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Thomas J. Pence , Indrek S. Wichman
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Essential Mathematics for Engineers and Scientists

This text is intended for students who have an undergraduate degree or extensive coursework in engineering or the physical sciences and who wish to develop their understanding of the essential topics of applied mathematics. The methods covered in these chapters form the core of analysis in engineering and the physical sciences. Readers will learn the solutions, techniques, and approaches that they will use as academic researchers or industrial R&D specialists. For example, they will be able to understand the fundamentals behind the various scientific software packages that are used to solve technical problems (such as the equations describing the solid mechanics of complex structures or the fluid mechanics of short-term weather prediction and long-term climate change), which is crucial to working with such codes successfully. Detailed and numerous worked problems help to ensure a clear and well-paced introduction to applied mathematics. Computational challenge problems at the end of each chapter provide students with the opportunity for hands-on learning and help to ensure mastery of the concepts. Adaptable to one- and two-semester courses.

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The authors wish to dedicate this work to their wives, Lora (Casper) Pence and Kadri (Roman) Wichman, for their love, support, and encouragement, and to their late fathers, James Thomas Pence and Sven Hjalmar Wichman, for life's lessons learned, mathematical and otherwise.

CONTENTS

PREFACE

The title of this book states that the mathematical subject matter covered herein is "essential." This assertion itself requires clarification. *For what* is the mathematics discussed here essential? *Why* is the mathematics discussed here essential? *For whom* is the mathematics discussed here essential (engineers and scientists, clearly, but that must also be explained)?

To begin answering these complicated questions it is necessary to discuss how and why they first arose.

The fact is that in the four decades between approximately 1980 and 2020 there has been an explosion in the uses of computers for the solution of engineering and scientific problems using techniques of computational modeling and simulation. Computers are also used for other purposes ranging from entertainment (videos, cell phones, etc.) to the apprehension of fugitives and for espionage (facial recognition), to large scale data processing in health care and social statistics, to the increased financialization and economization of daily life. Simply put, the computer is now ubiquitous.

One of the downsides of increased sophistication is the associated loss of contact with fundamentals. In everyday life this is observed as the inability to make repairs on ones's car or to replace a "heating, ventilation and air-conditioning system" in one's home. Once computers were able to solve long-standing difficult problems of engineering, what was left for the replaceable, interchangeable engineer? The computer made it all so easy. It never asked for vacations, bonuses or raises, and, best of all, unlike real-life engineers it never made any mistakes. Furthermore, if engineers and scientists are not needed, why should engineering and science even be taught?

We were led in the face of such circumstances to ask a more immediately relevant set of questions: What level of mathematical sophistication is necessary for our engineers and scientists (assuming they are still necessary) in an era where machine computation occupies the prominent place? A slightly different version of the same question is: How grounded should our students be in the fundamentals of mathematics? One cannot help but notice that the need to write one's own code has seemingly withered away as pre-packaged and commercial software programs became universally available. And so it became legitimate to ask even more questions: If a solution of some kind is always computable is there any longer a need to worry whether the mathematical algorithms and modeling assumptions are consistent? Is it worth one's time to ask whether the mathematical problem is well posed? Is there still a need on the part of the engineer or scientist to deploy strict algorithmic thinking during the problem-solving process or should his or her role end with the problem statement?

We learned by subsequent experience through many exaggerations, misstatements and mishaps that the consequences of shorting human technical ability and expertise can range from the comical to the harmful. Bridges collapsed, airplanes crashed, wind tunnels and other large-scale experimental facilities were still needed, and experimental diagnostics became even more sophisticated and accurate. Hence there arose a distinct need for oversight in the form of practicing engineers and scientists able to provide a competent technical scrutiny of proposed "solutions" to difficult technical problems. By necessity we were returned to asking modern variants of age-old questions: what *is* a problem, and what does it mean to *solve* it? We were forced to acknowledge that though we lived in an era technologically far superior to that in which much of our mathematics was developed, we had not yet outgrown the constraints it has imposed on us: everyday life requires mathematics as much, or more than it ever did.

With the change of scope and emphasis in modern engineering practice — our nearly ubiquitous dependence on machine computation — new and different ways were needed to present essential mathematical knowledge to students of engineering and the sciences. Even so, the path forward begins by casting a long glance over one's shoulder toward the solved problems from the venerable past. How did these problems arise? What questions did they seek to answer? How were the solutions generated? These questions provide valuable clues to the construction and solution of modern mathematical models. Although computers *have* altered the way we do our mathematics, and how we construct solutions, and how we consider the parts of a given problem in relation to the whole, it was a change in form, not content. As one simple example, the reader may reflect on the fact that though we are not about to discard our software codes we can nevertheless be more vigilant about testing them against established mathematical solutions. This goes from being merely important to crucial when we extend these codes and programs past the simpler problems for which they were initially designed. Man's reach exceeding his grasp, we seek the means for closing that gap whenever and however possible.

The preceding discussion, of course, was centered on the two questions, "*why* and *for what* is technical competence in advanced mathematics essential?"

Concerning "essential for whom?", the aim of this introductory graduate-level book is to visit, with sharpened tools and perspective honed by precedent and experience, a subset of the most important — essential — topics of mathematics that the aspiring graduate level engineer or scientist should know. Since this knowledge is foundational, it builds upward and outward. Each new subject matter, however, has a partial foundation of its own so the entire assembly might be taken by analogy to resemble a Roman aqueduct: a superstructure resting on separate but strongly linked segments, or foundations. The mathematics of engineering science builds upward firstly from a knowledge of exact mathematics unencumbered (except for the occasional example) by approximation, estimation and asymptotization. We believe that although there is available an assortment of approximate and asymptotic solution techniques for various classes of engineering and applied science problems, extended coverage of them in the first year of graduate study would amount to putting the cart before the horse.

Key Features

Key features of this book include the following

- The presumed background for this book is the equivalent of an undergraduate (B.S.) degree in engineering or in physical science. This implies a grounding in calculus through partial differential equations, and also familiarity with matrix theory including a basic understanding of eigenvalues and eigenvectors. It is also presumed that the reader has grappled with ODEs, PDEs, and eigenvalues in a first treatment. These subjects implicitly suggest that the student is familiar with introductory Fourier series, Laplace transforms, and vector analysis and notation. All of these topics are covered in "advanced" undergraduate textbooks as well as in several introductory graduate level texts. In a typical undergraduate technical major in mechanical engineering, for example, the student at the junior/senior level would have been exposed to treatments of heat conduction (for PDEs), vibrations (for ODEs and eigenvalues), and control theory (linear systems, Laplace transforms). In other areas of undergraduate study in different technical areas such as physics or chemistry, the student will have encountered junior/senior level courses in classical mechanics, statistical thermodynamics, electromagnetic theory and physical chemistry. Courses such as these inevitably cover many of the mathematical procedures with which we expect the reader to have background operational facility.
- The subject matter is divided into three major parts, each with at least four chapters. The three parts focus on Linear Algebra, Complex Variables, and Ordinary and Partial Differential Equations. The foundations of these topics are different but the connections and interrelations tying them together are numerous and deep. In order to solve problems numerically it is imperative to have a strong foundation in Linear Algebra. What are the objects of numerical solutions? Usually, ordinary differential equations (ODEs) and partial differential equations (PDEs). Complex Variables might be thought to stand outside these two areas, but the light shed on differential equations, integration, the significance of the derivative as well as the foundational notions of the Riemann sphere, the Cauchy-Riemann equations, and all of the techniques and skills learned in the act of evaluating integrals and assessing singularities (and noting their behavior) makes its' study extremely valuable. The basic concepts introduced in Complex Variables carry over into topics as concrete as geometric considerations for PDEs, and as abstract as the meaning of infinity or the notion of $\sqrt{-1}$.
- A major point of emphasis has been to discuss the Fredholm Alternative Theorem (FAT) and its implications for solving linear systems of equations (in Part I) and ordinary differential equations (in Part III). These two areas of emphasis are closely related because ODEs can be recast using finite differences as a system of linear algebraic equations.

Nevertheless, the FAT provides an excellent graduate level opportunity to discover the deeper connections between these topics. In addition, the limitations on the existence of solutions indicated by the FAT lead to new and broader insights, such as the concept of a modified Green's function described in Part III. Here the student will learn that an apparent limitation of a prior, perhaps more straightforward approach is a not-so-subtle manifestation of the need for a more fundamental principle (such as the modified Green's function). What first appear to be roadblocks often mark the entryway to a more foundational theory.

- The order of the material is not "random" (to use the current jargon) or arbitrary. Linear Algebra should be studied first followed by Complex Variables, and then ODEs and PDEs. There is a constant "build" as these topics are gradually developed, until, at the end of Chapter 13, much of the intellectual content of the previous twelve chapters is represented in the form of a final example in Chapter 13, Section 13.6. Indeed, detailed worked examples serve as a vehicle for introducing and advancing the theory. While certain of these may seem discipline specific by virtue of their particular formulation, they are in fact general with respect to the presentation of the underlying key mathematical concept. This is made clear in the narrative discussion. In addition to demonstrating procedures and treating previously raised issues, these examples serve to pose new questions. A version of this is the introduction of the pseudo-inverse in Section 3.5 which fulfills a specific need that, at first glance, appears to be a loose end but is then revealed to be a concept of fundamental importance. *The continued follow-up on issues raised by prior examples is a key feature of the text.*
- We seek to reinforce the essential concepts by circling back as the book evolves to reconsider important topics and methods in a new light or in a greater degree of generality. This is also part of the "build" that we have employed in the writing. For example, the Fredholm Alternative Theorem is first considered in the context of linear algebra with the same number of equations as unknowns (Chapter 3), then reconsidered in the context of linear algebra with an unequal number of equations and unknowns (Chapter 4), and then later in the context of ODE theory (Chapter 11). Education research has shown that students understand better both the logic and the necessity of the material under such an arrangement. This type of learning is called "active retrieval" [1]. The active retrieval of prior notions, examples and knowledge is also the central component of what these authors call "interleaved and varied practice." This approach fosters appreciation, assimilation and application of principles learned.
- This book might ideally be used as a graduate level text for an introductory full year course. However, it can also serve as a text in a one semester setting with a sampling of material from each of the three parts. At our institution we have used it in a semester course by covering the first two, or in some cases three, chapters of each part. The remaining material is more than sufficient for a schedule of personal study or for a second course, after which the student/reader will have acquired a strong foundation in the methods of exact analysis in engineering and applied science. After that, they will be

ready to study, if they wish, the full spectrum of asymptotic methods (series, integrals, WKB theory, steepest descent, etc.) as well as the classic techniques of approximate engineering analysis represented in Bender and Orszag's excellent and by no means dated book [58]. Topics such as the structure of phase space, solutions for nonlinear ODEs and PDEs, the influence of very large or very small parameters, are left for more advanced treatments, as are studies of the particulars of numerical methods.

- The authors believe that an important part of learning consists of the student's own engagement with the material, in quiet privacy, under a circle of light, phone and electronic gadgetry turned off. Some of this engagement comes from reading, which we could call *philosophical engagement*. A more *active* form of engagement amounts to working through the examples. And the most active and productive form of engagement, where learning truly occurs, comes by solving exercises. Exercises are provided for each section of each chapter. Some of the exercises reinforce standard learning whereas others require considerable effort. The reward is a leap of understanding with a heightened appreciation for the presented material. We have written a "computational challenge exercise" at the end of each chapter, which typically requires not only a thorough grasp of the material, but also the ability to treat amplified versions of previous problems in an algorithmic way. The resulting computational challenge provides a hands-on way to master the theory and it connects the essential mathematics presented herein to the computationally intensive era in which we live.
- Finally, there are several topics of exact engineering analysis that are not addressed, first because they are well covered in other, more introductory texts, and second because the level and depth and the large number of examples we use to cover topics here would make our book unwieldy. Thus, for example, we do not examine in detail variational methods, although they appear in Part III in our discussion of PDEs. Variational calculus leads often to nonlinear ODEs, which is another topic that is not represented herein. Choices were made, and topics were chosen to emphasize the essential nature of the subjects covered, the almost universal emphasis being on the exact solution of linear problems of applied mathematics. Once these subjects are understood the student or researcher can learn how to treat nonlinear equations, often by linearizing them(!) and almost as often by seeking a parameter around which to construct an asymptotic solution. In an age of computation, the predominant approach is a resort to numerical simulation, which inevitably involves linearization.

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The authors wish to acknowledge the impact that many persons have had on the writing of this book, on their learning of this subject matter, and, perhaps most importantly, on the entire concept of *learning how to learn*. The Gordian knot called *the learning process* is a mystery that has proved impossible to unravel, probably because it means something different to every person and every educator.

Before there was a book, there were notes: before there were notes, there was a course. That course has been taught for the past decade at Michigan State University (MSU) in alternating years by Pence and Wichman, and prior to that was team taught by Profs. Brereton, Naguib and Wichman. In the preparation of our notes for this book we especially thank Dr. Yen Nguyen for her valuable technical assistance. We acknowledge separately those persons who provided both instruction and inspiration for learning this subject matter, and for guiding us toward that nebulous place where learning how to learn begins to take root and flourish.

Tom Pence wishes to acknowledge, first of all, an outstanding coterie of math and science teachers in his small high school in Linden, Michigan, and specifically Tim Holcomb, Don Morrow, Jim Starrs and Judy Stoeri. Moving on to undergraduate studies, the Honors College at Michigan State University allowed TJP to explore a wide range of courses, both undergrad and grad, in math, physics and engineering mechanics by thoughtful and patient teachers in diverse areas including especially fluid mechanics (C. Y. Wang), continuum mechanics (S.D. Gavazza), computational mechanics (N. J. Altiero), variational mechanics (C.O. Horgan), statistical physics (T. Woodruff) and real analysis (E. Ingraham). Just as the high school teachers provided the preparation for the MSU courses, those courses in turn provided the necessary fortification for the next challenge – the Caltech first year graduate course gauntlet in applied mechanics, in this case courses in hydraulics (F. Marble), dynamics (W. Iwan), applied math methods (D.S. Cohen), nonlinear waves (G. B. Whitham) and elasticity (Eli Sternberg). This battery of courses provided not only essential mathematical tools, but also conveyed the important lesson that a common problem could be addressed by a variety of different analytical methods, each of which could yield up its own insights. This was essential for follow-up courses, especially those from J. N. Franklin and H.B. Keller. Research under the guidance of my thesis advisor, J. K. Knowles, emphasized the utility of being able to draw upon a variety of analytical tools. That point of view animates the spirit of this text. Special recognition is extended to Jim Knowles for his advice and inspiration.

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During TJP’s and ISW’s formative years of schooling and learning there was much reading also. We cannot but help mention the seminal works of the late mechanicians Clifford A. Truesdell and James Serrin, gentlemen scholars and applied mathematicians *extraordinaire* whom we have both admired from a distance.

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Tom Pence and Indrek Wichman profess mechanical engineering at Michigan State University. Since neither holds a degree in mathematics they are aptly characterized as gentleman math enthusiasts. Pence studied engineering mechanics at Michigan State (B.S.) and applied mechanics at Caltech (M.S., Ph.D.). Wichman studied mechanical engineering at Stony Brook University (B.S.), engineering mechanics at Virginia Tech (M.S.), and mechanical & aerospace engineering at Princeton (M.A., Ph.D.). Their professional goal is to someday be granted adjunct status in the MSU department of mathematics.