

Contents

Volume I

Introduction	<i>page</i> 1
1 Algebras and Modules	5
1.1 Group algebras	8
1.2 Twisted group algebras	14
1.3 G -algebras	26
1.4 Category algebras	29
1.5 Centre and commutator subspaces	34
1.6 The Hopf algebra structure of group algebras	40
1.7 Blocks and idempotents	45
1.8 Composition series and Grothendieck groups	51
1.9 Semisimple modules	54
1.10 The Jacobson radical	59
1.11 On the Jacobson radical of finite group algebras	70
1.12 Projective and injective modules	74
1.13 Wedderburn's Theorem	84
1.14 Splitting fields	89
1.15 Simple modules of finite group algebras	96
1.16 Central simple and separable algebras	101
1.17 Complexes and homology	109
1.18 Complexes and homotopy	117
2 Functors Between Module Categories	129
2.1 Induction and restriction	130
2.2 Frobenius reciprocity	136
2.3 Adjoint functors	140

2.4	Mackey's formula	145
2.5	Relative traces	151
2.6	Higman's criterion	154
2.7	Relative projectivity and adjoint functors	163
2.8	Morita Theory	167
2.9	Duality	178
2.10	Traces of endomorphisms	188
2.11	Symmetric algebras	190
2.12	Symmetry and adjunction	195
2.13	Stable categories	202
2.14	The Heller operator on stable categories	209
2.15	Stable categories of symmetric algebras	212
2.16	Relative traces for symmetric algebras	216
2.17	Stable equivalences of Morita type	222
2.18	Projective and injective resolutions	230
2.19	Derived categories	237
2.20	Derived functors and cohomology	239
2.21	Derived equivalences and Rickard complexes	245
3	Character Theory	251
3.1	Characters of modules and finite groups	251
3.2	Characters and duality	260
3.3	The orthogonality relations	262
3.4	Character tables of finite groups	271
3.5	Integrality of character values	277
3.6	Burnside's $p^a q^b$ -Theorem	282
3.7	Brauer's characterisation of characters	284
3.8	Splitting fields for group algebras	288
3.9	Integral group rings	289
4	Algebras over p-Local Rings	296
4.1	Local rings and algebras	296
4.2	Discrete valuation rings	301
4.3	Complete discrete valuation rings	305
4.4	Local algebras over complete local rings	311
4.5	Projective covers and injective envelopes	315
4.6	The Krull–Schmidt Theorem	322
4.7	Lifting idempotents and points	328
4.8	The Wedderburn–Malcev Theorem	339
4.9	Basic algebras and quivers	345
4.10	The Cartan matrix	352

Contents of Volume I

vii

4.11	Selfinjective algebras	355
4.12	Symmetric algebras over fields and local rings	358
4.13	Stable categories for \mathcal{O} -injective algebras	366
4.14	Stable equivalences of Morita type between \mathcal{O} -algebras	376
4.15	Stable equivalences of Morita type and automorphisms	385
4.16	The decomposition matrix	388
4.17	The decomposition map	392
5	Group Algebras and Modules over p-Local Rings	396
5.1	Vertices and sources	396
5.2	The Green correspondence	408
5.3	Group actions on matrix algebras	415
5.4	The Brauer construction	424
5.5	Pointed groups on G -algebras	430
5.6	Defect groups and source algebras of primitive G -algebras	438
5.7	Multiplicity modules	443
5.8	The Brauer construction applied to permutation modules	449
5.9	Brauer pairs and p -permutation G -algebras	455
5.10	Trivial source modules	461
5.11	p -permutation modules	466
5.12	Green's Indecomposability Theorem	472
5.13	Brauer characters	485
5.14	The decomposition map for group algebras	490
5.15	Generalised decomposition maps for group algebras	494
	<i>Bibliography</i>	505
	<i>Index</i>	510