A First Course in Differential Geometry

Surfaces in Euclidean Space

Differential geometry is the study of curved spaces using the techniques of calculus. It is a mainstay of undergraduate mathematics education and a cornerstone of modern geometry. It is also the language used by Einstein to express general relativity, and so is an essential tool for astronomers and theoretical physicists.

This introductory textbook originates from a popular course given to third-year students at Durham University for over 20 years, first by the late Lyndon Woodward and then by John Bolton (and others). It provides a thorough introduction by focussing on the beginnings of the subject as studied by Gauss: curves and surfaces in Euclidean space. While the main topics are the classics of differential geometry – the definition and geometric meaning of Gaussian curvature, the Theorema Egregium of Gauss, geodesics, and the Gauss–Bonnet Theorem – the treatment is modern and student-friendly, taking direct routes to explain, prove, and apply the main results. It includes many exercises to test students' understanding of the material, and ends with a supplementary chapter on minimal surfaces that could be used as an extension towards advanced courses or as a source of student projects.

John Bolton earned his Ph.D. at the University of Liverpool and joined Durham University in 1970, where he was joined in 1971 by **Lyndon Woodward,** who obtained his D.Phil. from the University of Oxford. They embarked on a long and fruitful collaboration, co-authoring over 30 research papers in differential geometry, particularly on generalisations of "soap film" surfaces. Between them, they have over 70 years' teaching experience, being well regarded as enthusiastic, clear, and popular lecturers. Lyndon Woodward passed away in 2000.

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Sections marked with † denote optional sections

Preface

We believe that the differential geometry of surfaces in Euclidean space is an ideal topic to present at advanced undergraduate level. It allows a mix of calculational work (both routine and advanced) with more theoretical material. Moreover, one may draw pictures of surfaces in Euclidean 3-space, so that the results can actually be visualised. This helps to develop geometrical intuition, and at the same time builds confidence in mathematical methods. One of our aims is to convey our enthusiasm for, and enjoyment of, this subject.

The book covers material presented for many years to advanced undergraduate Mathematics and Natural Sciences students at Durham University in a module entitled "Differential Geometry". This module constitutes one sixth of the academic content of their third year. The two main prerequisites are basic linear algebra and many-variable calculus.

We have three main targets.

(i) Gaussian curvature: we seek to explain this important function, and illustrate the geometrical information it carries. We further demonstrate its importance when we discuss the Theorema Egregium of Gauss.

(ii) Geodesics: these are the most important and interesting curves on a surface. They are the analogues for surfaces of straight lines in a plane.

(iii) The Gauss–Bonnet Theorem: among other things, this theorem shows that Gaussian curvature (which is defined using local properties of a surface) influences the global overall properties of that surface.

The Theorema Egregium and the Gauss–Bonnet Theorem are both very surprising, but readily understood and appreciated. They are also very important and influential from a historical perspective, having had a profound effect on the development of differential geometry as a whole.

We have tried to present the material needed to attain these targets using the minimum amount of theory, and have, for the most part, resisted the temptation to include extra material (but this resistance has crumbled spectacularly in Chapter 9!). This means that we have been rather selective in our choice of applications and results. However, each chapter contains some optional material, clearly signposted by a dagger symbol †, to provide flexibility in the module and to add interest and mental stimulation to the more committed student. The optional material also provides opportunities for additional reading as the module progresses.

There should be time to cover at least some of the optional material, and choices may be made between the technical, the slightly more advanced, and some interesting topics which are not specifically needed to attain our three targets mentioned above. There is also some optional material on surfaces in higher dimensional Euclidean spaces (and on general abstract surfaces), which is designed to whet the appetite of the students, and help the transition to more advanced topics.

In a forty lecture module, we would suggest that the material in the first four chapters should be covered in the first half of the module (and perhaps a start made on Chapter 5), with between four and six lectures on each of the first three chapters, and perhaps four lectures on the material in Chapter 4.

The pace picks up in the second half of the module. We suggest seven lectures for Chapter 5 and three for Chapter 6. Five lectures could be allowed for Chapter 7, and four for Chapter 8. However, this may only be achievable if students are asked to read for themselves the proofs of some of the results.

This may leave a couple of lectures to briefly discuss the contents of the optional Chapter 9 (on minimal and CMC surfaces). Although the material in this chapter is more advanced, it is included because the mathematics is so beautiful, and is suitable for self-study by an interested student. It could also form the starting point of a student project at senior undergraduate or beginning postgraduate level.

Our aim throughout is to make the material appealing and understandable, while at the same time building up confidence and geometrical intuition. Topics are presented in bite-sized sections, and concrete criteria or formulae are clearly stated for the various objects under discussion. We give as many worked examples as possible, given the time constraints imposed by the module, and have also included many exercises at the end of each of the chapters (and provided brief hints or solutions to some of them). On-line solutions to all the exercises are available to instructors on application to the publishers.

We have been heavily influenced by the excellent text *Differential Geometry of Curves and Surfaces* by Manfredo Do Carmo (Dover Books on Mathematics). However, we have omitted many of the more advanced topics found in that book, and at the same time have further elucidated, where we thought appropriate, the material we believe may be reasonably covered in our forty lecture module.

Finally, our sincere thanks to Roger Astley and his team at Cambridge University Press, who have been encouraging and patient throughout the rather long gestation period of the book.

Please enjoy the book.

Internal referencing

There are inevitably very many definitions which have to be included in a book of this nature. Rather than numbering these and referring back to them each time they are used, we thought it best to italicise the terms being defined and then include all these terms in the index.

Results and Examples are numbered in a single sequence within each section. A typical internal reference might be, for instance, Theorem 3 of §2.5. If no section reference is given, the result or example is in the current section. Equations to be referred to later in the book are numbered consecutively within each chapter (so, for instance, equation (3.7) is the seventh numbered equation in Chapter 3).