Imaging Optics

This comprehensive and self-contained text presents the fundamentals of optical imaging from the viewpoint of both ray and wave optics, within a single volume. Comprising three distinct parts, it opens with an introduction to electromagnetic theory, including electromagnetic diffraction problems and how they can be solved with the aid of standard numerical methods such as RCWA or FDTD. The second part is devoted to the basic theory of geometrical optics and the study of optical aberrations inherent in imaging systems, including large-scale telescopes and high-resolution projection lenses. A detailed overview of state-of-the-art optical system design provides readers with the necessary tools to successfully use commercial optical design software. The final part explores diffraction theory and concludes with vectorial wave propagation, image formation and image detection in high-aperture imaging systems. The wide-ranging perspective of this important book provides researchers and professionals with a comprehensive and rigorous treatise on the theoretical and applied aspects of optical imaging.

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Contents

Preface page ix
Acknowledgements xi

I Electromagnetic Theory in the Optical Domain

1 Electromagnetic Wave Propagation in Isotropic Media 3
1.1 Introduction 3
1.2 Maxwell’s Equations as Experimental Laws 3
1.3 Maxwell’s Equations in the Optical Domain 9
1.4 Electromagnetic Energy Density and Energy Transport 10
1.5 Potential Functions and the Electromagnetic Field Vectors 13
1.6 Harmonic Solutions and the Helmholtz Equation 18
1.7 Gaussian Beams 35
1.8 Wave Propagation at an Interface between Two Media 51
1.9 Transmission and Reflection in a Stratified Medium 68
1.10 Multilayer Reflection and Transmission Coefficients 72
1.11 The Scattering Matrix and the Impedance Matrix Formalism 81
1.12 Stratified Medium with Laterally Modulated Periodic Sublayers 87

2 Wave Propagation in Anisotropic Media 107
2.1 Introduction 107
2.2 Harmonic Electromagnetic Waves in an Anisotropic Medium 108
2.3 Plane Wave Solutions in Uniaxial and Biaxial Media 112
2.4 Solution of the Helmholtz Equation in an Anisotropic Medium 117
2.5 Energy Transport in a Medium with Linear Anisotropy 124
2.6 Eigenvalue Equations Involving the Ray Vector $\mathbf{s}$ 127
2.7 The Functions $v_p(\mathbf{k})$, $n_p(\mathbf{k})$, $v_r(\mathbf{s})$ and $n_r(\mathbf{s})$ 133
2.8 Conical Refraction 138
2.9 Optical Activity 143
2.10 Wave Propagation in an Anisotropic Medium Including Rotation 146
2.11 Energy Propagation in a General Anisotropic Medium 149
2.12 Reflection and Refraction at an Interface in Anisotropic Media 154

3 Surface Waves, Metamaterials and Perfect Imaging 160
3.1 Eigenmodes of a Metal–Dielectric Interface 161
3.2 Wave Propagation in Metamaterials 168
3.3 The Concept of the Perfect Lens 179
# CONTENTS

## II Geometrical Theory of Optical Imaging

4 Foundations of Geometrical Optics  
4.1 Introduction  
4.2 Geometrical Optics Derived from Maxwell’s Equations  
4.3 Characteristic Function of an Optical System  
4.4 Angle Characteristic Function of a Single Surface  
4.5 The First-order Angle Characteristic and the Paraxial Domain  
4.6 Stigmatic Imaging and the Angle Characteristic Function  
4.7 Construction of the Angle Characteristic Function of a System  
4.8 Isoplanatism and Aplanatism of an Optical System  
4.9 The Definition of Transverse and Wavefront Deviation  
4.10 Paraxial Optics and the Matrix Analysis of Optical Systems  
4.11 Radiometry and Photometry  

5 Aberration Analysis of Optical Systems  
5.1 Introduction  
5.2 Classification of Aberrations  
5.3 Calculation of the Seidel Aberration Coefficients  
5.4 Aberration of a Thin Lens  
5.5 Seidel Aberrations of a Plane-parallel Plate  
5.6 Chromatic Aberration  
5.7 Finite Ray-tracing  
5.8 Total Aberration at a Single Surface; Formulas of Hopkins and Welford  
5.9 Aperture- and Field-dependent Aberration Function of an Imaging System  
5.10 Paraxial and Finite Ray-tracing in Inhomogeneous Media  
5.11 Polarisation Ray-tracing in Anisotropic Media  

6 Analytic Design and Optimisation of Optical Systems  
6.1 Introduction  
6.2 Analytic Aberration-free Design of an Optical System  
6.3 Merit Function of an Optical System  
6.4 Optimisation of Optical Systems  
6.5 Optical Tolerancing  

7 Design Methods for Optical Imaging Systems  
7.1 Introduction  
7.2 The Achromatic Doublet  
7.3 The Photographic Landscape Lens  
7.4 The Portrait Lens  
7.5 Flat-field Imaging Systems  
7.6 The Astronomical Telescope  
7.7 Microscope Optics  
7.8 Aspheric Objectives for Optical Disc Systems  
7.9 Large-field Projection Systems with Diffraction-limited Quality  

## III Diffraction Theory of Optical Imaging

8 Vectorial and Scalar Theory of Diffraction and Focusing  
8.1 Foundation of Vector Diffraction
CONTENTS

8.2 Boundary Value Problems in Diffraction 559
8.3 The Debye–Wolf and Related Diffraction Theories 562
8.4 Scalar Diffraction Theories 568
8.5 The Validity of the Debye–Wolf Theory 580

9 The Aberrated Scalar and Vector Point-spread Function 582
9.1 Introduction 582
9.2 Pupil Function Expansion Using Zernike Polynomials 584
9.3 The Point-spread Function and the Nijboer–Zernike Diffraction Theory 591
9.4 The Extended Nijboer–Zernike Diffraction Theory 609
9.5 Vector Point-spread Function and the ENZ Diffraction Theory 614
9.6 Energy and Momentum Density and Their Flow Components 640

10 Frequency Analysis of Optical Imaging 657
10.1 Introduction 657
10.2 Optical Transfer Function of a Classical Wide-field Imaging System 660
10.3 Frequency Transfer by a Scanning Imaging System 710
10.4 The Three-dimensional Transfer Function 725
10.5 Light Scattering and Frequency Transfer 767

11 Theory of Vector Imaging 782
11.1 Vector Ray Tracing – The Generalised Jones Matrix Formalism 783
11.2 Vectorial Point-spread Function 787
11.3 Focusing of Partially Coherent, Partially Polarised Light 794
11.4 Properties of High-numerical-aperture Imaging Systems 804
11.5 High-aperture Scanning Light Microscopes Imaging a Point Object 814
11.6 Theory of Multiphoton Fluorescence Microscopes 832
11.7 Extension of the Imaging Theory to More Complicated Optical Systems 847
11.8 Imaging of Arbitrary Objects 855

Appendix A Fourier Analysis, Complex Notation and Vector Formulas 860
Appendix B Phase and Group Velocity of a Wave Packet 879
Appendix C The Kramers–Kronig Dispersion Relations 882
Appendix D Zernike Polynomials 888
Appendix E Magnetically Induced Optical Rotation (Faraday Effect) 907
Appendix F Vector Point-spread Function in a Multilayer Structure 913
Appendix G V. S. Ignatowsky: Diffraction by a Lens of Arbitrary Aperture 919

References* 945
Author Index 959
Subject Index 963
Preface

The idea of writing a specific book on ‘imaging optics’ came some ten years ago and was spurred by the experience which the present authors had acquired in teaching at a university and in guiding research of M.Sc. and Ph.D. students. We noticed that the more advanced optical subjects which have to be mastered by these students to successfully accomplish their studies are rather scattered over the literature. We felt that a comprehensive, well-organised book on the theory and practice of optical imaging, using the same notation and conventions for the various subjects, was lacking. The present book, which has been conceived in the past nine years, should make it easier for students to acquire specific knowledge in the field of optical imaging.

The book comprises three parts. The first introductory part provides the physical basis of optics by means of Maxwell’s equations and applies these equations to wave propagation in free space and to refraction and reflection at interfaces between media. A special topic is the propagation of fundamental and higher-order Gaussian beams. The principles needed for solving diffraction problems are explained with special attention to wave propagation and diffraction in stratified media. The rigorous coupled wave analysis and the finite-difference time-domain method are treated in some detail. The chapter on wave propagation in anisotropic media focuses on linear and circular birefringence as a preparation for polarisation aspects in imaging which are encountered in Part III of the book. Emphasis is put on the intriguing effect of conical refraction. Combined with the chapter on surface waves, the reader acquires a good overview of light diffraction and light scattering at an object surface or in an object volume of which an image has to be formed by the optical imaging system.

The second part of the book is devoted to geometrical optics, aberration theory and optical design. It provides the reader with a theoretical basis of ray optics and illustrates the limits on imaging quality based on this simplified light propagation model. Paraxial optics is treated by means of the matrix theory of refraction/reflection and ray propagation. An extensive chapter on aberration theory applied to a single surface, a single lens and to entire systems shows the practical limitations in imaging quality of an optical system. Throughout this chapter, the fundamental diffraction unsharpness and the image blurring due to geometrical aberrations are jointly evaluated. In some cases the (partial) suppression of aberration in an optical system can be achieved by analytic methods. These methods are presented in some detail, together with the more widely used numerical optimisation methods. Imaging quality of an optical system can be further reduced by manufacturing errors. A statistical analysis is presented of the influence of opto-mechanical and mounting errors of lens elements and surfaces such that the expected quality of a real-world imaging system can be evaluated as well as the spread around this value. The second part ends with a longer chapter on optical design methods applied to a wide variety of low- and high-aperture optical imaging systems.

The diffraction of light is the subject of the third part of the book. Based on Maxwell’s equations, the first chapter starts with an in-depth treatment of vector diffraction models which are then, step by step, reduced to the older scalar diffraction theories. Various intermediate stages of approximation between the rigorous vector model and the simplest scalar diffraction model are presented such that the reader can decide which approximation is adequate for a specific diffraction problem at hand. The point-spread function, a basic building block for the construction of the image intensity of a composite object, is discussed for an ideal and an aberrated imaging system. The classical scalar diffraction theory of Zernike and Nijboer is used for the diffraction analysis of low-aperture, aberrated imaging systems. The region of validity of this classical diffraction theory is then extended to a much larger focal volume (Extended Nijboer–Zernike theory) and provides the reader with semi-analytic results which can replace the numerical methods used for the evaluation of a general diffraction integral. The extension of this theory to ideal and aberrated point-spread functions of imaging systems with high-aperture serves to describe image formation in non-ideal high-resolution imaging systems. The chapter on point-spread functions ends with a detailed vector-based analysis of the propagation of energy, linear momentum and
angular momentum in a high-aperture focused beam. Spatial-frequency analysis of the imaging by an optical system is the subject of a chapter in which the influence of the object illumination on the image is also studied. The van Cittert–Zernike coherence theorem is presented and applied to a certain number of experimental configurations. The classical two-dimensional frequency analysis is extended to the imaging of three-dimensional (volumetric) objects or object surfaces. The influence of light scattering in the optical system on the spatial frequency transfer from object to image space concludes this chapter. The final chapter of Part III discusses the systematic analysis of (vector) imaging systems. The general state of polarisation of the light radiated by the object is defined as well as the possible anisotropy of the imaging system itself, of specially inserted birefringent elements, polarisers, etc. The light propagation from object to image space is described by means of a modular, matrix-based model of wave propagation. The detection of light in image space is performed by means of a polarisation-dependent detector array in a high-aperture imaging geometry.

A number of appendices have been added and they explain, in more detail than was possible in the main body of the book, a certain number of basic definitions or analytic/numerical tools which are frequently used throughout the book. We mention the first appendix which emphasises the role of Fourier methods in modern optics. The fourth appendix provides the reader with an overview of the properties and the applications of Zernike polynomials in optics. A special appendix contains the English translation of an influential publication in the Russian language by V.S. Ignatowsky, dating back to the beginning of the twentieth century. It presents an analysis of the electromagnetic field in the focal region of a high-aperture beam focused by a lens. This publication has inspired many later researchers in this field.

The overview of subjects which is given above also shows which material associated with (classical) imaging optics is definitely missing from this book. Only incidentally and without much detail, we mention a few of the modern methods in low- and high-aperture imaging. Many acronyms circulate in the literature for special imaging methods adapted to a special type of object, illumination, state of polarisation, spectral composition of the light, interferometric detection mode, etc. Simply because of the size of this book, these methods could not be included. Another interesting topic that is missing is the (unique) retrieval of object properties from one or several recordings of an image of the object. This subject, both from the experimental and the numerical point of view, shows interesting progress, also with respect to the high-aperture imaging geometry.

The writing of a book requires continuous concentration on the subject and in this respect the first author (J. B.) was privileged because of his retired status. The absence of time-consuming managerial tasks, of proposal writing and of regular work on national or international committees permitted a permanent focus on book writing. The second author (P. T.) could not benefit from these favourable circumstances and his contribution has remained relatively small. Sections 1.2 and 1.5 and Subsection 1.8.3 of Chapter 1 and the entire Chapters 8 and 11 of the book bear his signature. The remaining part of the book and the Appendices A to F have been written by the first author.

We are confident that this textbook will be a welcome companion on the desk of a masters or graduate student in general optics and in optical imaging in particular. Part of the book can also serve as teaching material for an advanced optics course to such students. Finally, the professional in optical research and development will have at his disposal a reference book covering a wide variety of subjects in advanced optical imaging.
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