

## Phased Arrays for Radio Astronomy, Remote Sensing, and Satellite Communications

Discover a modern approach to the analysis, modeling and design of high sensitivity phased arrays. Network theory, numerical methods and computational electromagnetic simulation techniques are uniquely combined to enable full system analysis and design optimization. Beamforming and array signal processing theory are integrated into the treatment from the start. Digital signal processing methods such as polyphase filtering and RFI mitigation are described, along with technologies for real time hardware implementation. Key concepts from interferometric imaging used in radio telescopes are also considered. A basic development of theory and modeling techniques is accompanied by problem sets that guide readers in developing modeling codes that retain the simplicity of the classical array factor method while incorporating mutual coupling effects and interactions between elements. Combining current research trends with pedagogical material suitable for a first-year graduate course, this is an invaluable resource for students, teachers, researchers, and practicing RF/microwave and antenna design engineers.

**Karl F. Warnick** is a Professor in the Department of Electrical and Computer Engineering at Brigham Young University, and a Fellow of the IEEE.

**Rob Maaskant** is an Associate Professor at Chalmers University of Technology and the Eindhoven University of Technology.

**Marianna V. Ivashina** is a Professor in the Department of Electrical Engineering at Chalmers University of Technology.

**David B. Davidson** is the Director of Engineering at the International Centre for Radio Astronomy Research and the Chair of Radio Astronomy Engineering, both at Curtin University, and Professor Extraordinary at the University of Stellenbosch. He is a Fellow of the IEEE.

**Brian D. Jeffs** is a Professor in the Department of Electrical and Computer Engineering at Brigham Young University

Cambridge University Press

978-1-108-42392-2 — Phased Arrays for Radio Astronomy, Remote Sensing, and Satellite Communications

Karl F. Warnick , Rob Maaskant , Marianna V. Ivashina , David B. Davidson , Brian D. Jeffs

Frontmatter

[More Information](#)

## EuMA High Frequency Technologies Series

Series Editor

Peter Russer, Technical University of Munich

Homayoun Nikookar, *Wavelet Radio*

Thomas Zwick, Werner Wiesbeck, Jens Timmermann, and Grzegorz Adamiuk (Eds),

*Ultra-wideband RF System Engineering*

Er-Ping Li and Hong-Son Chu, *Plasmonic Nanoelectronics and Sensing*

Luca Roselli (Ed), *Green RFID Systems*

Vesna Crnojević-Bengin, *Advances in Multi-band Microstrip Filters*

Natalia Nikolova, *Introduction to Microwave Imaging*

Karl F. Warnick, Rob Maaskant, Marianna V. Ivashina, David B. Davidson, and

Brian D. Jeffs, *Phased Arrays for Radio Astronomy, Remote Sensing, and*

*Satellite Communications*

### *Forthcoming*

Peter Russer, Johannes Russer, Uwe Siart, and Andreas Cangellaris, *Interference and*

*Noise in Electromagnetics*

Maurizio Bozzi, Apostolos Georgiadis, and Ke Wu, *Substrate Integrated Waveguides*

Luca Pierantoni, Fabio Coccetti, and Davide Mencarelli, *Radiofrequency*

*Nanoelectronics Engineering*

Alexander Yarovoy, *Introduction to UWB Wireless Technology and Applications*

Philippe Ferrari, Rolf Jakoby, Onur Karabey, Gustavo Rehder, and Holger Maune,

*Reconfigurable Circuits and Technologies for Smart Millimeter-Wave Systems*

Luca Roselli, *Enabling Technologies for The Internet of Things*

Costas Sarris and Andrew Austin, *Uncertainty Quantification in Computational*

*Electromagnetics*

Nuno Borges Carvalho, Alessandra Costanzo, Apostolos Georgiadis, and Franco

Mastri, *Wireless Power Transfer*

Cambridge University Press

978-1-108-42392-2 — Phased Arrays for Radio Astronomy, Remote Sensing, and Satellite Communications

Karl F. Warnick, Rob Maaskant, Marianna V. Ivashina, David B. Davidson, Brian D. Jeffs

Frontmatter

[More Information](#)

---

“Anyone interested in phased arrays should read this book - it provides an excellent insight into this technology and while aimed at principally at the imaging community has widespread application. The treatment of noise in a mutual coupled array is particularly useful.”

Professor Tony Brown,  
*Manchester University*

“Many headline discoveries in radio astronomy are products of phased arrays. This book deals with new-generation arrays born of the revolution in information processing systems and enabled by contemporary electromagnetic design tools. Covering real exemplar instruments, the book is broad in scope and detailed in its presentation of array design theory, allied signal processing and practical implementations. The authors’ cross-disciplinary approach extends to remote sensing and satellite applications, and they provide much-needed links to mainstream antenna engineering. Aimed at research engineers, the book is also invaluable to graduate students and professionals seeking an overview of leading-edge practice.”

Peter J. Hall,  
*Emeritus Professor of Radio Astronomy Engineering,  
Curtin University*

Cambridge University Press  
978-1-108-42392-2 — Phased Arrays for Radio Astronomy, Remote Sensing, and Satellite Communications  
Karl F. Warnick , Rob Maaskant , Marianna V. Ivashina , David B. Davidson , Brian D. Jeffs  
Frontmatter  
[More Information](#)

---

Cambridge University Press

978-1-108-42392-2 — Phased Arrays for Radio Astronomy, Remote Sensing, and Satellite Communications

Karl F. Warnick , Rob Maaskant , Marianna V. Ivashina , David B. Davidson , Brian D. Jeffs

Frontmatter

[More Information](#)

# Phased Arrays for Radio Astronomy, Remote Sensing, and Satellite Communications

KARL F. WARNICK

Brigham Young University, Provo, Utah, USA

ROB MAASKANT

Chalmers University of Technology, Gothenburg, Sweden, and Eindhoven University of Technology, Netherlands

MARIANNA V. IVASHINA

Chalmers University of Technology, Gothenburg, Sweden

DAVID B. DAVIDSON

University of Stellenbosch, South Africa, and Curtin University, Perth, Australia

BRIAN D. JEFFS

Brigham Young University, Provo, Utah, USA



CAMBRIDGE  
UNIVERSITY PRESS

Cambridge University Press

978-1-108-42392-2 — Phased Arrays for Radio Astronomy, Remote Sensing, and Satellite Communications

Karl F. Warnick , Rob Maaskant , Marianna V. Ivashina , David B. Davidson , Brian D. Jeffs

Frontmatter

[More Information](#)

## CAMBRIDGE UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi – 110025, India

79 Anson Road, #06–04/06, Singapore 079906

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning, and research at the highest international levels of excellence.

[www.cambridge.org](http://www.cambridge.org)

Information on this title: [www.cambridge.org/9781108423922](http://www.cambridge.org/9781108423922)

DOI: 10.1017/9781108539258

© Cambridge University Press 2018

This publication is in copyright. Subject to statutory exception and to the provisions of relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Cambridge University Press.

First published 2018

Printed in the United Kingdom by TJ International Ltd. Padstow Cornwall

*A catalogue record for this publication is available from the British Library.*

ISBN 978-1-108-42392-2 Hardback

Cambridge University Press has no responsibility for the persistence or accuracy of URLs for external or third-party Internet Web sites referred to in this publication and does not guarantee that any content on such Web sites is, or will remain, accurate or appropriate.

Contents

|   |   |                |
|---|---|----------------|
|   | <i>Preface</i>  | <i>page xi</i> |
|   | <i>Acknowledgements</i>   | xiii           |
|   | <i>Notation and Units</i>   | xv             |
| 1 | <b>Phased Arrays for High-sensitivity Receiver Applications</b>             | 1              |
|   | 1.1 Contemporary Design Methods for Phased Arrays                           | 2              |
|   | 1.2 Phased Arrays in Radio Astronomy  | 4              |
|   | 1.3 Phased Arrays for Passive Remote Sensing                                | 18             |
|   | 1.4 Phased Arrays for Satellite Communications                              | 22             |
|   | 1.5 System Requirements, Figures of Merit, and Antenna Terms                | 24             |
|   | 1.6 Summary   | 32             |
| 2 | <b>Active Antenna Receivers</b>   | 39             |
|   | 2.1 Voltage, Current, and Field Phasors                                     | 39             |
|   | 2.2 Coordinate Systems  | 40             |
|   | 2.3 Transmitting Antennas   | 42             |
|   | 2.4 Receiving Antennas  | 51             |
|   | 2.5 Equivalent Circuit Models   | 53             |
|   | 2.6 Spectral Output Noise Power   | 57             |
|   | 2.7 Single Active Antenna SNR Model   | 70             |
|   | 2.8 Summary   | 76             |
| 3 | <b>Antenna Examples</b>   | 80             |
|   | 3.1 Isotropic Radiator  | 80             |
|   | 3.2 Hertzian Dipole   | 81             |
|   | 3.3 Linear Antenna  | 83             |
|   | 3.4 Loop Antenna  | 86             |
|   | 3.5 Comparison of Dipole and Loop Antennas                                  | 89             |
|   | 3.6 Patch Antennas  | 92             |
|   | 3.7 Aperture Antennas   | 96             |
|   | 3.8 Summary   | 103            |
| 4 | <b>Transmitting Arrays, Network Analysis, and Pattern Overlap Integrals</b> | 106            |
|   | 4.1 The Array Factor and Classical Array Analysis                           | 107            |

|      |   |  |
|------|---|--|
| viii | <b>Contents</b>                                     |  |
|      | 4.2   | Network Analysis Methods for Phased Arrays 120                             |
|      | 4.3   | Transmitting Array Model 122   |
|      | 4.4   | Network Theory Model and the Impedance Matrix Representation 123           |
|      | 4.5   | Active Impedances 124  |
|      | 4.6   | Embedded Element Patterns 125  |
|      | 4.7   | Beamformer Weight Vector 130   |
|      | 4.8   | Pattern Overlap Integrals 132  |
|      | 4.9   | Directivity Optimization 137   |
|      | 4.10  | The Overlap Matrix and Mutual Resistance 141                               |
|      | 4.11  | Antenna Losses and Radiation Efficiency 143                                |
|      | 4.12  | Gain for Transmitting Arrays 144   |
|      | 4.13  | Modeling Antenna Arrays in the Network Theory Framework 145                |
|      | 4.14  | The Lossless, Resonant, Minimum Scattering Approximation 146               |
|      | 4.15  | Summary 149  |
| 5    | <b>Array Receiver Theory and Modeling</b>           | 154  |
|      | 5.1   | Receiving Array Network Model 154  |
|      | 5.2   | Receiving Pattern Directivity and Reciprocity for Active Arrays 161        |
|      | 5.3   | Signal and Noise Correlation Matrices 163                                  |
|      | 5.4   | Signal and Noise Model for Receiving Arrays 166                            |
|      | 5.5   | Fundamental Noise Theorem for Phased Arrays 175                            |
|      | 5.6   | Active Receiving Array SNR Model 175                                       |
|      | 5.7   | Summary 176  |
| 6    | <b>Figures of Merit for Active Receiving Arrays</b> | 179  |
|      | 6.1   | Array Gain (SNR Gain) 181  |
|      | 6.2   | Antenna Terms for Active Receiving Arrays 185                              |
|      | 6.3   | Isotropic Noise Response 186   |
|      | 6.4   | Active Antenna Available Gain 189  |
|      | 6.5   | Active Antenna Available Power 190   |
|      | 6.6   | Receiving Efficiency 191   |
|      | 6.7   | Active Antenna Effective Area 194  |
|      | 6.8   | Antenna Efficiency and Aperture Efficiency for Active Receiving Arrays 196 |
|      | 6.9   | Reciprocity for Receiving Arrays and the Equivalent Transmitting Array 196 |
|      | 6.10  | Active Antenna Noise Temperature 197                                       |
|      | 6.11  | Receiver Noise Temperature and Noise Figure 199                            |
|      | 6.12  | Noise Matching Efficiency 202  |
|      | 6.13  | Minimization of Receiver Noise 204   |
|      | 6.14  | Sensitivity Model for an Active Receiving Array 207                        |
|      | 6.15  | Array Y-factor Measurement Technique 208                                   |
|      | 6.16  | LRMSA Example: Half-Wave Dipole Array 211                                  |
|      | 6.17  | Summary 217  |



|           | Contents  | ix  |
|-----------|---|-----|
| <b>7</b>  | <b>Design and Optimization of Phased Array Antennas</b>                     | 221 |
|           | 7.1 General Considerations for Aperture Arrays                              | 222 |
|           | 7.2 System and Design Considerations for Aperture Arrays                    | 226 |
|           | 7.3 Phased Array Feed Design Aspects  | 235 |
|           | 7.4 Design Optimization Methods   | 244 |
| <b>8</b>  | <b>Numerical Modeling of Phased Array Antennas</b>                          | 253 |
|           | 8.1 Numerical Methods and Full Array System Modeling                        | 254 |
|           | 8.2 The Standard Method of Moments (MoM) Approach                           | 255 |
|           | 8.3 Surface Impedance Formulation of Imperfect Conductors                   | 259 |
|           | 8.4 Fast Direct MoM Methods – Macro Basis Function Approaches               | 262 |
|           | 8.5 Fast Iterative MoM Methods  | 273 |
|           | 8.6 Physical Optics Approximation   | 278 |
|           | 8.7 Feed–reflector Interaction Analysis                                     | 280 |
|           | 8.8 Differential Equation Based Methods – FDTD and FEM                      | 285 |
|           | 8.9 System SNR Modeling in the Noise Wave Representation                    | 286 |
|           | 8.10 Comments and Caveats on Numerical Modeling                             | 290 |
| <b>9</b>  | <b>Analog Front End, Array Elements, and Receiver Electronics</b>           | 300 |
|           | 9.1 Frequency and Bandwidth   | 300 |
|           | 9.2 Resonant Antennas   | 301 |
|           | 9.3 Broadband Antennas  | 302 |
|           | 9.4 Electrically Small Antennas and Bandwidth Limitations                   | 307 |
|           | 9.5 Baluns and Feeding  | 309 |
|           | 9.6 Planar Arrays and Microstrip Patch Antennas                             | 310 |
|           | 9.7 Receiver Electronics  | 313 |
|           | 9.8 Low Noise Amplifiers for Astronomical Arrays                            | 313 |
|           | 9.9 Cryogenic PAFs  | 314 |
|           | 9.10 Front End to Back End Signal Transport                                 | 318 |
|           | 9.11 Downconversion and Sampling  | 319 |
| <b>10</b> | <b>Array Signal Processing, Calibration, and Beamforming</b>                | 325 |
|           | 10.1 Beamforming  | 325 |
|           | 10.2 Array Calibration  | 333 |
|           | 10.3 Beamformer Weight Calculation Algorithms                               | 336 |
|           | 10.4 Relationships Among Beamforming Algorithms                             | 355 |
|           | 10.5 Array Sensitivity Map  | 356 |
|           | 10.6 Polarimetric Calibration and Beamforming                               | 359 |
|           | 10.7 RFI Mitigation with Array Receivers                                    | 369 |
|           | 10.8 Analog and Digital Beamforming Implementations                         | 389 |
| <b>11</b> | <b>Interferometric Arrays and Synthesis Imaging</b>                         | 396 |
|           | 11.1 Introduction   | 396 |
|           | 11.2 The Classical Theory of Interferometry: Overview and Literature Survey | 397 |

|    |   |     |
|----|---|-----|
| x  | <b>Contents</b>   |     |
|    | 11.3 Radio Interferometry Measurement Equation (RIME)   | 412 |
|    | 11.4 Recent Developments and Open Challenges            | 415 |
|    | 11.5 Summary  | 416 |
| 12 | <b>Real Time Digital Signal Processing</b>              | 420 |
|    | 12.1 Introduction                                       | 420 |
|    | 12.2 Interferometry and Spectroscopy – Frequency Domain | 421 |
|    | 12.3 Beamformers  | 425 |
|    | 12.4 Polyphase Filters                                  | 427 |
|    | 12.5 Hardware Implementations                           | 431 |
|    | 12.6 Summary  | 441 |
|    | <i>Index</i>  | 444 |

## Preface

Many excellent books on phased array antennas are already in print. The theoretical and mathematical content commonly found in books on this topic has stabilized in the last decade or two, but the past ten years has seen a dramatic departure from classical analytical tools in the methodologies used to design and optimize phased arrays. Our goal is to gather in one place recent advances in the mathematical framework for phased array analysis and create a book for which the theoretical treatment reflects the state-of-the-art in the academic literature and is equal to the task of designing antenna arrays for applications with demanding performance requirements.

One of the themes of this book is the design of phased arrays based on computer simulation. The last few decades have seen enormous progress in computational electromagnetics (CEM), which has revolutionized our ability to analyze antennas rigorously. The framework developed in this book can be used either with analytical approximations for the antenna response, or with computational electromagnetics tools for more accurate results.

Another theme is the dedicated analysis of active receiving arrays using network theory and the signal correlation matrix formulation. Most antenna textbooks focus almost exclusively on the antenna as a transmitting system, and then mention that the reciprocity theorem extends this analysis to receiving systems. Modern phased array systems involve active front ends, with low noise amplifiers in close proximity to the antenna or even integrated into the antenna feed. The system in this case is nonreciprocal, and some conventional antenna concepts no longer apply. Many systems use digital beamforming, which provides additional degrees of freedom when compared to classical analog beamformers. These systems require a new vocabulary and new figures of merit to properly describe them; where possible, we reconcile these with traditional antenna terms.

The authors have been closely involved in modern developments in phased arrays, computational electromagnetics, and array signal processing for applications in challenging fields such as radio astronomy for many years. This text represents the first synthesis of these modern design methods as a book; this is complemented by a review of classical methods such as the array factor approximation. Big-picture array design considerations, types of antenna elements, and components in the analog front end are reviewed. Both aperture type arrays and phased array feeds, used in conjunction with a reflector, are covered. Array signal processing is considered in detail and the theory of interferometric arrays (widely used in radio astronomy) is outlined. The book

concludes with some selected topics on real time digital signal processing. While many of the methods presented in this book were developed in the astronomical phased array community, the set of tools presented is equally applicable to other fields requiring high performance phased arrays. In particular, we outline remote sensing and satellite communications applications.

This book is inherently interdisciplinary. Some of the most far-reaching insights in the phased array community have come from exploring the connection between antenna design, microwave network analysis, and signal processing theory. Modern systems require joint optimization of all subsystems, from the antenna elements to the analog electronics and digital signal processing. This conjunction of disciplines is readily apparent in the table of contents and material in this book. Bringing together disciplines in new ways always brings uncomfortable collisions of notation, terminology, and mindset. We have, with just a few not easily avoidable exceptions, ironed out these inconsistencies in the material. As the book was conceived, we purposefully declined the easier path of an edited collection, as we hoped for a more pedagogically streamlined and logically organized treatment than collections of papers generally can achieve. The highest purpose of this book is to present a seamless synthesis of antenna theory, network analysis, signal processing theory, and application-specific concepts.

Finally, this is also a textbook. Most chapters include problem sets suitable for a course in phased array design and modeling. Many of the problems build on each other to aid the reader in creating a simple but powerful software tool that uses the overlap matrix formulation, network analysis, and correlation matrix framework to implement the array modeling and design methods covered in this book. It will help the reader in comprehending the content of the book to understand that there are two ways to use the analytical framework: (1) with analytical approximations for antenna element patterns, based on the lossless, resonant, minimum scattering approximation (LRMSA), and (2) using numerical simulations based on computational electromagnetics tools for the antenna array. These two approaches yield an equivalent network source for the array. The equivalent source from either approach can be used to initialize the network theory, overlap matrix, and correlation matrix formulations for the array system's signal response, external noise, antenna loss noise, and receiver noise by which the full array system is modeled.

We hope that this book inspires a new generation of researchers to combine techniques from the different disciplines brought together in the book in new and creative ways, and to continue to push forward the frontier of research in phased array receivers and other multiantenna systems in an expanding range of applications.

Karl F. Warnick  
Rob Maaskant  
Marianna V. Ivashina  
David B. Davidson  
Brian D. Jeffs

## Acknowledgements

We acknowledge support from our families while pursuing the research upon which much of this book is based, the outstanding students we have been privileged to work with and who have contributed many key ideas and innovative developments, and the incredible working environment provided by our colleagues at Brigham Young University and its sponsoring organization, the Church of Jesus Christ of Latter-day Saints.

We gratefully recognize Michael A. Jensen for pioneering work on the network analysis of multiantenna systems, his personal mentoring and friendship, and for many formative and enlightening conversations with him over the years.

The work upon which this book is based has been supported by nearly 20 years of continuous funding from the U.S. National Science Foundation. We thank the program managers we have worked with during this time for their courtesy, hard work, and professionalism.

Karl F. Warnick and Brian D. Jeffs

I gratefully acknowledge support from my family during my research career, as well as the contributions of many excellent students, and the support of my colleagues and research administrators at Stellenbosch University.

I would specifically like to recognize Johannes H. Cloete, with whom I worked for around 25 years at Stellenbosch University, for his dedication to research in antenna systems, metrology and electromagnetic analysis, and for bringing radio astronomy into my field of interest.

The South African National Research Foundation has supported my work for over two decades, most recently via a generous grant for the Square Kilometre Array (SKA) Research Chair, and I acknowledge this support, as well as many very useful interactions with the engineers and scientists of the South African SKA project.

David B. Davidson

We are grateful to our remarkably knowledgeable colleagues at ASTRON for involving us in their unique radio astronomy instrumentation building practice as well as for the many inspiring discussions which crossed the boundaries of multiple disciplines. We greatly thank ASTRON's management and Wim van Cappellen in particular for offering us the opportunity to carry out world-leading research in our science field for almost a decade, and for three years after leaving ASTRON by financially supporting Ivashina's VINNMER Fellowship jointly with Onsala Space Observatory. We further thank our students for their dedication and hard work, as well as our international colleagues within the SKA consortium. As always, there are so many people to thank, but special mention must go to: Professors Anton Tijhuis, Raj Mittra, the late Per-Simon Kildal, Christophe Craeye, Arnold van Ardenne, and John Conway; Drs. Kees van 't Klooster, Bert Woestenburger, Jan-Geralt bij de Vaate, Tobia Carozzi, and Miroslav Pantaleev; our former roommate Michel Arts; our former PhD students Oleg Iupikov, André Young, and David Prinsloo; and MSc student Bill Yang. Special thanks go to TICRA (Cecilia and Knud), DTU-Space (Niels), and ESA (Kees and Benedetta) for a very pleasant recent collaboration on the SKA-inspired array antenna research for Earth observation applications.

We also acknowledge the financial support of our funding agencies: the Netherlands Organisation for Scientific Research NWO, the Swedish VINNOVA Agency, the Swedish Research Council VR, the Swedish National Space Board, the European Space Agency, and the European Commission.

A cooperation lasting over a decade with the expert co-authors of this book has been a key source of inspiration and driving source of many scientific endeavors; it has resulted in a life-proven friendship which is of great value to us!

Last but not least, we mention our lovely children Daniëlla (2004) and Anton (2014) for – as it appears – accepting our peculiar domestic working culture and inheriting our passion for research and innovation.

Rob Maaskant and Marianna V. Ivashina

## Notation and Units

Throughout this book, the following notation is used. Spatial vectors are indicated as  $\overline{E}$  (in this case, the electric field). Vectors in the linear algebra sense are indicated as  $\mathbf{x}$ , and matrices as  $\mathbf{A}$ , using bold lower and upper case roman font respectively. The individual elements of a vector or matrix are indicated as  $x_i$  or  $A_{ij}$  respectively. Otherwise, the notation is as generally encountered in engineering books on this topic. A summary is presented below.

The time convention used for phasor quantities is  $e^{j\omega t}$ , hence, the phasor wave  $e^{-jkr}$  propagates in the direction of increasing  $r$ . (Physics books often adopt the  $e^{-i\omega t}$  convention, in which case the sign changes in the wave's phasor representation.) Phasors represent peak values.

|                 |  |
|-----------------|--|
| $\nabla \times$ | the curl operation   |
| $\nabla \cdot$  | divergence operation   |
| $\times$        | vector cross product of two vectors  |
| $\overline{E}$  | three-dimensional (field) vector $E$   |
| $\epsilon_0$    | permittivity of free space ( $\approx 8.854 \times 10^{-12}$ F/m)  |
| $\epsilon_r$    | relative permittivity of a dielectric material (dimensionless)   |
| $\mu_0$         | permeability of free space ( $4\pi \times 10^{-7}$ H/m)  |
| $\mu_r$         | relative permeability of a magnetic material (dimensionless)   |
| $c$             | the speed of light in free space ( $\approx 2.9979 \times 10^8$ m/s)   |
| $\bar{r}$       | compact vector notation for the point $(x, y, z)$ , in rectangular coordinates, $\bar{r} = x\hat{x} + y\hat{y} + z\hat{z}$ |
| $\hat{r}$       | unit vector in the direction $\bar{r}$ , equivalent to the spherical point $(\theta, \phi)$                                |
| $k$             | electromagnetic wavenumber (rad/m)   |
| $\bar{k}$       | electromagnetic wave vector  |
| $\lambda$       | electromagnetic wavelength   |
| $O(M^n)$        | of the order of $M^n$ , formally,<br>$N = O(M^n) \Rightarrow \lim_{M \rightarrow \infty} \log N / \log M = n$              |
| $(u, v, w)$     | orthogonal coordinates normalized to wavelength<br>in the easterly, northerly, and source directions respectively          |
| $(l, m, n)$     | direction cosines  |
| $E[\cdot]$      | statistical expected value operation   |
| $\Omega$        | Solid angle, or an alternate notation for the spherical point $(\theta, \phi)$   |
| $d\Omega$       | differential solid angle, in spherical coordinates, $d\Omega = \sin \theta d\theta d\phi$                                  |

xvi      **Notation and Units**

|                                 |   |
|---------------------------------|---|
| $d\bar{r}$                      | differential volume element, surface element, or line element (depending on the domain of integration)  |
| $dS$                            | differential area element   |
| $d\bar{A}$                      | differential area element with unit vector, $\hat{n} dS$  |
| $\mathbf{v}, \mathbf{y}$ , etc. | $N$ -dimensional vector quantity (e.g., array output signals)   |
| $\mathbf{R}, \mathbf{A}$ , etc. | $N \times N$ matrix, bold upper case roman font   |
| $\mathbf{v}^H, \mathbf{R}^H$    | complex conjugate transpose of vector $\mathbf{v}$ and matrix $\mathbf{R}$ respectively   |
| $\mathbf{v}^T, \mathbf{R}^T$    | conventional transpose of vector $\mathbf{v}$ and matrix $\mathbf{R}$ respectively  |
| $\mathbf{v} \odot \mathbf{y}$   | Hadamard element-wise array product, $\mathbf{v} \odot \mathbf{y} = [v_1 y_1, \dots, v_N y_N]^T$  |
| $\mathbf{A} \otimes \mathbf{B}$ | matrix Kronecker product  |
| $\text{vec}\{\mathbf{A}\}$      | matrix to vector operator. In $\mathbf{a} = \text{vec}\{\mathbf{A}\}$ , $\mathbf{a}$ is a vector formed by stacking (column-ordered scanning) all the columns of $\mathbf{A}$ |
| $\text{unvec}\{\mathbf{a}\}$    | vector to matrix operator. $\text{unvec}\{\text{vec}\{\mathbf{A}\}\} = \mathbf{A}$  |
| $\ \mathbf{y}\ $                | Conventional $L_2$ vector norm, $\ \mathbf{y}\  = \sqrt{\sum_i  y_i ^2}$  |
| $\approx, \propto, \equiv$      | respectively “approximately equal to,” “proportional to,” and “is defined as”   |
| $C^N$                           | $N$ -dimensional complex vector space   |
| $\cup, \cap$                    | set (or vector space) union and intersection operations respectively  |
| $\mathbf{v} \in C^N$            | vector $\mathbf{v}$ is an element of $C^N$  |
| $S \subset C^N$                 | vector space $S$ is a subset (subspace) of $N$ -dimensional vector space $C^N$  |
| $S^\perp$                       | vector subspace perpendicular (complementary) to $S$ . If $S$ and $S^\perp \subset C^N$ then $S \cap S^\perp = \phi$ , the empty set, and $S \cup S^\perp = C^N$              |

This book uses the SI system of units.

| Name                     | Symbol        | Unit   | Equivalent units or notes                                   |
|--------------------------|---------------|--|---|
| Electric field intensity | $E$           | V/m  |   |
| Magnetic field intensity | $H$           | A/m  |   |
| Brightness               | $\mathcal{B}$ | $\text{W m}^{-2} \text{ Hz}^{-1} \text{ rad}^{-2}$ |   |
| Spectral flux density    | $S$           | $\text{W m}^{-2} \text{ Hz}^{-1}$                  | 1 Jansky (Jy) = $10^{-26} \text{ W m}^{-2} \text{ Hz}^{-1}$ |
| Poynting flux density    | $S$           | $\text{W m}^{-2}$                                  | In one polarization   |
| Temperature              | $T$           | K  | Physical or equivalent temperature                          |
| Temporal frequency       | $f$           | Hz   | ( $f$ is also reflector focal length)                       |
| Angular frequency        | $\omega$      | $\text{rad s}^{-1}$                                | $2\pi f$  |
| Bandwidth                | $B$           | Hz   | Typically for one filterbank output                         |