same introductory level. I have rather taken the extended space one enjoys in a book to include subject matter that in my mind should have been included logically and coherently in the course but could not be included for lack of time. I have been able in this way to include most of the points I regretted omitting from the course. This description actually only corresponds to the first six chapters of this book. Indeed, I have taken this opportunity to include some more recent subject matter from the field of cold atoms physics, which I feel is quite relevant for our understanding of superconductivity, as I describe in more detail below.

Coherently with this introductory spirit, I have tried to be quite explicit in my writing, providing all the necessary details for understanding by the same kind of students as the ones I was teaching. On balance between going into detail or skipping the "obvious," I have chosen the former. Naturally, there are limits to this, as providing too much detail makes the text burdensome. This choice has also been mostly valid at the beginning of chapters, with the finishing parts being usually being devoted to more advanced matter and going accordingly at a somewhat more accelerated pace.

In the same spirit, I have done my best to make this book as self-contained as possible. The understanding requires only basic knowledge of electromagnetism, quantum mechanics, statistical physics and little of solid state physics (and some mathematics). But otherwise I have rather chosen to start from first principles, without for example referring to linear response or scattering theory.

Regarding the organization of the chapters, my aim has been to go directly at the microscopic understanding of superconductivity provided by BCS theory. Whether accepted or rejected, it is the reference frame of our present-day comprehension. Another ingredient that I feel is quite important when teaching science nowadays, particularly when exploring such a strange phenomenon as superconductivity, is to describe the evolution of ideas that has led to the present understanding. This is an essential point for explaining how the present scientific knowledge has been built, which allows for its future evolution on a sound basis. Ideally, this should be done by following the historical evolution of the field, but

this comes rapidly in contradiction with good teaching practices because the real history is often quite long and somewhat tortuous. Hence one needs to identify shortcuts to obtain a clear logical presentation. Nevertheless, I have kept in mind this spirit, trying to explain wherever possible where the matter comes from.

This approach appears particularly in the first chapter, which presents the early ingredients of superconductivity and closes with a section summarizing the ideas leading to BCS theory. The second chapter deals with the basics of BCS theory. The last sections of this chapter are more advanced and are addressed to readers who are somewhat dissatisfied with the BCS wave function, which is fairly frequent. As such, they may be skipped upon first reading. The third chapter extends BCS theory to the nonzero temperature situation. The last sections of this chapter are an appropriate place to deal in depth with the microscopics of the pairing interaction, and then to open the door leading to symmetries and mechanisms other than the standard BCS ones, which are present in unconventional superconductors. Again, these sections may be skipped at first reading. The portion of the manuscript devoted to BCS closes with the fourth chapter, which is mostly devoted to the response of a superconductor to an electromagnetic perturbation, according to BCS theory. One aim of this chapter is to show that BCS theory indeed describes a system that is superconducting. This is a worthwhile purpose, although the road is fairly long.

The next chapter deals with the fascinating manifestations of quantum mechanics at the macroscopic scale that appear in superconductors, through flux quantization and the Josephson effects. These give rise in particular to remarkable applications. The sixth chapter is devoted to the beautiful Ginzburg–Landau theory, which leads to the introduction of vortices, so crucial for strong current applications of superconductivity. It does not come in the proper historical order, but I feel that this is pedagogically the appropriate position.

The following chapters arise from the opportunity presented by the recent remarkable progress in the physics of ultracold atoms, allowing one to display the BEC–BCS crossover. This establishes a direct physical link between these two related aspects of superfluidity and enlightens our understanding of pairing in superconductivity. Hence, all the more since I have had a direct interest in this field, it has been quite tempting to include a chapter on the BEC–BCS crossover, both for logical reasons and also because this is beautiful physics. This has not been such an easy matter, since it implied, at a simple level, the introduction of several new concepts. In particular, this has inspired first a complete chapter on Bose–Einstein condensation, a necessary ingredient to fully understand the BEC–BCS crossover. This chapter is also welcome for enhancing one's understanding of superconductivity, since it allows one to present in detail the physics of superfluidity that underlies superconductivity.

The final chapter is by far the most difficult matter of this book and also was the most difficult to write. Hence its position at the end of this book is quite appropriate. Nevertheless, its presence is logically necessary. Indeed, it serves to complete the unsatisfactory handling of BCS theory regarding the time-dependent nature of the pairing interaction. This completion is rewarded by some remarkable agreements between experiment and theory, confirming the validity of our understanding of superconductivity in the corresponding compounds. On the other hand, this chapter is also justified by the very recent discovery of hydride superconductors with very high critical temperatures, for which the formalism

presented in this chapter seems to be the appropriate one. However, although I have done my best to stay in the spirit of the preceding chapters, this one is not as self-contained as the preceding ones. A proper complete presentation of the matter would have led to far too technical and far too long explanations for this book. Hence some stages require a leap of faith, which I have tried to patch as well as I could.

On a final note, I would like to thank Xavier Leyronas for all the pleasant time I had sharing with him teaching superconductivity. And I want to take this opportunity to express my gratitude and thoughts to my friend, Dierk Rainer, from whom I learned so much.