

MHD Waves in the Solar Atmosphere

This volume presents a full mathematical exposition of the growing field of coronal seismology which will prove invaluable for graduate students and researchers alike. Roberts' detailed and original research draws upon the principles of fluid mechanics and electromagnetism, as well as observations from the *TRACE* and *SDO* spacecraft and key results in solar wave theory. The unique challenges posed by the extreme conditions of the Sun's atmosphere, which often frustrate attempts to develop a comprehensive theory, are tackled with rigour and precision; complex models of sunspots, coronal loops and prominences are presented, based on a magnetohydrodynamic (MHD) view of the solar atmosphere, and making use of Faraday's concept of magnetic flux tubes to analyse oscillatory phenomena. The rapid rate of progress in coronal seismology makes this essential reading for those hoping to gain a deeper understanding of the field.

BERNARD ROBERTS is Emeritus Professor of Solar Magnetohydrodynamics at the University of St Andrews. His important contributions to the field over the past forty years have been recognized with an election to the Royal Society of Edinburgh in 1997, a Saltire Scottish Science Award in 1998, and, in 2010, the Royal Astronomical Society's prestigious Chapman Medal, awarded for 'investigations of outstanding merit in solar-terrestrial physics'.

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Bernard Roberts

University of St Andrews, Scotland



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To my wife Margaret and our sons Alastair, James, Michael and Richard.

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Preface

In this account we concentrate on providing a theoretical development that addresses the phenomenon of magnetic waves in the Sun's atmosphere. There is no surprise that the solar atmosphere abounds with magnetic waves. After all, the presence of a magnetic field endows the medium with an elasticity that is proportional to the strength of the magnetic field, so any compression of the plasma results in a restoring response, and any twisting of the magnetic field leads to an attempt to return to an untwisted state: magnetic waves result. Now we are familiar with sound waves in the air around us. The added complexity in the solar atmosphere is the presence of magnetic field in a conducting medium, quite different from our everyday experience of sound in the open air or in a room where magnetism is relatively unimportant. Admittedly, in the Sun the magnetic field is relatively less important in the solar interior, where pressure forces tend to overwhelm whatever magnetic fields are there. But magnetism becomes more important in the visible layer of the solar atmosphere – the photosphere – where the field is marshalled into concentrations, becoming most evident in pores and sunspots. Moreover, in the higher atmosphere itself – in the solar corona – the magnetic field becomes all important, dominating much of the other forces (such as gravity) that may arise and effectively underwriting all that occurs there.

In our account here we explore mathematical models of some of the physical effects that may arise in the solar atmosphere. Our treatment is based upon a magnetohydrodynamic view of the solar atmosphere in which detailed plasma effects are taken into account only in as far as they modify the large scale picture of phenomena. Consequently, our description is based upon a marriage of fluid mechanics and electromagnetism, and even here we discount the rapid effects of Maxwell's electrodynamics being content to include the magnetic force as given by the Lorentz force, though Faraday's law of induction interlinks magnetism and fluid so that phenomena are a result of magnetism and fluid dynamics acting together. Moreover, much of our description is based upon Faraday's concept of a *magnetic flux tube* (or its cousin, the *magnetic slab*). In different layers of the solar atmosphere, the nature of a flux tube may be different, though wherever a flux tube occurs it gives rise to a communication channel, a one dimensional connection between one region and another in an otherwise three-dimensional medium. Accordingly, the guiding of magnetic waves plays a central role in our description.

Waves are interesting in their own right as a physical phenomena, but they gain considerably in importance when we realise that waves carry information about the medium in

which they propagate. Understanding this affords us a seismology, a study of the medium from an exploration of the waves the medium supports. This notion, variously termed *coronal seismology* or *MHD seismology*, is of recent origin as new space observations of various solar objects allow us to deduce properties of that object, such as its magnetic field strength or the thickness of a layer of inhomogeneity or the size of the coefficient of damping of a wave, properties that may be difficult to ascertain by other means. The success of terrestrial seismology, studying the propagation of waves in the Earth's interior, provides a general motivation, as does the success of helioseismology, the study of the Sun's interior from sound vibrations detected at the solar surface.

I can readily recall my own early involvement in coronal seismology. In the summer of 1982, I was visiting the Astronomy Department in ETH, Zurich, working principally with Professor J. O. Stenflo on photospheric magnetic fields. On one occasion, Dr. Arnold Benz (a well known solar radio astronomer) asked me to join him for lunch and, without preamble, posed to me the question: 'how do you explain a one second oscillation in the solar corona?'. Without hesitation, I replied it must be 'a magnetic wave within a coronal magnetic field', a coronal loop oscillating therein. 'But why such a *short* time as a second?', challenged Benz. 'That must be the *transverse* timescale of the oscillation, the period or timescale being determined by the transverse spatial scale and not the longitudinal one', I responded. Benz then told me, casually, that this was a major problem in coronal physics, with many authors using the length of a loop to determine timescales. Anyway, I agreed to write out some of the details in my response to our lunchtime exchange, dotting the i's and crossing the t's and putting in the π 's too. To guide my work and to explain my initial response to Benz's question, I had in my head the wave diagrams computed by my then research student, Patricia Edwin, who was producing wave diagrams for a variety of solar circumstances. These diagrams were subsequently published in Edwin and Roberts (1982, 1983). When Pat started her research under my direction in St Andrews I had given her the task of adding a magnetic field to the outside of a magnetic structure (a slab), extending my own earlier work on this topic from a field-free environment to a magnetic environment. Also, I had asked her to carry out an investigation in cylindrical coordinates and not the Cartesian coordinates I had used earlier, this being appropriate for a magnetic flux tube. It is amusing to recall that at the time, in 1980, I had suggested this area of investigation partly because Pat was doing her PhD on a part-time basis, and I thought (wrongly, it transpired) that there was no urgency in this area and therefore the topic was entirely suitable for a part-time researcher!

The theory developed in response to Benz's question was subsequently published, first as a short article in the journal *Nature* (in Roberts, Edwin and Benz 1983) and then in a more detailed exposition in *The Astrophysical Journal* (in Roberts, Edwin and Benz 1984). I for one sat back expecting a minor revolution based upon these articles, which put forward the notion of coronal seismology rather fully. But no such development initially occurred, although the two articles were well cited. And there matters may have rested were it not for the launch in space of the *TRACE* instrument in 1998, which shortly thereafter saw directly oscillating loops and was able to measure their characteristics, which fortunately accorded reasonably well with the theories we had developed. Sadly, Patricia Edwin did not live to see the full development of the ideas expressed in our early papers, dying in 2007, though

she must have noted with great interest the surge of activity in this area of our work that followed the launch of *TRACE*. Central to this development were the observations reported independently in 1999 by Dr. Markus Aschwanden (a former student of Benz's) and colleagues in California and Professor Valery Nakariakov (then a postdoctoral researcher in St Andrews working with me) and co-researchers. The *direct* detection of waves in coronal magnetic structures provided an enormous boost to the subject.

In our account here we have laid out in some mathematical detail the nature of magnetohydrodynamic waves in magnetic structures, beginning with the well known case of a uniform medium. Many books have described, in varying degrees of detail, the waves of a uniform medium, but it is important here in setting a benchmark by which the waves of a non-uniform medium may usefully be compared. Thereafter we turn to an examination of the various complexities introduced by *magnetic structuring*, covering the simplest case of a surface wave on a single interface before exploring the waves of a magnetic slab and the cylindrical magnetic flux tube. The waves in these two geometrical objects, the slab and the tube, are closely related in many respects. However, any treatment of a tube involves Bessel functions, a complication avoided in the slab. Thus, a knowledge of the results for a slab can often guide an exploration of a tube. Complications in addition to geometry, such as gravitational stratification, are then explored, before turning to the role of damping and nonlinearity.

The level of mathematical detail we present corresponds to that which we perceive as necessary for anyone trying to understand what lies behind the main results. We view our treatment as hopefully providing a foundation for anyone wishing to work in the field of MHD waves in solar physics. Often the available review literature is content to omit such details making it difficult if not impossible to understand what is being presented. Finally, we end our treatment by presenting a number of solar illustrations which, in one way or another, help illuminate the general theory we have laid out earlier. Each illustration presented is taken from some aspect of solar physics, and represents to some extent a personal attempt to discuss available solar observations in the light of MHD wave theory. It is hoped – and indeed expected – that others will further extend the partial viewpoints that have occurred to me.

It is a pleasure to record here the postgraduate students and post-doctoral researchers who have worked with me over the years. This is always a two way process, with each of us learning something from the other. The postgraduates include (in chronological order) Alec M. Milne, Andrew R. Webb, Patricia M. Edwin, W. Robert Campbell, David J. Evans, Alan J. Miles, Rekha Jain, Partha S. Joarder, Alan Johnston, Patrick Ferguson, Alastair MacDonald, Cheryl A. Mundie, Jason Smith, Mark Daniell, Keith Bennett, David Boddie, Claire Foullon, Lorna James, Gavin R. Donnelly, Michael P. McEwan, and Cicely K. Macnamara. Postdoctoral and other researchers include Iain C. Rae, Sami K. Solanki, Lorna M. Small, Kris Murawski, Valery Nakariakov, Ramon Oliver, Nagendra Kumar, Istvan Ballai, Robertus Erdélyi, Jean Claude Thelen, Temury Zaqarashvili and Toni J. Díaz. I have learnt much from collaborations with colleagues, especially Joe Hollweg, Marty Lee and Terry Forbes (in Durham, New Hampshire), Michael Ruderman and Robertus Erdélyi (in Sheffield), Valery Nakariakov and Erwin Verwichte (in Warwick), Marcel Goossens (in Leuven), and Jose Luis Ballester and Ramon Oliver (in Palma, Mallorca). Colleagues

in St Andrews were always free for discussions, whatever their other commitments, and I especially benefited over many years from talks with Eric Priest, Alan Hood, Andrew Wright and Peter Cargill. A special mention is warranted for Professor Eugene Parker (Chicago), whose style of writing in solar physics has always been an inspiration for me and has helped show me how best to present physical ideas. I must confess to not always being the best of students, but his inspiration was always there.

Finally, last but not least is the continuing inspiration afforded me by my hill walking companions Bob Grundy, Rod Cross and Frank Story, who have kept my mind stimulated through many an interesting discussion or debate, in science, religion, history, politics, economics, or whatever. And the walks were fun too. My sincerest thanks, and long may it continue!

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