

Introduction to the series by the editors

1

1

Approaches to learning and teaching Mathematics

This series of books is the result of close collaboration between Cambridge University Press and Cambridge International, both departments of the University of Cambridge. The books are intended as a companion guide for teachers, to supplement your learning and provide you with extra resources for the lessons you are planning. Their focus is deliberately not syllabus-specific, although occasional reference has been made to programmes and qualifications. We want to invite you to set aside for a while assessment objectives and grading, and take the opportunity instead to look in more depth at how you teach your subject and how you motivate and engage with your students.

The themes presented in these books are informed by evidence-based research into what works to improve students' learning and pedagogical best practices. To ensure that these books are first and foremost practical resources, we have chosen not to include too many academic references, but we have provided some suggestions for further reading.

We have further enhanced the books by asking the authors to create accompanying lesson ideas. These are described in the text and can be found in a dedicated space online. We hope the books will become a dynamic and valid representation of what is happening now in learning and teaching in the context in which you work.

Our organisations also offer a wide range of professional development opportunities for teachers. These range from syllabus- and topic-specific workshops and large-scale conferences to suites of accredited qualifications for teachers and school leaders. Our aim is to provide you with valuable support, to build communities and networks, and to help you both enrich your own teaching methodology and evaluate its impact on your students.

Each of the books in this series follows a similar structure. In the first chapter, we have asked our authors to consider the essential elements of their subject, the main concepts that might be covered in a school curriculum, and why these are important. The next chapter gives you a brief guide on how to interpret a syllabus or subject guide, and how to plan a programme of study. The authors will encourage you to think too about what is not contained in a syllabus and how you can pass on your own passion for the subject you teach.

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The main body of the text takes you through those aspects of learning and teaching which are widely recognised as important. We would like to stress that there is no single recipe for excellent teaching, and that different schools, operating in different countries and cultures, will have strong traditions that should be respected. There is a growing consensus, however, about some important practices and approaches that need to be adopted if students are going to fulfil their potential and be prepared for modern life.

In the common introduction to each of these chapters we look at what the research says and the benefits and challenges of particular approaches. Each author then focuses on how to translate theory into practice in the context of their subject, offering practical lesson ideas and teacher tips. These chapters are not mutually exclusive but can be read independently of each other and in whichever order suits you best. They form a coherent whole but are presented in such a way that you can dip into the book when and where it is most convenient for you to do so.

The final two chapters are common to all the books in this series and are not written by the subject authors. Schools and educational organisations are increasingly interested in the impact that classroom practice has on student outcomes. We have therefore included an exploration of this topic and some practical advice on how to evaluate the success of the learning opportunities you are providing for your students. The book then closes with some guidance on how to reflect on your teaching and some avenues you might explore to develop your own professional learning.

We hope you find these books accessible and useful. We have tried to make them conversational in tone so you feel we are sharing good practice rather than directing it. Above all, we hope that the books will inspire you and enable you to think in more depth about how you teach and how your students learn.

Paul Ellis and Lauren Harris

Series Editors

2

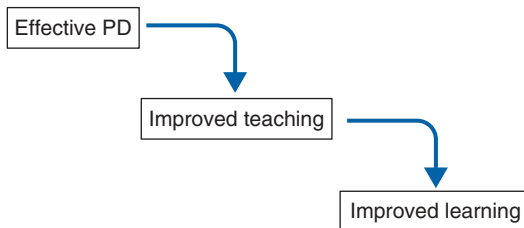
Purpose and context

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International research into educational effectiveness tells us that student achievement is influenced most by what teachers do in classrooms. In a world of rankings and league tables we tend to notice performance, not preparation, yet the product of education is more than just examinations and certification. Education is also about the formation of effective learning habits that are crucial for success within and beyond the taught curriculum.

The purpose of this series of books is to inspire you as a teacher to reflect on your practice, try new approaches and better understand how to help your students learn. We aim to help you develop your teaching so that your students are prepared for the next level of their education as well as life in the modern world.

This book will encourage you to examine the processes of learning and teaching, not just the outcomes. We will explore a variety of teaching strategies to enable you to select which is most appropriate for your students and the context in which you teach. When you are making your choice, involve your students: all the ideas presented in this book will work best if you engage your students, listen to what they have to say, and consistently evaluate their needs.



Cognitive psychologists, coaches and sports writers have noted how the aggregation of small changes can lead to success at the highest level. As teachers, we can help our students make marginal gains by guiding them in their learning, encouraging them to think and talk about how they are learning, and giving them the tools to monitor their success. If you take care of the learning, the performance will take care of itself.

When approaching an activity for the first time, or revisiting an area of learning, ask yourself if your students know how to:

- approach a new task and plan which strategies they will use
- monitor their progress and adapt their approach if necessary
- look back and reflect on how well they did and what they might do differently next time.

2

Approaches to learning and teaching Mathematics

Effective learners understand that learning is an active process. We need to challenge and stretch our students and enable them to interrogate, analyse and evaluate what they see and hear. Consider whether your students:

- challenge assumptions and ask questions
- try new ideas and take intellectual risks
- devise strategies to overcome any barriers to their learning that they encounter.

As we discuss in the chapters on **Active learning** and **Metacognition**, it is our role as teachers to encourage these practices with our students so that they become established routines. We can help students review their own progress as well as getting a snapshot ourselves of how far they are progressing by using some of the methods we explore in the chapter on **Assessment for Learning**.

Students often view the subject lessons they are attending as separate from each other, but they can gain a great deal if we encourage them to take a more holistic appreciation of what they are learning. This requires not only understanding how various concepts in a subject fit together, but also how to make connections between different areas of knowledge and how to transfer skills from one discipline to another. As our students successfully integrate disciplinary knowledge, they are better able to solve complex problems, generate new ideas and interpret the world around them.

In order for students to construct an understanding of the world and their significance in it, we need to lead students into thinking habitually about why a topic is important on a personal, local and global scale. Do they realise the implications of what they are learning and what they do with their knowledge and skills, not only for themselves but also for their neighbours and the wider world? To what extent can they recognise and express their own perspective as well as the perspectives of others? We will consider how to foster local and global awareness, as well as personal and social responsibility, in the chapter on **Global thinking**.

As part of the learning process, some students will discover barriers to their learning: we need to recognise these and help students to overcome them. Even students who regularly meet success face their own challenges. We have all experienced barriers to our own learning at some point in our lives and should be able as teachers to empathise and share our own methods for dealing with these. In the

Purpose and context

chapter on **Inclusive education** we discuss how to make learning accessible for everyone and how to ensure that all students receive the instruction and support they need to succeed as learners.

Some students are learning through the medium of English when it is not their first language, while others may struggle to understand subject jargon even if they might otherwise appear fluent. For all students, whether they are learning through their first language or an additional language, language is a vehicle for learning. It is through language that students access the content of the lesson and communicate their ideas. So, as teachers, it is our responsibility to make sure that language isn't a barrier to learning. In the chapter on **Language awareness** we look at how teachers can pay closer attention to language to ensure that all students can access the content of a lesson.

Alongside a greater understanding of what works in education and why, we as teachers can also seek to improve how we teach and expand the tools we have at our disposal. For this reason, we have included a chapter in this book on **Teaching with digital technologies**, discussing what this means for our classrooms and for us as teachers. Institutes of higher education and employers want to work with students who are effective communicators and who are information literate. Technology brings both advantages and challenges and we invite you to reflect on how to use it appropriately.

This book has been written to help you think harder about the impact of your teaching on your students' learning. It is up to you to set an example for your students and to provide them with opportunities to celebrate success, learn from failure and, ultimately, to succeed.

We hope you will share what you gain from this book with other teachers and that you will be inspired by the ideas that are presented here. We hope that you will encourage your school leaders to foster a positive environment that allows both you and your students to meet with success and to learn from mistakes when success is not immediate. We hope too that this book can help in the creation and continuation of a culture where learning and teaching are valued and through which we can discover together what works best for each and every one of our students.

3

The nature of the subject

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Mathematics is a unique subject within the school curriculum. Not only does it underpin many other subject areas, but mathematical truth is fundamentally different from other types of knowledge. Mathematics has a rich and varied history – for as long as people have been reasoning, they have been doing Mathematics. We believe that we have a duty to offer all students the opportunity to engage with rich Mathematics that will allow them to appreciate its unique nature for themselves.

Mathematical truth

Mathematical ideas in some form have existed for thousands of years.

The ancient Greeks were among the first to engage with the sort of thinking that we now recognise as mathematical proof. In ancient Greek society, people spent time debating philosophical ideas. They came to realise that, unlike other types of thought, mathematical truth was resistant to sceptical attack, because once they had proved something, no one else could disprove it. As a consequence, the study of Mathematics became an important part of a classical education. The first proofs were geometrical. Euclid proposed a set of axioms that were widely agreed on, and much of early academic Mathematics consisted of making logical deductions from his starting points in order to prove new theorems.

Humankind has always been fascinated by marking the passage of time. As part of our intrinsic desire to make sense of the world around us, throughout human history we have tried to explain the motion of the stars and planets. We have also taken an interest in scientific questions closer to home, such as why things float, why people get sick, how to build better bridges and how much wine can be held in a barrel. Many developments in the history of Mathematics came about as a result of people needing to explain phenomena, and sharing some of their stories can help to bring Mathematics alive for our students.

Here are some suggestions of historical mathematical figures whose stories might be of interest:

- Archimedes, who solved the problem of working out the density of the king's crown while taking a bath.
- Al-Khwarizmi, who developed methods for solving equations and after whom the word 'algorithm' was named.

3

Approaches to learning and teaching Mathematics

- Isaac Newton, who formulated laws that govern motion and gravitational effects, and developed calculus.
- Galileo Galilei, who was imprisoned by the church for expounding the theory of heliocentricity.
- René Descartes, who linked algebra and geometry through his invention of Cartesian coordinates.
- Pierre de Fermat and Blaise Pascal, whose correspondence formed the beginnings of probability theory.

The Mathematics that each of these people did covered a wide range of ideas, but the common thread between them is proof – taking agreed starting points and making deductions to develop new theorems.

If we want our students to work like mathematicians, we need to be explicit about the different types of knowledge seen in a Mathematics classroom. Dave Hewitt introduced the terms ‘arbitrary’, to refer to that which students need to be told, and ‘necessary’, that which they can deduce for themselves.

For example, if you draw a shape with four right angles and four equal sides, the fact that it’s called a square is ‘arbitrary’. Different languages and cultures will use words other than ‘square’ to describe the same object. However, the fact that its diagonals intersect at 90 degrees is a property of a square that students can discover for themselves – it is a ‘necessary’ truth. No matter when or where a square is drawn, its diagonals will always have this property. This type of mathematical truth is universal and it is important that students are offered opportunities to appreciate such universality.

Mathematics as a foundation for other subjects

A teacher challenged her students to come up with a job that didn’t include any Mathematics at all. One student thought he had cracked this with the suggestion ‘priest’, but was met with the response ‘Who would be responsible for the parish accounts then?’ Mathematics is inherent in our daily lives. Whether the context is finance, measurement or making