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Balakumar Balachandran , Edward B. Magrab
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Vibrations

This new edition explains how vibrations can be used in a broad spectrum of applications and how to meet the challenges faced by engineers and system designers.

The text integrates linear and nonlinear systems and covers the time domain and the frequency domain, responses to harmonic and transient excitations, and discrete and continuous system models. It focuses on modeling, analysis, prediction, and measurement to provide a complete understanding of the underlying physical vibratory phenomena and their relevance for engineering design.

Knowledge is put into practice through numerous examples with real-world applications in a range of disciplines, detailed design guidelines applicable to various vibratory systems, and over 40 online interactive graphics which provide a visual summary of system behaviors and enable students to carry out their own parametric studies. Thirteen new tables act as a quick reference for self-study, detailing key characteristics of physical systems and summarizing important results.

This is an essential text for undergraduate and graduate courses in vibration analysis, and a valuable reference for practicing engineers.

Balakumar Balachandran is a Minta Martin Professor of Engineering at the University of Maryland. He has authored and co-authored many books, chapters, and journal articles related to dynamics and vibrations, and he has several patents to his credit. He is a fellow of the American Society of Mechanical Engineers and the American Institute of Aeronautics and Astronautics.

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Vibrations

Third Edition

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To Malini, Ragini, and Nitin
and
In memory of T. R. Balachandran (1933–2017)

For
June Coleman Magrab
Still my muse after all these years

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EXAMPLES

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INTERACTIVE GRAPHICS

Interactive graphics are available online www.cambridge.org/vibrations

SYMBOLS

a_i, b_i	Fourier series coefficients
a_{1n}, a_{2n}, b_{1n}, b_{2n},	boundary condition parameters for beam
$\boldsymbol{a}$	acceleration vector
$\boldsymbol{a}_G$	absolute acceleration of center of mass of a system
$a(t)$, $a(\tau)$	acceleration
$a_{ss}(t)$	steady-state portion of $a(t)$
c, c_j	damping coefficient, translation motion
c_b	longitudinal speed in beam
c_c	critical damping coefficient
c_d	fluid damping coefficient
c_{eq}	equivalent viscous damping
c_{jn}	damping coefficient
c_r	pulse duration-bandwidth product
c_t	damping coefficient, rotational motion
c_{32}	ratio of damping coefficients, c_3/c_2
$c_i(\Omega_i)$	coefficients in response of a single degree-of-freedom system to periodic forcing
$\boldsymbol{e}_1$, $\boldsymbol{e}_2$	body-fixed unit vectors
$f(t)$, $f(\tau)$	external forcing
$f(x,t)$, $f(\eta,\tau)$	external forcing on beam
f_n	natural frequency, Hz
f_{nc}	non-conservative force per unit length on a beam
g	gravitational constant
h	elevation from ground or thickness of a beam
$\boldsymbol{i}$, $\boldsymbol{j}$, $\boldsymbol{k}$	unit vectors fixed in inertial reference plane
k, k_j, k_s	translation spring constant, spring stiffness
k_e	equivalent spring constant
k_f	stiffness of elastic foundation
k_{jn}	stiffness coefficient
k_t, k_{tj}	torsion spring constant
k_{32}	stiffness ratio, k_3/k_2
m, m_i, M_s	mass of a particle or a rigid body
m_e	equivalent mass
m_o	mass of a beam

m_r	mass ratio, m_2/m_1
$p(x,t)$	axial force on beam
$\boldsymbol{p}$, $\boldsymbol{p}_i$	linear momentum vector
q_j	generalized coordinate
$\dot{q}_j$	generalized velocity
r	radius of gyration of beam cross-section
$\boldsymbol{r}$, $\boldsymbol{r}_i$	position vector
$u(t)$	unit step function
s, s_j	Laplace transform parameter, roots of a polynomial in the parameter s
t	time
t_o	initial time, time delineating a characteristic of a waveform, characteristic time of a beam
t_r	rise time
$\boldsymbol{v}$	velocity vector
$v(t)$, $v(\tau)$	velocity
$v_{ss}(t)$	steady-state portion of $v(t)$
$w(x,t)$, $w(\eta, \tau)$	transverse displacement of beam
$x(0)$, $\dot{x}(0)$, $x_j(0)$, $\dot{x}_j(0)$	initial displacement, initial velocity
$x(t)$, $x(\tau)$, $x_j(t)$	displacement
$x_{\max}$	magnitude of $x(\tau_m)$
x_o	static-equilibrium position
$x_{ss}(t)$	steady-state portion of $x(t)$
$\dot{x}(t)$, $\dot{x}(\tau)$, $\dot{x}_j(t)$	velocity
$\ddot{x}(t)$, $\ddot{x}(\tau)$, $\ddot{x}_j(t)$	acceleration
$y(t)$	displacement of base or nondimensional displacement
$\ddot{y}(t)$	acceleration of base
$z(t)$	relative displacement between mass and base
A	cross-sectional area of beam
A_o	magnitude of displacement due to initial conditions
$[A]$	state matrix
B_j	nondimensional torsion stiffness
B_w	bandwidth of a filter
C_n	beam eigenfunction parameter
$[C]$	damping matrix
D	Rayleigh dissipation function
$D(\Omega)$	denominator of $H(\Omega)$
E	Young's modulus
E_d, E_{diss}	dissipation energy
E_T	total energy in a signal
$E(\omega)$	signal energy as a function of frequency

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$\boldsymbol{F}$, $\boldsymbol{F}_i$	force vector
$\boldsymbol{F}_s$	internal force vector
$F_T(t)$	force transmitted to the base
F_o	magnitude of $f(t)$
$F(x)$	spring force
$F(s)$	Laplace transform of forcing $f(t)$ or $f(\tau)$
G	shear modulus
G_B	beam energy function
G_0, G_L	beam energy function at boundaries
$G_{ij}(j\Omega)$	frequency-response function of inertial element i to force applied to inertial element j
$G(\Omega)$	frequency-response function
$H(\Omega)$	amplitude response function, single degree-of-freedom system
$H_{ij}(\Omega)$	amplitude response function of inertial element i to force applied to inertial element j
$H_{st}(\Omega)$	amplitude response function, system with structural damping
$H_{mb}(\Omega)$	amplitude response function, system with base excitation
$H_{ub}(\Omega)$	amplitude response function, system with rotating unbalanced mass
$\boldsymbol{H}$	angular momentum
I	moment of inertia of beam cross-section about bending axis
J	mass moment of inertia about axis of rotation
J_G	mass moment of inertia about center of mass
J_O	mass moment of inertia about point "O"
J_j	mass moment of inertia of m_j
J_o	mass moment of inertia of m_o
K_f	nondimensional stiffness of elastic foundation
K_j	nondimensional spring stiffness
K_s	stiffness ratio, $k_sL^3/(EI)$
$[K]$	stiffness matrix
L	length of an element
L_T	Lagrangian for a beam
$\boldsymbol{M}$	net moment about a fixed point or the center of mass
M	magnitude of $\boldsymbol{M}$
$M(t)$	external moment
M_{jn}	inertia coefficient
M_j	mass attached to beam boundaries
M_o	mass attached to beam
M_{so}	mass ratio, M_s/m_o
$[M]$	inertia matrix, mass matrix

N_n	square of the norm of the beam eigenfunction
N_d	number of periods to reach τ_d
$\hat{P}$	nondimensional axial force on beam
P_o	percentage overshoot
Q	quality factor
$Q(\phi)$	spatial beam function
Q_i	generalized force
R	reduction in transmissibility
$R(\phi)$	spatial beam function
S, S_j	sensitivity
$S(\phi)$	spatial beam function
T	period of oscillation at frequency ω, kinetic energy
$T(\phi)$	spatial beam function
T_d	period of oscillation at frequency ω_d
TR	transmissibility ratio
T_o, T_1	spring tension
U	potential energy of beam
V	potential energy, shear force on beam
V_o	initial velocity
W	work
$W(x)$, $W(\eta)$	transverse displacement of a beam
$W(0)$, $W'(0)$, $W''(0)$, $W'''(0)$	beam shape function and its derivatives evaluated at $\eta = 0$
$W_n(\eta)$	beam mode shape
$X(s)$	Laplace transform of $x(t)$ or $x(\tau)$
X_j	magnitude of displacement response to harmonic force
X_{ij}	elements of $\{X\}$
X_o	initial displacement
$\{X\}$	displacement column vector
$\{X\}_j$	jth mode shape corresponding to Ω_j
α	nonlinear spring stiffness coefficient, coefficient in proportional damping matrix
α	angular acceleration vector
β	structural damping constant, coefficient in proportional damping matrix
δ	logarithmic decrement
δ_{nm}	Kronecker delta
$\delta(t)$	delta function
δ_{st}	static displacement of a spring
ε	coefficient of restitution, percentage error

ζ	damping ratio, damping factor
ζ_j	modal damping factor
η	nondimensional beam coordinate, x/L
$\{\eta(t)\}, \{\dot{\eta}(t)\}, \{\ddot{\eta}(t)\}$	modal amplitude or displacement, modal velocity, modal acceleration column matrices
$\theta, \ \dot{\theta}, \ \ddot{\theta}$	angular displacement, angular velocity, angular acceleration
$\theta(\Omega)$	phase angle response to harmonic excitation, steady state
$\theta_t(\Omega)$	phase angle response to harmonic excitation, transient
$\theta_{st}(\Omega)$	phase angle response to harmonic excitation, system with structural damping
κ	curvature
λ_j	roots of a polynomial in the parameter λ
μ	coefficient of friction, kinematic viscosity, overlap factor in turning
ρ	mass density
σ	bending stress in a beam
τ	nondimensional time, $\omega_n t$ or $\omega_{n1} t$
τ_d	nondimensional time it takes for a system to decay to a specified level
τ_m	nondimensional time at which $x(\tau)$ is a maximum
τ_r	nondimensional rise time
$\phi, \dot{\phi}, \ddot{\phi}$	angular displacement, angular velocity, angular acceleration
φ	phase angle associated with ζ
φ_d	phase angle associated with initial conditions
$\psi_n(\tau)$	temporal separation of variables function
$\psi(\Omega)$	phase response, system with harmonic base excitation
ω	excitation frequency, rad/s
ω_c	cutoff frequency, rad/s
ω_d	damped natural frequency, rad/s
ω_{dj}	damped natural frequency of jth mode
ω_n	natural frequency of single degree-of-freedom system, rad/s
ω_{nj}	uncoupled natural frequency of jth spring-mass system, rad/s
ω_r	frequency ratio, ω_{n2}/ω_{n1}
ω	angular velocity vector
$[\Phi]$	modal matrix
$\Omega, \Omega_i, \Omega_o$	nondimensional frequency ratio, ω/ω_n or ω/ω_{n1}, ω_i/ω_n, ω_o/ω_n
Ω	nondimensional frequency coefficient for a beam
Ω_c	center frequency ratio of a filter
Ω_{cl}	lower cutoff frequency ratio of a filter

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Ω_{cu}	upper cutoff frequency ratio of a filter
$\Omega_{\max}$	frequency at which $H(\Omega)$ is a maximum
Ω_n	nondimensional natural frequency coefficient for a beam at the nth natural frequency

PREFACE TO THE THIRD EDITION

Vibration is a classical subject whose principles have been known and studied for many centuries and presented in many books. Over the years, the use of these principles to understand and design systems has seen considerable growth in the diversity of systems that are designed with vibrations in mind: mechanical, aerospace, electromechanical and microelectromechanical devices and systems, biomechanical and biomedical systems, ships and submarines, and civil structures. As the performance envelope of an engineered system is pushed to higher limits, nonlinear effects also have to be taken into account.

AIMS OF THE BOOK

This book has been written to enable the use of vibration principles in a broad spectrum of applications and to meet the wide range of challenges faced by system analysts and designers. To this end, the authors have the following goals.

- To provide an introduction to the subject of vibrations for undergraduate students in engineering and the physical sciences.
- To present vibration principles in a general context and to illustrate the use of these principles through carefully chosen examples from different disciplines.
- To use a balanced approach that integrates principles of linear and nonlinear vibrations with modeling, analysis, prediction, and measurement so that physical understanding of the vibratory phenomena and their relevance for engineering design can be emphasized.
- To deduce design guidelines that are applicable to a wide range of vibratory systems.

In writing this book, the authors have used the following guidelines. The material presented should have, to the extent possible, a physical relevance to justify its introduction and development. The examples should be relevant and wide ranging. There should be a natural integration and progression between linear and nonlinear systems, between the time domain and the frequency domain, among the responses of systems to harmonic and transient excitations, and between discrete and continuous system models. There should be a minimum emphasis placed on the discussion of numerical methods and procedures, per se, and instead, advantage should be taken of tools such as Matlab® and Mathematica® for generating the numerical solutions to complement the analytical solutions. In addition, numerical tools should be used in concert with analysis to extend studies on linear systems to include nonlinear elements. Finally, there should be a natural and integrated interplay and presentation between analysis, modeling, measurement, prediction, and design so that a reader does not develop artificial distinctions among them.

NEW MATERIAL

In this third edition, the authors have significantly enhanced the previous editions by creating the following new materials:

- Added numerous examples of model construction
- Introduced two ways in which a linear spring can be used to create zero stiffness
- Expanded the treatment of the Maxwell model
- Increased the number of waveforms analyzed for both periodic and transient excitations
- Doubled the number of vibration absorbers considered
- Presented a general way to determine the natural frequencies and mode shapes of 19 different undamped linear two degree-of-freedom systems and subsequently showed how to use these results to obtain the frequency-response functions for damped systems
- Added several new examples that illustrate novel applications of vibration analysis and design.

REARRANGEMENT OF MATERIAL

We have rearranged some of the material to make it more cohesive and better focused, to allow special cases to be easily considered, and to allow for the results to be more easily generalized. This is especially true for the material covering single degree-of-freedom systems subject to harmonic excitation and to transient excitation, to systems with two degrees of freedom, and to beams.

TABLES

We have created 13 new tables in which we have collected, summarized, and in many cases extended, the results appearing in the main body of the text. The material presented in the tables has been organized so that the similarity in the vibration model features, system response, or other characteristics of seemingly different physical systems can be emphasized. These tables have many purposes: they are used to summarize the important results, they serve as a reference source and as a study guide, they extend basic results, and, in some cases, they can be used to create exercise problems.

INTERACTIVE GRAPHICS

We have created over 40 real-time interactive graphics that are keyed to the book's text and figures. The interactive graphics require no programming experience, only the use of a mouse or touch pad. (It does require one to download a free program from Wolfram.) Additionally, appearing in appropriate places in the text are guidelines that direct the reader on what to note in each and the major conclusions that can be reached through the use of an interactive graphic. The interactive graphics materials are intuitive to use and self-explanatory. All figures are enhanced using different colors, line types and labels and in many of them the authors display numerical values of special quantities of interest such as a maximum/minimum value or an optimum value. There are numerous advantages and benefits of these interactive graphics materials, including the following: they are easy to use since no knowledge of a programming language is required; real-time parametric investigations can be conducted and "what-if" scenarios can be explored while making comparisons with special cases; they complement textbook material where graphics can only provide specific instances of the results; and they can be used to enhance homework assignments by asking questions the answers to which can help further the understanding of the material.

With the inclusion of the interactive graphics, the analytical and numerical results are now extensively complemented with the ability to easily visualize and explore them. These interactive graphics also allow the student to verify the design guidelines that appear throughout the book and to realize that the interactive graphics can be used as a design aid for applications beyond the classroom. By conducting parametric studies with the aid of the interactive graphics, a reader can also get a better appreciation for the range of vibratory behaviors possible. Many of these interactive graphics can be used to reveal interesting phenomena, which the authors believe will help further a reader's understanding of vibrations.

FEATURES RETAINED FROM THE PREVIOUS EDITION

In addition to the new enhancements mentioned above, this book retains the following features.

- **Newton's laws and Lagrange's equations** are used to develop models of systems. Since an important part of this development requires kinematics, kinematics is reviewed in Appendix A.
- **Laplace transforms** are used to develop analytical solutions for linear vibratory systems and, from the Laplace domain, extend these results to the frequency domain. The responses of these systems are discussed in both the time and frequency domains to emphasize their duality.

- **Notions of transfer functions and frequency-response functions** also are used throughout the book to help the reader develop a comprehensive picture of vibratory systems.
- **Design for vibration (DFV) guidelines** are introduced and are based on vibration principles developed throughout the book. The guidelines appear at the appropriate places in each chapter. These design guidelines serve the additional function of summarizing the preceding material by encapsulating the most important elements as they relate to some aspect of vibration design.
- **Introduction** to each chapter provides a discussion on what specifically will be covered in that chapter.
- **Examples** have been chosen so that they are of different levels of complexity, cover a wide range of vibration topics and, in most cases, have practical applications to real-world problems.
- **Exercises** have been organized to correlate with the most appropriate section of the text.
- **Appendices** are included on the following:
  - Preliminaries from dynamics
  - Laplace transform pairs
  - Solution methods to second-order ordinary differential equations
  - Matrices
  - Complex numbers and variables
  - State-space formulation
  - Natural frequencies and mode shapes of bar, shafts, and strings
  - Derivation details related to beam vibrations.
- **A glossary** is included to list in one place the definitions of the major terms used in the book.

CONTENTS AND ORGANIZATION

The book is organized into nine chapters, with the topics covered ranging from pendulum systems and spring-mass-damper prototypes to beams. In the first chapter, a brief introduction to the subject of vibrations is provided, related history is reviewed, and examples of scenarios where this subject is relevant are provided.

In the second chapter, the inertia, stiffness, and damping elements that are used to construct a vibratory system model are introduced, the notion of equivalent spring stiffness is presented in different physical contexts, the modeling of nonlinear springs is addressed, damping models are discussed, and many examples of modeling physical systems are shown. In Chapter 3, the equation governing a single degree-of-freedom vibratory system is derived by using the principles of linear momentum balance and angular momentum balance and the Lagrange equations. The notions of natural frequency and damping factor are introduced and mass excitation, base excitation, and unbalanced mass excitation are examined. The linearization of governing equations for nonlinear systems is also discussed. In Chapter 4, the responses of linear single degree-of-freedom systems to initial conditions are examined for the Kelvin–Voigt material and for a Maxwell material, and the effects of nonlinear springs and damping are determined. In addition, response stability, nonlinear springs, and nonlinear dampers are discussed.

In Chapter 5, the responses of single degree-of-freedom systems subjected to periodic excitations are considered and the notions of resonance, frequency-response functions, and transfer functions are introduced. The relation between the information in the time domain and the frequency domain is examined in detail. The concepts used for vibration isolation and accelerometers are presented and the notion of equivalent damping is introduced. Alternative forms of the frequency-response function are discussed. The forced response of a nonlinear oscillator is also treated. In Chapter 6, the responses of single degree-of-freedom systems to different types of transient excitations are analyzed in terms of their frequency spectra relative to the amplitude response function of the system. The notion of rise time, overshoot, and settling time are presented. The transient response of a nonlinear oscillator is also examined.

Multiple degree-of-freedom systems are treated in Chapters 7 and 8 leading up to continuous systems in Chapter 9. In Chapter 7, the derivation of governing equations of motion of a system with multiple degrees of freedom is addressed by using the principles of linear momentum balance and angular momentum balance and Lagrange's equations. The natural frequencies and mode shapes of undamped systems are studied and the notion of a vibratory mode is explained. The linearization of governing system for nonlinear systems is treated and the stability and vibrations of rotating shafts on flexible mounts is presented in detail.

In Chapter 8, the general solution for the responses of systems with two degrees of freedom subjected to initial conditions and arbitrary forcing is presented by using the normal-mode approach. The limitation of this approach with regard to the type of damping that can be considered is addressed. The notions of resonance, frequency-response functions, and transfer functions for a multiple degree-of-freedom system are discussed with respect to their application for system identification and for the design of vibration absorbers and for vibration isolation. The vibration-absorber material includes the traditional treatment of linear vibration absorbers and a brief introduction to the design of several distinctly different types of nonlinear vibration absorbers, which include bar-slider systems, pendulum absorbers, and particle-impact dampers. Techniques that can be used to determine optimal choice of absorber and isolator parameters are also presented.

In Chapter 9, the free and forced oscillations of thin elastic beams are treated for a large number of boundary conditions, in-span attachments, and beam geometry. Considerable attention is paid to the determination of natural frequencies and mode shapes for these configurations, which include effects of axial forces and an elastic foundation. In this chapter, the power of the Laplace transform approach to solve the beam response for complex boundary conditions and in-span attachments becomes apparent. In addition, an appendix on the natural frequencies and mode shapes associated with the free oscillations of strings, bars, and shafts, each for various combinations of boundary conditions including an attached mass and an attached spring is included.

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