

## Contents

	<i>Preface</i>	<i>page xi</i>
<b>1</b>	<b>Monoidal and Braided Categories</b>	<b>1</b>
	1.1 Monoidal Categories	1
	1.2 Examples of Monoidal Categories	7
	1.2.1 The Category of Sets	7
	1.2.2 The Category of Vector Spaces	7
	1.2.3 The Category of Bimodules	7
	1.2.4 The Category of $G$ -graded Vector Spaces	8
	1.2.5 The Category of Endo-functors	13
	1.2.6 A Strict Category Associated to a Monoidal Category	15
	1.3 Monoidal Functors	16
	1.4 Mac Lane's Strictification Theorem for Monoidal Categories	25
	1.5 (Pre-)Braided Monoidal Categories	28
	1.6 Rigid Monoidal Categories	38
	1.7 The Left and Right Dual Functors	43
	1.8 Braided Rigid Monoidal Categories	48
	1.9 Notes	54
<b>2</b>	<b>Algebras and Coalgebras in Monoidal Categories</b>	<b>55</b>
	2.1 Algebras in Monoidal Categories	55
	2.2 Coalgebras in Monoidal Categories	65
	2.3 The Dual Coalgebra/Algebra of an Algebra/Coalgebra	70
	2.4 Categories of Representations	78
	2.5 Categories of Corepresentations	82
	2.6 Braided Bialgebras	87
	2.7 Braided Hopf Algebras	95
	2.8 Notes	101
<b>3</b>	<b>Quasi-bialgebras and Quasi-Hopf Algebras</b>	<b>103</b>
	3.1 Quasi-bialgebras	103
	3.2 Quasi-Hopf Algebras	110
	3.3 Examples of Quasi-bialgebras and Quasi-Hopf Algebras	119

viii	<i>Contents</i>	
	3.4 The Rigid Monoidal Structure of ${}_H\mathcal{M}^{\text{fd}}$ and $\mathcal{M}_H^{\text{fd}}$	125
	3.5 The Reconstruction Theorem for Quasi-Hopf Algebras	128
	3.6 Sovereign Quasi-Hopf Algebras	131
	3.7 Dual Quasi-Hopf Algebras	135
	3.8 Further Examples of (Dual) Quasi-Hopf Algebras	141
	3.9 Notes	146
<b>4</b>	<b>Module (Co)Algebras and (Bi)Comodule Algebras</b>	147
	4.1 Module Algebras over Quasi-bialgebras	147
	4.2 Module Coalgebras over Quasi-bialgebras	154
	4.3 Comodule Algebras over Quasi-bialgebras	162
	4.4 Bicomodule Algebras and Two-sided Coactions	168
	4.5 Notes	176
<b>5</b>	<b>Crossed Products</b>	177
	5.1 Smash Products	177
	5.2 Quasi-smash Products and Generalized Smash Products	185
	5.3 Endomorphism $H$ -module Algebras	188
	5.4 Two-sided Smash and Crossed Products	191
	5.5 $H^*$ -Hopf Bimodules	196
	5.6 Diagonal Crossed Products	201
	5.7 L–R-smash Products	214
	5.8 A Duality Theorem for Quasi-Hopf Algebras	220
	5.9 Notes	223
<b>6</b>	<b>Quasi-Hopf Bimodule Categories</b>	225
	6.1 Quasi-Hopf Bimodules	225
	6.2 The Dual of a Quasi-Hopf Bimodule	230
	6.3 Structure Theorems for Quasi-Hopf Bimodules	235
	6.4 The Categories ${}_H\mathcal{M}_H^H$ and ${}_H\mathcal{M}$	239
	6.5 A Structure Theorem for Comodule Algebras	246
	6.6 Coalgebras in ${}_H\mathcal{M}_H^H$	249
	6.7 Notes	251
<b>7</b>	<b>Finite-Dimensional Quasi-Hopf Algebras</b>	253
	7.1 Frobenius Algebras	253
	7.2 Integral Theory	261
	7.3 Semisimple Quasi-Hopf Algebras	268
	7.4 Symmetric Quasi-Hopf Algebras	273
	7.5 Cointegral Theory	279
	7.6 Integrals, Cointegrals and the Fourth Power of the Antipode	288
	7.7 A Freeness Theorem for Quasi-Hopf Algebras	299
	7.8 Notes	303
<b>8</b>	<b>Yetter–Drinfeld Module Categories</b>	305
	8.1 The Left and Right Center Constructions	305

<i>Contents</i>		ix
8.2	Yetter–Drinfeld Modules over Quasi-bialgebras	310
8.3	The Rigid Braided Category ${}^H_H\mathcal{YD}^{\text{fd}}$	318
8.4	Yetter–Drinfeld Modules as Modules over an Algebra	325
8.5	The Quantum Double of a Quasi-Hopf Algebra	330
8.6	The Quasi-Hopf Algebras $D^\omega(H)$ and $D^\omega(G)$	335
8.7	Algebras within Categories of Yetter–Drinfeld Modules	342
8.8	Cross Products of Algebras in ${}^H\mathcal{M}$ , ${}^H\mathcal{M}_H$ , ${}^H_H\mathcal{YD}$	347
8.9	Notes	351
<b>9</b>	<b>Two-sided Two-cosided Hopf Modules</b>	<b>353</b>
9.1	Two-sided Two-cosided Hopf Modules	353
9.2	Two-sided Two-cosided Hopf Modules versus Yetter–Drinfeld Modules	355
9.3	The Categories ${}^H_H\mathcal{M}_H^H$ and ${}^H_H\mathcal{YD}$	360
9.4	A Structure Theorem for Bicomodule Algebras	362
9.5	The Structure of a Coalgebra in ${}^H_H\mathcal{M}_H^H$	363
9.6	A Braided Monoidal Structure on ${}^H_H\mathcal{M}_H^H$	369
9.7	Hopf Algebras within ${}^H_H\mathcal{M}_H^H$	371
9.8	Biproduct Quasi-Hopf Algebras	376
9.9	Notes	379
<b>10</b>	<b>Quasitriangular Quasi-Hopf Algebras</b>	<b>381</b>
10.1	Quasitriangular Quasi-bialgebras and Quasi-Hopf Algebras	381
10.2	Further Examples of Monoidal Algebras	386
10.3	The Square of the Antipode of a QT Quasi-Hopf Algebra	388
10.4	The QT Structure of the Quantum Double	394
10.5	The Quantum Double $D(H)$ when $H$ is Quasitriangular	400
10.6	Notes	406
<b>11</b>	<b>Factorizable Quasi-Hopf Algebras</b>	<b>407</b>
11.1	Reconstruction in Rigid Monoidal Categories	407
11.2	The Enveloping Braided Group of a QT Quasi-Hopf Algebra	414
11.3	Bosonisation for Quasi-Hopf Algebras	419
11.4	The Function Algebra Braided Group	421
11.5	Factorizable QT Quasi-Hopf Algebras	433
11.6	Factorizable Implies Unimodular	440
11.7	The Quantum Double of a Factorizable Quasi-Hopf Algebra	443
11.8	Notes	450
<b>12</b>	<b>The Quantum Dimension and Involutionary Quasi-Hopf Algebras</b>	<b>451</b>
12.1	The Integrals of a Quantum Double	451
12.2	The Cointegrals of a Quantum Double	457
12.3	The Quantum Dimension	462
12.3.1	The Quantum Dimension of $H$	462
12.3.2	The Quantum Dimension of $D(H)$	466
12.4	The Trace Formula for Quasi-Hopf Algebras	469

x	<i>Contents</i>	
	12.5 Involutory Quasi-Hopf Algebras	472
	12.6 Representations of Involutory Quasi-Hopf Algebras	474
	12.7 Notes	479
<b>13</b>	<b>Ribbon Quasi-Hopf Algebras</b>	481
	13.1 Ribbon Categories	481
	13.2 Ribbon Categories Obtained from Rigid Monoidal Categories	488
	13.3 Ribbon Quasi-Hopf Algebras	496
	13.4 A Class of Ribbon Quasi-Hopf Algebras	505
	13.5 Some Ribbon Elements for $D^\omega(H)$ and $D^\omega(G)$	508
	13.6 Notes	512
	<i>Bibliography</i>	515
	<i>Index</i>	525