Econometric Exercises, Volume 7

Bayesian Econometric Methods

SECOND EDITION

Bayesian Econometric Methods examines principles of Bayesian inference by posing a series of theoretical and applied questions and providing detailed solutions to those questions. This second edition adds extensive coverage of models popular in finance and macroeconomics, including state space and unobserved components models, stochastic volatility models, ARCH, GARCH, and vector autoregressive models. The authors have also added many new exercises related to Gibbs sampling and Markov Chain Monte Carlo (MCMC) methods.

The text includes regression-based and hierarchical specifications, models based upon latent variable representations, and mixture and time series specifications. MCMC methods are discussed and illustrated in detail – from introductory applications to those at the current research frontier – and MATLAB computer programs are provided on the website accompanying the text. Suitable for graduate study in economics, the text should also be of interest to students studying statistics, finance, marketing, and agricultural economics.

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To Lise

To the Reverend but not the Queen

To Melissa, Madeline, and Drew: Still, and Always.

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Preface to the Series

The past two decades have seen econometrics grow into a vast discipline. Many different branches of the subject now happily coexist with one another. These branches interweave econometric theory and empirical applications and bring econometric method to bear on a myriad economic issues. Against this background, a guided treatment of the modern subject of econometrics in volumes of worked econometric exercises seemed a natural and rather challenging idea.

The present series, *Econometric Exercises*, was conceived in 1995 with this challenge in mind. It has since become an exciting reality with the publication of the first installment of a series of volumes of worked econometric exercises. How can these volumes work as a tool of learning that adds value to the many existing textbooks of econometrics? What readers do we have in mind as benefiting from this series? What format best suits the objective of helping these readers learn, practice, and teach econometrics? These questions we now address, starting with our overall goals for the series.

Econometric Exercises is published as an organized set of volumes. Each volume in the series provides a coherent sequence of exercises in a specific field or subfield of econometrics. Solved exercises are assembled together in a structured and logical pedagogical framework that seeks to develop the subject matter of the field from its foundations through to its empirical applications and advanced reaches. As the Schaum series has done so successfully for mathematics, the overall goal of *Econometric Exercises* is to develop the subject matter of econometrics through solved exercises, providing a coverage of the subject that begins at an introductory level and moves through to more advanced undergraduate- and graduate-level material.

Problem solving and worked exercises play a major role in every scientific subject. They are particularly important in a subject like econometrics in which there is a rapidly growing literature of statistical and mathematical technique and an ever-expanding core to

the discipline. As students, instructors, and researchers, we all benefit by seeing carefully worked-out solutions to problems that develop the subject and illustrate its methods and workings. Regular exercises and problem sets consolidate learning and reveal applications of textbook material. Clearly laid out solutions, paradigm answers, and alternate routes to solution all develop problem-solving skills. Exercises train students in clear analytical thinking and help them in preparing for tests and exams. Teachers, as well as students, find solved exercises useful in their classroom preparation and in designing problem sets, tests, and examinations. Worked problems and illustrative empirical applications appeal to researchers and professional economists wanting to learn about specific econometric techniques. Our intention for the *Econometric Exercises* series is to appeal to this wide range of potential users.

Each volume of the series follows the same general template. Chapters begin with a short outline that emphasizes the main ideas and provides an overview of the most relevant theorems and results. The introductions are followed by a sequential development of the material by solved examples and applications, and by computer exercises when appropriate. All problems are solved and they are graduated in difficulty with solution techniques evolving in a logical, sequential fashion. Problems are asterisked when they require more creative solutions or reach higher levels of technical difficulty. Each volume is self-contained. There is some commonality in material across volumes to reinforce learning and to make each volume accessible to students and others who are working largely, or even completely, on their own.

Content is structured so that solutions follow immediately after the exercise is posed. This makes the text more readable and avoids repetition of the statement of the exercise when it is being solved. More importantly, posing the right question at the right moment in the development of a subject helps to anticipate and address future learning issues that students face. Furthermore, the methods developed in a solution and the precision and insights of the answers are often more important than the questions being posed. In effect, the inner workings of a good solution frequently provide benefit beyond what is relevant to the specific exercise.

Exercise titles are listed at the start of each volume, following the table of contents, so that readers may see the overall structure of the book and its more detailed contents. This organization reveals the exercise progression, how the exercises relate to one another, and where the material is heading. It should also tantalize readers with the exciting prospect of advanced material and intriguing applications.

The series is intended for a readership that includes undergraduate students of econometrics with an introductory knowledge of statistics, first- and second-year graduate students of econometrics, as well as students and instructors from neighboring disciplines (such as statistics, psychology, or political science) with interests in econometric methods. The volumes generally increase in difficulty as the topics become more specialized.

The early volumes in the series (particularly those covering matrix algebra, statistics, econometric models, and empirical applications) provide a foundation to the study of

econometrics. These volumes will be especially useful to students who are following the first-year econometrics course sequence in North American graduate schools and need to prepare for graduate comprehensive examinations in econometrics and to write an applied econometrics paper. The early volumes will equally be of value to advanced undergraduates studying econometrics in Europe, to advanced undergraduate and honors students in the Australasian system, and to masters and doctoral students in general. Subsequent volumes will be of interest to professional economists, applied workers, and econometricians who are working with techniques in those areas, as well as students who are taking an advanced course sequence in econometrics and statisticians with interests in those topics.

The *Econometric Exercises* series is intended to offer an independent learning-by-doing program in econometrics and it provides a useful reference source for anyone wanting to learn more about econometric methods and applications. The individual volumes can be used in classroom teaching and examining in a variety of ways. For instance, instructors can work through some of the problems in class to demonstrate methods as they are introduced; they can illustrate theoretical material with some of the solved examples; and they can show real data applications of the methods by drawing on some of the empirical examples. For examining purposes, instructors may draw freely from the solved exercises in test preparation. The systematic development of the subject in individual volumes will make the material easily accessible both for students in revision and for instructors in test preparation.

In using the volumes, students and instructors may work through the material sequentially as part of a complete learning program, or they may dip directly into material in which they are experiencing difficulty, to learn from solved exercises and illustrations. To promote intensive study, an instructor might announce to a class in advance of a test that some questions in the test will be selected from a certain chapter of one of the volumes. This approach encourages students to work through most of the exercises in a particular chapter by way of test preparation, thereby reinforcing classroom instruction.

Further details and updated information about individual volumes can be obtained from the *Econometric Exercises* website:

www.cambridge.org/ECEX

As series editors, we welcome comments, criticisms, suggestions, and, of course, corrections from all our readers on each of the volumes in the series as well as on the series itself. We bid you as much happy reading and problem solving as we have had in writing and preparing this series.

York, Tilburg, New Haven
June 2005

Karim M. Abadir
Jan R. Magnus
Peter C. B. Phillips

Preface

Bayesian econometrics has enjoyed an increasing popularity in many fields. This popularity has been evidenced through the recent publication of several textbooks at the advanced undergraduate and graduate levels, including those by Poirier (1995), Bauwens, Lubrano, and Richard (1999), Koop (2003), Lancaster (2004), Geweke (2005), and Greenberg (2013). The purpose of the present volume is to provide a wide range of exercises and solutions suitable for students interested in Bayesian econometrics at the level of these textbooks.

The Bayesian researcher should know the basic ideas underlying Bayesian methodology (i.e., Bayesian theory) and the computational tools used in modern Bayesian econometrics (i.e., Bayesian computation). The Bayesian should also be able to put the theory and computational tools together in the context of substantive empirical problems. We have written this book with these three activities – theory, computation, and empirical modeling – in mind. We have tried to construct a wide range of exercises on all of these aspects.

Loosely speaking, Chapters 1 through 9 focus on Bayesian theory, whereas Chapters 11 and 12 focus primarily on recent developments in Bayesian computation. The remaining chapters focus on particular models (usually regression-based). Inevitably, these chapters combine theory and computation in the context of particular models. Although we have tried to be reasonably complete in terms of covering the basic ideas of Bayesian theory and the computational tools most commonly used by the Bayesian, there is no way we can cover all the classes of models used in econometrics. Accordingly, we have selected a few popular classes of models (e.g., regression models with extensions and panel data models) to illustrate how the Bayesian paradigm works in practice. Particularly in Chapters 12 through 20 we have included substantive empirical exercises – some of them based closely on journal articles. We hope that the student who works through these chapters will have a good feeling for how serious Bayesian empirical work is done

and will be well placed to write a Ph.D. dissertation or a journal article using Bayesian methods.

For the student with limited time, we highlight that a division in this book occurs between the largely theoretical material of Chapters 1 through 9 and the computational and model-specific material in Chapters 10 through 20. A student taking a course on Bayesian statistical theory could focus on Chapters 1 through 9, whereas a student taking a Bayesian econometrics course (or interested solely in empirical work) could focus more on Chapters 10 through 20 (skimming through the more methodologically oriented material in the early chapters).

Although there have been some attempts to create specifically Bayesian software, in our estimation, most Bayesians still prefer to create their own programs using software such as MATLAB, R, Python, or Julia. We have used MATLAB to create answers to the empirical problems in this book. Our code for various exercises is provided on the following web site:

http://web.ics.purdue.edu/~jltobias/bem.html

While we use MATLAB in our own coding we wish to note that, at least at the time of this writing, a free alternative is available called Octave, which can be downloaded from:

www.gnu.org/software/octave/

The statistics package within Octave and a number of other useful packages can also be freely installed, and we highly encourage those interested in pursuing this option to do so. In our own experimentation, much of the code we provide can be executed without difficulty within Octave with the support of these additional packages and our supporting m-files.

A few notational conventions are applied throughout the book, and it is worthwhile reviewing some of these prior to diving into the exercises. In regression-based problems, which constitute the majority of the exercises in the later chapters, lowercase letters such as y and x_i are reserved to denote scalar or vector quantities whereas capitals such as X or X_j are used to denote matrices. In cases for which the distinction between vectors and scalars is critical, this will be made clear within the exercise. In the regression-based problems, y is assumed to denote the $n \times 1$ vector of stacked responses for the dependent variable, y_i the ith element of that vector, x_i a k-vector of covariate data, and X the $n \times k$ matrix obtained from stacking the x_i over i. Latent variables, which are often utilized in the computational chapters of the book, are typically designated with a "*" superscript, such as y_i^*. In Chapters 1 through 9, many exercises are presented that are not directly related to linear regression models or models that can be viewed as linear on suitably defined latent data. In these exercises, the distinction between random variables and realizations of those variables is sometimes important. In such cases, we strive to use capital letters to denote random variables, which are unknown *ex ante*, and lowercase letters to denote their realizations, which are known *ex post*. So, in the context

of discussing a posterior distribution (which conditions on the data), we will use $\overline{y}$, but if we are interested in discussing the sampling properties of the sample mean, $\overline{Y}$ would be the appropriate notation. In multiline derivations, "$\times$" is used to denote multiplication, and we will also use $=$ and $\propto$ to refer to the preceding line of the derivation so that, for example, the third line may be equal to the second, while the fourth line is proportional to the third. We will also use iid throughout to denote independent and identically distributed random variables, "pdf" to denote a probability density function and "cdf" to denote the cumulative distribution function. In the later chapters of the book, we will also frequently use the abbreviation "M–H" algorithm to refer to the Metropolis–Hastings algorithm. Finally, specific parameterizations of various densities are provided in the Appendix.

On the issue of parameterization, the reader who is somewhat familiar with the Bayesian literature may realize that researchers often employ different parameterizations for the same model, with no particular choice being "correct" or "ideal." A leading example is the linear regression model, in which the researcher can choose to parameterize this model in terms of the error variance or the error precision (the reciprocal of the variance). In this book, we try to remain consistent in terms of parameterization within individual chapters, though some departures from this trend do exist. These differences arise from our own individual tastes and styles toward approaching these models, and they are superficial rather than substantive. In our view it is quite valuable to expose the student to the use of different parameterizations, since this is the reality that he or she will face when exploring the Bayesian literature in more detail. In all cases, the parameterization employed is clearly delineated within each exercise.

We would like to thank the editors of the *Econometrics Exercises* series – Karim Abadir, Jan Magnus, and Peter Phillips – for their helpful comments and support during the planning and writing of this book. Hoa Jia, Babatunde Abidoye, Jingtao Wu, and Dimitris Korobilis deserve special recognition for reviewing numerous exercises and helping to reduce the number of typographical errors in the first edition. Jiang Huang provided helpful support in the typesetting of several exercises in the second edition and Daniel Kebede provided valuable assistance in reviewing several of the exercises new to this edition. Tim Bond and Kevin Mumford kindly shared American Community Survey (ACS) data for use in some exercises. The list of other colleagues and students who have helped us – through designing, solving, and pointing out errors in our problems or solutions – is too long to enumerate here. We would, however, like to thank our students at the Australian National University; University of California, Irvine; Purdue University; Iowa State University; Leicester University; Scottish Graduate Programme in Economics; University of Toronto; and the University of Technology Sydney for their participation, wise insights, and enthusiasm.