1 Introduction

1.1 INTRODUCTION

Vibrations are oscillations about an equilibrium position. They occur in many aspects of our everyday experiences. They are an integral part of human life: low-frequency oscillations of the lungs and the heart; high-frequency oscillations of the ear; oscillations of the larynx to create speech; and oscillations induced by rhythmical body motions such as walking, jumping, and dancing.

Sometimes vibrations are used for our benefit in such devices as loudspeakers, vibratory feeders, paint mixers, electrodynamic shakers, massage chairs and beds, plate compactors, ultrasonic devices used in non-invasive diagnostics, sirens and alarms for warnings, and stimulation of bone growth. Illustrations of these types of devices are shown in Figure 1.1. Other beneficial uses of vibrations include atomic clocks that are based on atomic vibrations and ultrasonic instrumentation used in eye and other types of surgeries.

There are situations in which vibrations are unwanted and can be due to manmade or natural sources. Examples of unwanted manmade disturbances are vehicular traffic, construction site machinery, factory metal forming machines, aircraft and helicopter fly-overs, railroads, rotating machinery, wind farms, and electric transformers. Examples of these types of sources are shown in Figure 1.2. Examples of disturbances from natural phenomena are earthquakes, thunder, ocean waves, and wind.

Vibrations are also undesirable when performing measurements with precision instruments such as an electron microscope and when fabricating microelectromechanical systems. In vehicle design, noise due to vibrating panels needs to be reduced. However, vibrations that can be responsible for unpleasant sounds, which is called noise, are also responsible for the music that we hear.

Frequently, systems must be designed to withstand vibration environments. Examples include buildings designed to withstand earthquakes and wind; offshore drilling platforms

Figure 1.1. Examples of the beneficial uses of vibrations: (a) paint mixer, (b) electrodynamic shaker, (c) massage chair, (d) loudspeaker, and (e) parts feeder.
Source: Photo of paint mixer courtesy of Radia, Plymouth MN www.radiaproducts.com/products/paint-mixers-shakers/; Photo of electrodynamic shaker courtesy of Bruel & Kjaer Sound and Vibration Measurement A/S; Photo of massage chair courtesy of Titan World LLC/Titan World LLC; Photo of loudspeaker courtesy of Mitek Audio, Mitek USA; Photo of parts feeder by Richdsu https://commons.wikimedia.org/wiki/File:Bowl_Feeder.jpg and reprinted under Creative Commons Attribution 3.0.

to withstand ocean waves; chimneys and bridges to withstand wind; aircraft, helicopters, ships, and railroad cars to withstand their internally generated disturbances; machine tools; washing machines; and fragile packages sent from one destination to another. Examples of these types of systems are shown in Figure 1.3.

Some systems are designed specifically to reduce or minimize the effects of a particular type of vibratory disturbance such as automobile suspension systems, helicopter rotors, submarine stealth, vibration isolation tables, and disc drives. Examples of these types of systems are shown in Figure 1.4.

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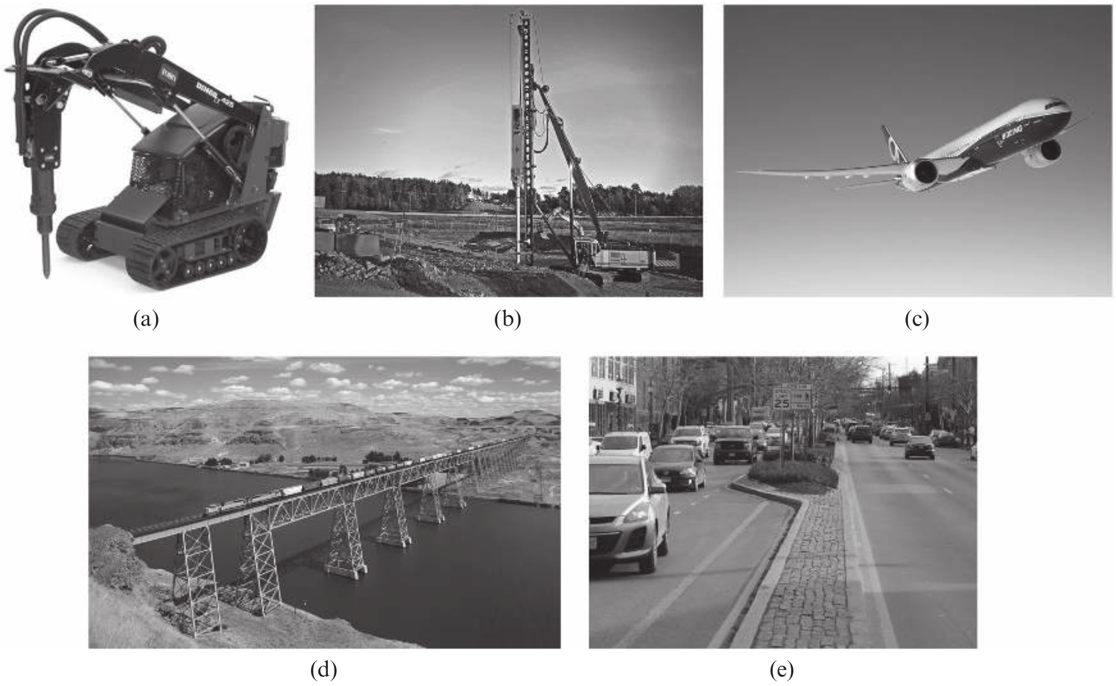

Figure 1.2. Examples of unwanted manmade environmental vibratory disturbances: (a) hydraulic breaker (jackhammer), (b) pile driver, (c) airplane, (d) train, and (e) vehicular traffic.
Source: Photo of hydraulic breaker courtesy of The Toro Company; Photo of pile driver courtesy of Hammer & Steel Inc. Hazelwood, MO; Photo of Boeing 777X-9 ascending copyright © Boeing and reprinted with permission of Boeing/Boeing Images; Photo of train by David Gubler from https://commons.wikimedia.org/wiki/File:UP_EMD_SD9043AC_Joso_Bridge,_USA.jpg and reprinted under Creative Commons Attribution 3.0.

Vibrations manifest themselves in many different ways. Temporally, they can occur intermittently, continuously, or randomly, and physically they can manifest themselves as a displacement, an unbalanced force or moment, an acoustic wave, or a pressure wave.

Based on the discussion so far, one can roughly place the design of devices and systems in cases in which some aspect of vibrations is involved into one of three categories. These categories are: (i) cases in which one seeks to control vibrations and puts them to beneficial use; (ii) systems whose oscillatory motions must be prevented or minimized from impacting itself, another system, or its environment; and (iii) systems that must be able to withstand a vibratory environment and perform as intended.

When it is known that a system will have to operate in an environment that will subject the system to vibrations or that the system itself will be a source of vibrations, one must consider these vibrations in the system's design stage. If this is not done, one may end up with systems that

- Experience catastrophic failure
- Undergo excessive wear

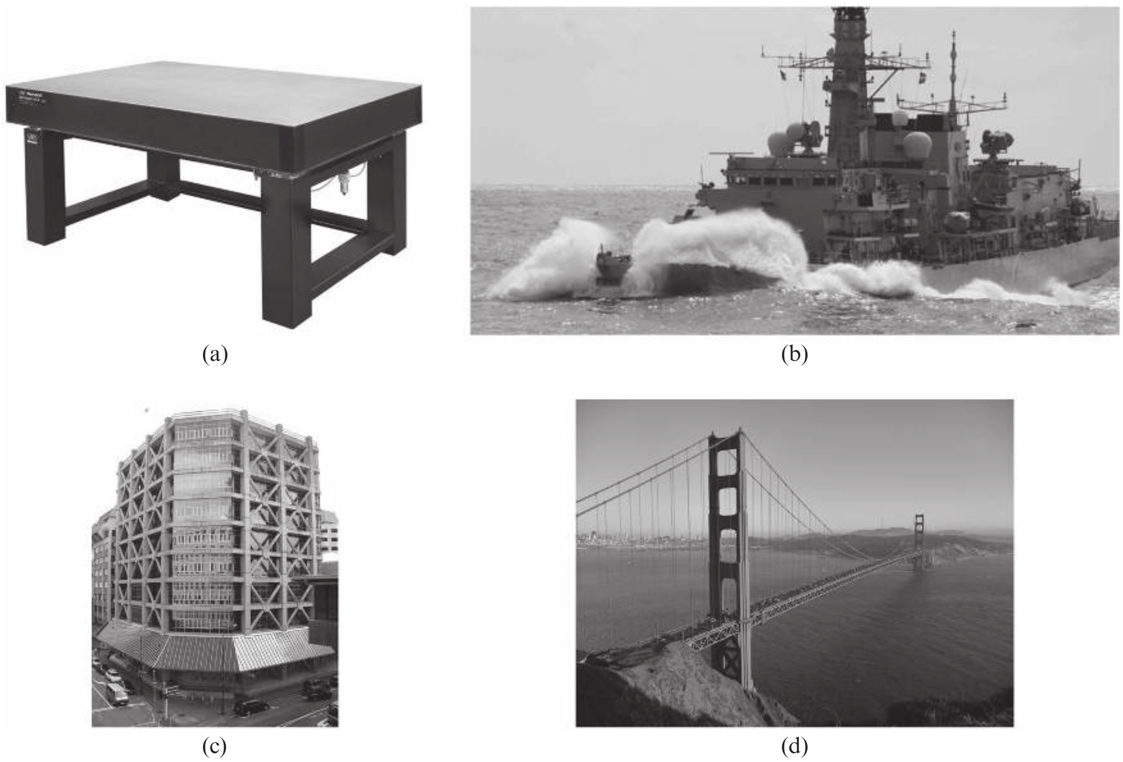

Figure 1.3. Examples of systems designed to withstand vibration environments: (a) isolation table, (b) ship, (c) earthquake resistant building, and (d) suspension bridge.
Source: Photo of isolation table, permission to use granted by Newport Corporation, all rights reserved; Photo of ship used under the UK Open Government License v3.0; Photo of earthquake resistant building by Alastair McLean, courtesy of Te Ara, The Encyclopedia of New Zealand; Photo of suspension bridge by Octagon in https://commons.wikimedia.org/wiki/File:Golden_Gate_Bridge_.JPG and reprinted under ShareAlike 3.0 Unported.

- Experience excessive displacements and stresses
- Are hard to control
- Produce unacceptable disturbances to the surroundings, both vibratory and acoustic
- Waste energy.

On the other hand, systems that are designed with vibrations in mind tend to

- Operate as intended
- Undergo minimum wear
- Have long life
- Are controllable
- Interfere minimally with their environment
- Perform useful tasks
- Operate with high mechanical efficiency.

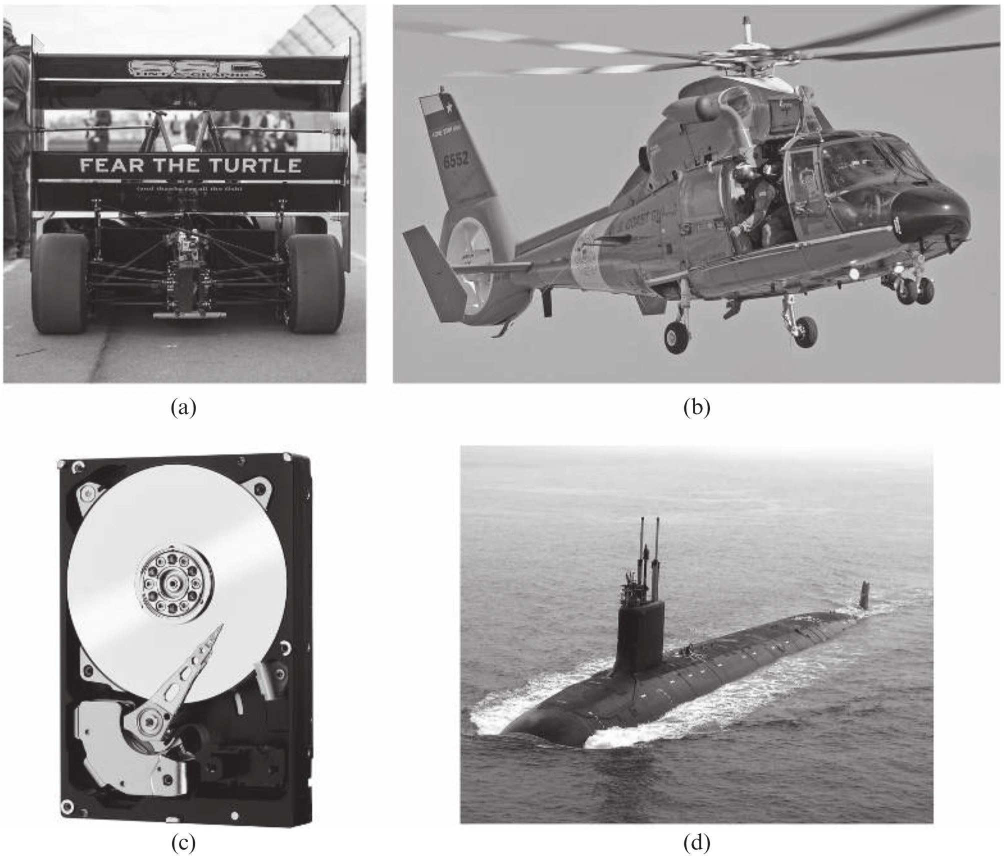

Figure 1.4. Examples of systems designed specifically to reduce the effects of a particular type of vibratory disturbance: (a) automobile suspension system, (b) helicopter rotors, (c) disc drive, and (d) submarine silencing.
Source: Photo of racing car courtesy of Terps Racing, University of Maryland, College Park, MD; Photo of helicopter rotors, US Coast Guard photo by Adam Eggers; Photo of disc drive courtesy of Western Digital; Photo of submarine US Navy photo.

Additionally, when undesirable vibratory motions are reduced

- Human comfort is increased
- Humans and machinery operate more efficiently and safely
- Systems tend to be more reliable and predictable
- There is a decrease in material fatigue and in human fatigue.

Vibrations, a Subset of Dynamics
As mentioned earlier, vibrations involve oscillatory motions that occur about an equilibrium position of a system. As such, vibrations are considered a subset of the subject area called dynamics, which covers all types of motions. Preliminaries of dynamics are covered in Appendix A. These preliminaries are needed for the determination of the displacement,

velocity, and acceleration of a mass element; the number of degrees of freedom of a system; the kinetic energy and work of a system; and the governing equations of motion.

1.2 A BRIEF HISTORY OF VIBRATIONS

It is likely that the early interest in vibrations was due to the development of musical instruments such as whistles and drums. As early as 4000 BCE, it is believed that in India and China there was an interest in understanding music, which is described as a pulsating effect due to rapid change in pitch. The origin of the harmonica can be traced back to 3000 BCE, when in China a bamboo reed instrument called a *sheng* was introduced. From archeological studies of the royal tombs in Egypt, it is known that stringed instruments have also been around from about 3000 BCE. A first scientific study into such instruments is attributed to the Greek philosopher and mathematician Pythagoras (582–507 BCE). He showed that if two like strings are subjected to equal tension, and if one is half the length of the other, the tones they produce are an octave (a factor of two) apart. It is interesting to note that although music is considered a highly subjective and personal art, it is closely governed by vibration principles such as those determined by Pythagoras and others who followed him.

The vibrating string was also studied by Galileo Galilei (1564–1642), who was the first to show that pitch is related to the frequency of vibration. Galileo also laid the foundations for studies of vibrating systems through his observations made in 1583 regarding the motions of a lamp hanging from a cathedral in Pisa, Italy. He found that the period of motion was independent of the amplitude of the swing of the lamp. This property holds for all vibratory systems that can be described by linear models. The pendulum system studied by Galileo has been used as a paradigm to illustrate the principles of vibrations for many centuries. Galileo and many others who followed him laid the foundations for vibrations, which is a discipline that is generally grouped under the umbrella of mechanics. A brief summary of some of the major contributors and their contributions is provided in Table 1.1. The biographies of many of the individuals listed in this table can be found in the *Dictionary of Scientific Biography*.[1] It is interesting to note from Table 1.1 that the early interest of the investigators was in pendulum and string vibrations, followed by a phase where the focus was on membrane, plate, and shell vibrations, and a subsequent phase in which vibrations in practical problems and nonlinear oscillations received considerable attention.

Lord Rayleigh's book *The Theory of Sound*,[2] which was first published in 1877, is one of the early comprehensive publications on vibrations. In fact, many of the mathematical developments that are commonly taught in a vibrations course can be traced back to the 1800s and before. However, since then, the use of these principles to understand and

[1] C. C. Gillispie, editor, *Dictionary of Scientific Biography*, 18 Volumes, Scribner, New York, 1970–1990.
[2] Rayleigh, J.W. Strutt, Lord, *The Theory of Sound*, 2 Volumes, Macmillan, London, 1877, 1878.

Table 1.1. **Major contributors to the field of vibrations and their contributions**

Contributor	Areas of contributions
Galileo Galilei (1564–1642)	Pendulum frequency measurement, vibrating string
Marin Mersenne (1588–1648)	Vibrating string
John Wallis (1616–1703)	String vibration: observations of modes and harmonics
Christian Huygens (1629–1695)	Nonlinear oscillations of pendulum
Robert Hooke (1635–1703)	Pitch–frequency relationship; Hooke's law of elasticity
Isaac Newton (1642–1727)	Laws of motion; calculus
Gottfried Leibnitz (1646–1716)	Calculus
Joseph Sauveur (1653–1716)	String vibration: coined the name "fundamental harmonic" for lowest frequency and "harmonics" for higher frequency components
Brook Taylor (1685–1731)	Vibrating string frequency computation; Taylor's theorem
Daniel Bernoulli (1700–1782)	Principle of liner superposition of harmonics; string and beam vibrations
Leonhard Euler (1707–1783)	Angular momentum principle; complex numbers; Euler's equations; beam, plate, and shell vibrations
Jean d'Alembert (1717–1783)	d'Alembert's principle; equations of motion; wave equation
Charles Coulomb (1736–1806)	Torsional vibrations; friction
Joseph Lagrange (1736–1813)	Lagrange's equations; frequencies of open and closed organ pipes
E. F. F. Chladni (1756–1824)	Plate vibrations: nodal lines
Jacob Bernoulli (1759–1789)	Beam, plate, and shell vibrations
J. B. J. Fourier (1768–1830)	Fourier series
Sophie Germain (1776–1831)	Equations governing plate vibrations
Simeon Poisson (1781–1840)	Plate, membrane, and rod vibrations; Poisson's effect
G. R. Kirchhoff (1824–1887)	Plate and membrane vibrations
R. F. A. Clebsh (1833–1872)	Vibrations of elastic media
Lord Rayleigh (1842–1919)	Energy methods: Rayleigh's method; Strutt diagram; vibration treatise
C. G. P. De Laval (1845–1913)	Vibrations of unbalanced rotating disc: practical solutions
Gaston Floquet (1847–1920)	Stability of periodic oscillations: Floquet theory
Henri Poincaré (1854–1912)	Nonlinear oscillations; Poincaré map; stability; chaos
A. M. Liapunov (1857–1918)	Stability of equilibrium
Aurel Stodola (1859–1943)	Beam, plate, and membrane vibrations; turbine blades
Balthasar van der Pol (1889–1959)	Nonlinear oscillations: van der Pol oscillator
Jacob Pieter Den Hartog (1901–1989)	Nonlinear systems with Coulomb damping; vibration of rotating and reciprocating machinery; vibration textbook

design systems has seen considerable growth in the diversity of systems that are designed with vibrations in mind: mechanical, electromechanical and microelectromechanical devices and systems, biomechanical and biomedical systems, ships and submarines, and civil structures.

1.3 ABOUT THIS BOOK

Vibration analysts and designers face a broad spectrum of applications and challenges. To meet these challenges, we have done the following with the material in this book: (1) we present vibration principles in a general context and illustrate their use through carefully chosen examples from different disciplines; (2) we use an approach that integrates principles of linear and nonlinear vibrations with modeling, analysis, prediction, and measurement so that physical understanding of the vibratory phenomena and their relevance for engineering design can be emphasized; (3) we deduce design guidelines that are applicable to a wide range of vibratory systems; (4) we present tables summarizing major results and include additional results for reference; and (5) we make available interactive graphics that provide an environment from which the reader can explore and analyze in real time all major results.

Topics

The major topics covered in the remaining eight chapters are as follows

- In Chapter 2, the inertia, stiffness, and damping elements that are used to construct a vibratory system model are introduced, the notion of equivalent spring stiffness is presented in different physical contexts, the modeling of nonlinear springs is addressed, and many examples of modeling physical systems are shown.
- In Chapter 3, the equation governing a single degree-of-freedom vibratory system is derived using the principles of linear momentum balance and angular momentum balance and the Lagrange equations. The notions of natural frequency and damping factor are introduced and mass excitation, base excitation, and unbalanced mass excitation are examined. The linearization of governing equations for nonlinear systems is also discussed.
- In Chapter 4, the responses of linear single degree-of-freedom systems to initial conditions are examined for the Kelvin–Voigt material and for a Maxwell material and the effects of nonlinear springs and damping are determined. In addition, stability, nonlinear springs, and nonlinear dampers are covered.
- In Chapter 5, the responses of single degree-of-freedom systems subjected to periodic excitations are considered and the notions of resonance, frequency-response functions, and transfer functions are introduced. The relation between the information in the time domain and the frequency domain is examined in detail. The concepts used for

vibration isolation and accelerometers are presented and the notion of equivalent damping is introduced. The forced response of a nonlinear oscillator is also treated.
- In Chapter 6, the responses of single degree-of-freedom systems to different types of transient excitations are analyzed in terms of their frequency spectra relative to the amplitude response function of the system. The notions of rise time, overshoot, and settling time are presented. The transient response of a nonlinear oscillator is examined.
- In Chapter 7, the derivation of governing equations of motion of a system with multiple degrees of freedom is addressed by using the principles of linear momentum balance and angular momentum balance and Lagrange's equations. The natural frequencies and mode shapes of undamped systems are studied and the notion of a vibratory mode is explained. The linearization of governing system for nonlinear systems is treated and stability is addressed.
- In Chapter 8, the general solution to a two degree-of-freedom system subjected to initial conditions and arbitrary forcing is presented using the normal-mode approach. The limitation of this approach with regard to the type of damping that can be considered is addressed. The frequency-response function for a general two degree-of-freedom system is discussed in detail. The notion of a vibration absorber is presented for several distinctly different types of systems. The responses of nonlinear systems are also considered.
- In Chapter 9, the free and forced oscillations of thin elastic beams are treated for a large number of boundary conditions, in-span attachments, and beam geometry. Considerable attention is paid to the determination of natural frequencies and mode shapes for these configurations.

Interactive Graphics

In subsequent chapters all major aspects of the material are supplemented with real-time interactive graphics, which are available on the publisher's website. The interactive graphics require no programming experience, only that the reader download a free program from Wolfram. Appearing in appropriate places in the text are guidelines that direct the reader on what to note in each interactive graphic and the major conclusions that can be reached from the use of that interactive graphic.

These interactive graphics are introduced at the appropriate places in the respective chapters and have been created with the following aims.

- They complement and augment the material in the text and enhance understanding of the topics.
- They provide real-time ability to perform parametric investigations and explore "what-if" scenarios: typically, a wide range of parameters and configuration combinations can be explored, with comparisons made for special cases.
- They are intuitive to use and self-explanatory in the context of the material that they are illustrating.

- When appropriate, they provide numerical values for important quantities of interest such as the maximum or minimum or optimum.

With regard to the interactive graphics, the reader is encouraged to explore Interactive Graphic 1.1. This interactive graphic is used to illustrate the fundamental aspects of the response of a single degree-of-freedom vibratory system for different amounts of damping and for different types of excitation to the mass: initial displacement, impact force, and harmonically varying force.

INTERACTIVE GRAPHIC 1.1: NORMALIZED RESPONSE OF A SPRING-MASS-DAMPER SYSTEM TO VARIOUS EXCITATIONS AND DAMPING

Summary Tables

Numerous summary tables have been created to

- Summarize the important results
- Serve as a reference source
- Serve as a study guide
- Extend basic results
- Serve, in some cases, as exercises
- Organize certain material to illustrate and emphasize the similarity in the vibration model's features, the system response, or other characteristic of different physical systems.