

Index

- absolutely indecomposable, 473
 $\text{add}(M)$, 81
additive commutator, 35
adjoint functors, 140
adjunction
 counit, 140
 isomorphism, 140
 tensor-Hom, 137
 unit, 140
algebra, 5
 automorphism group, 26
 basic, 345
 block, 47
 category, 29
 central simple, 101
 equivalent, 103
 defined over a subfield, 95
 G , 26
graph of an, 348
homomorphism, 5
Hopf, 43
induced, 442
interior G -, 26
local, 311
Morita equivalent, 167
opposite, 6
point of an, 335
primitive G -, 438
selfinjective, 355
semisimple, 85
separable, 105
simple, 85
split, 89
split local, 311
splitting field, 89
symmetric, 37, 190, 359
twisted category, 32
algebraic integer, 277
almost split sequence, 371
Alperin's weight conjecture
 for finite groups, 100
anti-automorphism, 43
antipode, 43
augmentation, 43
 homomorphism, 40
 ideal, 40
Auslander–Reiten conjecture, 204, 375
Auslander–Reiten duality, 371
Aut, 26
 Aut_0 , 385
 Aut_1 , 385
automorphism
 group, 26
 inner, 26
 outer, 26
basic algebra, 345
 of A , 347
biadjoint functors, 140
bialgebra, 43
bimodule, 6
block, 47
Brauer character, 486
 irreducible, 486
Brauer construction, 425, 429
Brauer group, 173
Brauer group of a field, 103
Brauer homomorphism, 424, 425, 429
Brauer pair, 456
 inclusion, 458

- Brauer's characterisation of characters, 284
- Brauer's reciprocity, 394, 492
- Burnside's p^aq^b -Theorem, 282
- Burry–Carlson–Puig Theorem, 436
- canonical evaluation map, 77
- Cartan matrix, 352
- category
 - derived, 237
 - finite, 29
 - injective object, 83
 - projective object, 83
 - small, 29
 - split morphism, 83
 - triangulated, 204, 211
- category algebra, 29
- Cauchy sequence, 305, 307
- centre of a ring, 5
- central function, 190, 251
- central group extension, 20
- central simple algebra, 101
 - equivalent, 103
- $\text{CF}(A)$, 251
- chain homotopy, 117
- chain map, 110
 - degreewise split, 122
 - homotopic, 117
- character, 252
 - generalised, 253
 - irreducible, 253
 - monomial, 269
 - of a direct sum, 254
 - of a dual module, 260, 261
 - of a permutation module, 257
 - of a tensor product, 255
 - regular, 256
 - scalar product, 264
 - table, 271
 - triple, 424
- class function, 251
- class sum correspondence, 290
- Clifford's Theorem, 58
- $\text{Cl}_k(G)$, 252
- coalgebra, 43
- coassociativity, 43
- cochain homotopy, 117
- cohomology, 111
- commutator space, 35
- complete
 - local ring, 305
 - module, 307
- completion
 - of a local ring, 306
 - of a module, 307
- complex
 - acyclic, 111
 - bounded, 110
 - bounded above, 110
 - bounded below, 110
 - chain, 109
 - homology, 111
 - chain map, 110
 - cochain, 109
 - cohomology, 111
 - cone of, 122
 - contractible, 117
 - differential, 109
 - exact, 111
 - homotopy category, 117
 - quasi-isomorphic, 111
 - Rickard, 247
 - shift automorphism, 110
 - split, 125
 - tilting, 246, 247
- composition
 - factor, 51
 - series, 51
 - equivalent, 51
 - length, 51
- cone of a complex, 122
- conjugacy class sum, 34, 290
- connecting homomorphism, 112
- contractible, 117
- crossed product, 28
- decomposition map, 394, 490
- decomposition matrix, 388
- defect group, 439
 - primitive G -algebra, 439
- defect pointed group, 435
- degreewise split, 122
- derived
 - functor, 240
- derived category, 237
 - bounded, 237
 - bounded above, 237
 - bounded below, 237
- determinant, 188
 - of a module, 252
- differential, 109
- direct factor, 45
- discrete valuation ring, 301

- division algebra, 10
- duality, 179
 - adjunction, 182, 183
 - algebra, 185
- $e(\chi)$, 263
- Eckmann–Shapiro, 242
- elementary group, 284
- epimorphism, 83
- exponent of a group, 288
- Ext, 240
- Feit’s conjecture, 399
- first cohomology class, 17
- First Orthogonality Relations, 264
- 5-Lemma, 113
- flat module, 64
- Fourier transformation, 264
- Frobenius group, 275
- Frobenius’s Reciprocity, 136, 214, 269
- G -algebra, 26
- G -algebra
 - homomorphism, 27
- G -graded algebra, 28
- G_p' , 486
- G -set, 11
 - 2-transitive, 270
- G -stably C -separable, 344
- generalised decomposition
 - map, 495
 - matrix, 497, 498
 - number, 495, 498
- graded object, 109
 - shift automorphism, 109
- graded-commutative, 245
- graph of an algebra, 348
- Green correspondence, 408
- Green correspondent, 408
- Green’s Indecomposability Theorem, 473
- Grothendieck group, 53, 265
 - scalar product, 93
- group
 - cohomology, 244
- group algebra, 8
 - antipode, 43
 - comultiplication, 43
 - count, 43
 - twisted, 15
 - unit, 43
- group graded algebra, 28
- Heller operator, 209
- Higman’s criterion, 155, 166
- Hochschild cohomology, 243
 - Tate analogue, 370
- homogeneous component, 58
- homology, 111
- homotopy, 117
 - category, 117
 - equivalent, 117
 - inverse, 117
- homotopy equivalence, 117
- Hopf algebra, 43
- $\text{IBr}_k(G)$, 486
- ideal
 - nilpotent, 60
- idempotent, 9
 - central primitive, 263
 - orthogonal, 9
 - primitive, 9
 - primitive decomposition, 9, 322
- inclusion of pointed groups, 433
- indecomposable
 - absolutely, 473
 - module, 12
- induced algebra, 442
- induced character, 258
- induction
 - for class functions, 258
 - functor, 130
 - tensor, 134
- injective
 - envelope of a module, 321
 - module, 81
 - object, 83
- Inn, 26
- inner automorphism, 26
- interior G -algebra, 26
 - structural homomorphism, 26
- interior G -algebra
 - homomorphism, 27
- invertible bimodule, 175
- IPr , 491
- $\text{Irr}_k(G)$, 253
- isotypic component, 58, 264
- Jacobson radical, 59
- Jordan–Hölder, 51
- k -split, 205
- acyclic, 231

- complex, 231
- injective, 205
- surjective, 205
- k*-stable category, 203
- Krull–Schmidt Theorem, 325
- Künneth’s formula, 249
- $\ell(A)$, 100
- $L^0(A)$, 393
- $L^0(G)$, 499
- left adjoint functor, 140
- left derived functor, 240
- Lifting Theorem for idempotents, 329
- local pointed group, 430
- local ring, 296
- Loewy layers, 69
- Loewy length, 69
- long exact homology sequence, 111
- M*-group, 269
- Mackey’s Formula, 146
- Maschke’s Theorem, 73, 106
- modular isomorphism problem, 294
- module, 6
 - p*-permutation, 466
 - absolutely indecomposable, 473
 - Artinian, 7
 - basis of a free, 7
 - category, 7
 - character of a , 252
 - complete, 307
 - finitely generated, 7
 - finitely generated projective, 77
 - flat, 64, 78
 - free, 7
 - generating subset, 7
 - indecomposable, 12
 - injective, 81
 - left, 6
 - Loewy layers of a , 69
 - Loewy length of a , 69
 - Noetherian, 7
 - permutation, 12
 - projective, 74
 - pure submodule, 303
 - radical of a , 68
 - rank, 7
 - rank of a free, 297
 - regular, 7
 - right, 6
 - semisimple, 54
 - simple, 12
 - socle of a , 68
 - source, 397
 - structural homomorphism, 89
 - torsion-free, 302
 - transitive permutation, 12
 - trivial source, 461
 - uniserial, 54, 69
 - vertex, 397
 - monomorphism, 83
 - Morita
 - context, 168
 - Morita equivalence, 61, 86, 167
 - Morita’s Theorem, 167
 - multiplicity module, 399, 443
 - Nakayama
 - automorphism, 358
 - functor, 187
 - Nakayama’s Lemma, 60
 - nilpotent
 - element, 60
 - group, 269
 - ideal, 60
 - normalised 2-cocycle, 22
 - Okuyama’s method, 382
 - ΩA , 224
 - 1-coboundary, 17
 - 1-cocycle, 15
 - ordinary representation theory, 251
 - Out, 26
 - Out₀, 385
 - Out₁, 385
 - outer automorphism, 26
 - p*'-element, 97
 - p*'-part of a group element, 98
 - p*-adic integers, 306
 - p*-element, 97
 - p*-elementary group, 284
 - p*-modular system, 296
 - p*-part of a group element, 98
 - p*-permutation module, 466
 - p*-quasi-elementary group, 285
 - p*-regular class function, 487
 - p*-weight of a finite group, 101
 - path
 - algebra, 350
 - category, 349

- perfect, 227
 - bimodule, 227
 - field, 309
- $\text{perm}_K(G)$, 285
- permutation module, 12
 - transitive, 12
- perpendicular space, 192
- $\text{Pic}(A)$, 176, 178, 385
- Picard group, 176, 178, 385
- point, 335
 - multiplicity of a, 335
- pointed group, 430
- primitive decomposition, 9, 322
- $\text{Pr}_{\mathcal{O}}(A)$, 393
- progenerator, 81
- $\text{proj}(A)$, 81
- projective
 - cover of a bimodule, 321
 - cover of a module, 318
 - ideal, 219
 - module, 74
 - object, 83
- projective representation, 416
- Puig correspondence, 415
- pure submodule, 303
- $\text{qelem}_K(G)$, 285
- quasi-elementary group, 285
- quasi-isomorphism, 111
- quiver, 349
- radical, 59
 - of a finite p -group algebra, 70
 - of a module, 68
 - of a subalgebra, 63
 - series, 68
- rank of a free module, 7
- regular character, 189, 256
- relative projective, 154
 - subalgebra, 159
- relative trace, 151, 166
 - Mackey formula, 152
- relatively k -injective
 - resolution, 231
- relatively k -projective
 - resolution, 231
- relatively injective, 163, 164, 202
 - subalgebra, 159
- relatively projective, 163, 164, 202
- relatively separable, 160, 340
- representation
 - of a group, 10
- resolution, 231
 - injective, 231
 - projective, 231
- restriction
 - for class functions, 258
 - functor, 130
- retraction, 154
- Rickard complex, 247
- right adjoint functor, 140
- right derived functor, 240
- scalar product, 492
 - of characters, 264
- Schanuel's Lemma, 76, 82
- Schur functor, 171
- Schur's Lemma, 55
- second cohomology class, 17, 32
- Second Orthogonality Relations, 267
- section, 154
- selfinjective algebra, 355
- semicovering, 478
- separable equivalence, 373
- separably equivalent algebras, 160
- shift automorphism, 109, 110
- shift functor, 123
- ΣA , 224
- simple
 - module, 12
- skew group algebra, 28
- Skolem–Noether Theorem, 174
- socle
 - of a module, 68
 - series, 68
- solvable group, 282
- source algebra, 441
- source idempotent, 441
- split morphism, 83, 154
- splitting field
 - of a finite group, 89
 - of an algebra, 89
- stable category, 203, 366
- stable centre, 219
- stable equivalence of Morita type, 222, 248
- stable Picard group, 229, 385
- $\text{StPic}(A)$, 229, 385
- subnormal, 473
- supersolvable group, 269
- symmetric algebra, 37, 190, 359

Index

515

- symmetrising form, 190, 191
- perpendicular space, 192
- Tate
 - Ext, 370
 - Hochschild cohomology, 370
 - duality, 371
- tensor induction, 134
- tensor product
 - of complexes, 114
- tilting complex, 246, 247
- Tor, 240
- trace of a matrix, 37, 188
- triangulated category, 204, 211
- trivial intersection, 223
- trivial module, 40
- trivial source module, 461
- twisted category algebra, 32
- twisted group algebra, 15
- 2-coboundary, 17
- 2-cocycle, 15, 32
 - identity, 15, 32
 - normalised, 22
- 2-transitive, 270
- valuation, 302
- Wedderburn's Theorem, 86
 - for split algebras, 90
- Wedderburn–Malcev Theorem, 339
- weight
 - of a finite group, 101