QUANTUM FIELD THEORY FOR ECONOMICS AND FINANCE

An introduction to how the mathematical tools from quantum field theory can be applied to economics and finance, this book provides a wide range of quantum mathematical techniques for designing financial instruments. The ideas of Lagrangians, Hamiltonians, state spaces, operators and Feynman path integrals are demonstrated to be the mathematical underpinning of quantum field theory and are employed to formulate a comprehensive mathematical theory of asset pricing as well as of interest rates, which are validated by empirical evidence. Numerical algorithms and simulations are applied to the study of asset pricing models as well as of nonlinear interest rates. A range of economic and financial topics is shown to have quantum mechanical formulations, including options, coupon bonds, nonlinear interest rates, risky bonds and the microeconomic action functional. This is an invaluable resource for experts in quantitative finance and in mathematics who have no specialist knowledge of quantum field theory.

BELAL EHSAN BAAQUIE is a professor at the International Centre for Education in Islamic Finance. He received his training in theoretical physics at Caltech and Cornell University, specializing in quantum field theory. He later developed an interest in finance and economics, and started applying quantum mathematics to these fields. He has written two books on quantum finance: *Quantum Finance* (Cambridge University Press, 2007) and *Interest Rates and Coupon Bonds in Quantum Finance* (Cambridge University Press, 2009), in addition to several other books focusing on topics from quantum mechanics and mathematics to books on leading ideas in science.

QUANTUM FIELD THEORY FOR ECONOMICS AND FINANCE

BELAL EHSAN BAAQUIE
The International Centre for Education in Islamic Finance

CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE
UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi – 110025, India

79 Anson Road, #06–04/06, Singapore 079906

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of
education, learning, and research at the highest international levels of excellence.

www.cambridge.org
Information on this title: www.cambridge.org/9781108423151
DOI: 10.1017/9781108399685

First published 2018

Printed in the United Kingdom by TJ International Ltd. Padstow Cornwall

A catalogue record for this publication is available from the British Library.

Library of Congress Cataloging-in-Publication Data
Names: Baaquie, B. E., author.
Title: Quantum field theory for economics and finance / Belal Ehsan Baaquie,
The International Centre for Education in Islamic Finance, Singapore.
Description: Cambridge, United Kingdom ; New York, NY : Cambridge University
Press, 2018. | Includes bibliographical references and index.
Identifiers: LCCN 2017061436 | ISBN 9781108423151 (Hardback : alk. paper)
Subjects: LCSH: Economics–Mathematical models. | Finance–Mathematical
models. | Quantum field theory.
Classification: LCC HB135 .B28 2018 | DDC 330.01/530143–dc23
LC record available at https://lccn.loc.gov/2017061436

ISBN 978-1-108-42315-1 Hardback

This book is dedicated to all the scholars, thinkers and visionaries who have been striving and contributing to enhance the knowledge and wisdom of humanity.

Contents

Cambridge University Press
978-1-108-42315-1 — Quantum Field Theory for Economics and Finance
Belal Ehsan Baaquie
Frontmatter
More Information

Cambridge University Press
978-1-108-42315-1 — Quantum Field Theory for Economics and Finance
Belal Ehsan Baaquie
Frontmatter
More Information

Foreword

I am aware of the author's work in applying theories of physics to finance since 2003, and the present book is a logical outcome of the author's line of thinking. The presentation of quantum field theory (QFT) given in this book is based on four strategic decisions.

(1) From the very outset it introduces the notion of quantum mathematics. This immediately attracts the attention of readers, with regard two points. First, they realize that in order to feel at home in QFT, they must devote enough time and attention to mastering these techniques. Second, once they have them well in hand, they can also use them outside of physics because they are just mathematical techniques.

(2) The book avoids giving applications of QFT to physics as this does not in the least help to understand QFT as a mathematical discipline.

(3) Throughout the book the formalism of the Feynman path integral is used, which intuitively is indeed the most appealing formalism of QFT.

(4) Last but not least, the book provides applications of QFT to a variety of economic and financial problems. One must realize that this is indeed quite different from calculations tied to high energy physics. Why? Needless to say, the whole machinery of QFT was created for applications to high energy physics; thus, one just follows the track and there is no need to raise any questions. On the contrary, QFT was not created to price options. Thus, instead of just following the track, at each step we have to modify and adapt our understanding of the mathematical tools of QFT.

The book has three distinctive features that are worth highlighting.

(1) There are many books on QFT, but this is a ground-breaking book that connects QFT with concepts in economics and finance.

(2) Almost half the book is devoted to studying models of economics and finance. As the book proceeds with different topics of QFT, chapters on economics and finance are introduced to show the close mathematical connections between these domains of knowledge.

(3) Many of the applications to economics and finance are based on models that can be empirically tested. To me, the most remarkable aspect of the book is that empirical tests show that these models are surprisingly accurate.

Going through the applications of QFT is a highly rewarding exercise as it tests our degree of understanding and expands our view of QFT. When readers grasp the logic of the applications, it will bolster their self-confidence and make them feel at home with QFT, and empower them to apply the mathematics of QFT to new fields of inquiry.

Bertrand M. Roehner
Laboratoire de Physique Théorique et Hautes Energies (LPTHE)
University of Paris 6
Paris, France

Preface

Quantum field theory is undoubtedly one of the most accurate and important scientific theories in the history of science. Relativistic quantum fields are the theoretical backbone of the Standard Model of particles and interactions. Relativistic and non-relativistic quantum fields are extensively used in myriad branches of theoretical physics, from superstring theory, high energy physics and solid state physics to condensed matter, quantum optics, nuclear physics, astrophysics and so on.

The mathematics that emerges from the formalism of quantum mechanics and quantum field theory is quite distinct from other branches of mathematics and is termed *quantum mathematics*. Quantum mathematics is a synthesis of linear algebra, calculus of infinitely many independent variables, functional analysis, operator algebras, infinite-dimensional linear vector spaces, the theory of probability, Lie groups, geometry, topology, functional integration and so on.

One of the mathematical bedrocks of quantum mechanics and quantum field theory is the Feynman path integral [Baaquie (2014)]. Unlike functional integration in general, the Feynman path integral is a functional integral with another key feature, which is that the path integral is constructed out of an underlying (infinite-dimensional) linear vector space. Operators are defined on this vector space, including the central operator of theoretical physics, which is the Hamiltonian.

The first application of calculus – made by Newton – was in the study of the dynamics of particles; calculus subsequently has gone on to become the universal language of quantitative modeling. Similarly, although quantum mathematics emerges from the study of quantum phenomena that are intrinsically indeterminate, the mathematical structure is not tied to its origins. Examples discussed below show that the mathematics of quantum field theory extends far beyond only quantum systems and can also be applied to a wide variety of subjects that span natural and social sciences. It is my view that quantum mathematics will, in time, supersede calculus and become the universal framework for quantitative modeling and mathematical thinking.

Important applications of quantum mathematics outside quantum physics have been made in vastly different fields, resulting in many ground-breaking results. Quantum mathematics has been applied to many *classical problem*; two famous examples are (1) the solution of classical phase transitions by Wilson, which led to his Nobel prize in physics in 1982 [Wilson (1983)] and (2) the complete classification of knots and links in three dimensions by Witten, for which he was awarded the Fields medal in 1989 [Witten (1989)]. More recently, superstring theory has led to a plethora of new results in pure mathematics using quantum mathematics. In fact, it would be no exaggeration to state that superstring theory has opened hitherto uncharted domains of pure mathematics of higher dimensions [Polchinski (1998); Zwiebach (2009)].

The formalism of quantum finance has been developed in this spirit and is based on the application of quantum mathematics to finance [Sornette (2003); Baaquie (2004, 2010)]. Two-dimensional quantum fields have been applied by Baaquie (2010) for analyzing interest rates and coupon bonds. Applications to economics has been made by Baaquie (2013a), and Baaquie and Yu (2018) have utilized a two-dimensional quantum field to describe and model futures asset prices. The bedrock of the application of quantum mathematics to both finance and economics is the employment of the Feynman path integral for modeling the behavior of interest rates and of spot as well as futures asset prices

The application of ideas from physics to economics and finance has led to the creation of a new field called econophysics, and to which quantum finance belongs [Mantegna and Stanley (1999); Roehner (2002a)]. Applications to psychology [Baaquie and Martin (2005)], to the social sciences [Haven and Khrennikov (2013)] and to decision sciences [Busemeyer and Bruza (2012)], to name a few, show the increasing utility of quantum mathematics in quantitative studies of social phenomena. Many universities, institutes and centers are teaching courses on the applications of quantum mathematics, and researchers are finding new and unexpected applications of quantum mathematics. For instance, the Institute of Quantum Social and Cognitive Science "promotes and develops high level research on the identification of quantum structures in non-physical domains, in particular, in socio-economic and cognitive sciences. The employment of the mathematical formalism of quantum mechanics outside the microscopic world is a growing research field and it has rapidly attracted the interest of the scientific community and the media."[1]

Quantum mathematics needs to be made accessible to a wide readership – beyond science, mathematics and engineering – so that students and researchers from all fields of study, including the social sciences, can employ the mathematical

[1] www2.le.ac.uk/departments/business/research/units/iqscs.

tools of quantum mathematics. Only with the knowledge of quantum mathematics being widespread can it fulfill its potential and, like calculus, become the sine qua non of all fields of quantitative modeling.

The mathematics of *quantum mechanics* has been discussed by Baaquie (2014) in *Path Integrals and Hamiltonians*. In contrast to the study of quantum mechanics, this book is an introduction to the mathematics of *quantum field theory*. What distinguishes quantum field theory from quantum mechanics is the coupling of infinitely many variables, or infinitely many degrees of freedom. The main purpose of this book is to introduce the mathematics of quantum field theory to researchers in finance and economics. The topics chosen are geared toward imparting the mathematical tools of quantum field theory that can facilitate further studies of finance and economics. This book provides a quick and simple primer to quantum field theory and can also be used as an introductory graduate text for readers from science, mathematics and engineering who are not specializing in theoretical physics.

A quantum field has quantum indeterminacy, whereas a classical stochastic field has classical randomness. The subtle difference between these is the subject of measurement theory in quantum mechanics [Baaquie (2013b)]. All the applications of quantum fields to economics and finance are in fact the application of stochastic fields; however, since the mathematics of stochastic and quantum fields are identical, the generic term "quantum field" is used for all applications of quantum fields to domains outside quantum physics.

How can one introduce quantum mathematics to students, readers and researchers unfamiliar with quantum field theory?

Unlike topics in mathematics, such as calculus, that have a well-defined syllabus, quantum fields cannot be so neatly modularized. Given the vast and increasingly complex mathematics of quantum fields, it is virtually impossible for one book to cover the entire terrain of quantum field theory.

This book presumes a working knowledge of linear algebra, calculus and probability theory. All the derivations are done from first principles and are comprehensive; there is no need to refer to any material outside this book. In order to introduce quantum field theory to readers from "distant" subjects, some of the leading examples of quantum fields are studied in detail. Starting from simpler examples, the various chapters lay the groundwork for analyzing more advanced topics. These examples encode many of the leading ideas of quantum field theory and are the building blocks of more advanced models.

To make the applications of quantum mathematics to economics and finance more tangible, the chapters on economics and finance are interwoven with chapters on quantum fields. In this manner, the reader can directly examine and connect the ideas of quantum field theory with its application, and in particular can see how these ideas are carried over to economics and finance. About 60% of the material of the book is directly an exposition of quantum field theory, with the remaining chapters being focused on its various applications to economics and finance.

The manner of presentation of the two pillars of the book – quantum fields on the one hand and economics and finance on the other– is quite different. Quantum field theory needs no empirical evidence for its utility and validity since the entire domain of particle physics stands as a testament to its empirical success. Hence the focus in the chapters on quantum field theory is on the various mathematical ideas and derivations, and only a fleeting connection is made with other subjects. An integral and pure presentation of quantum field theory is necessary to show that it is free from a bias toward any specific application. In fact, if one skips the Chapters on economics and finance, which are marked by an asterisk, the book then reads as an introductory graduate text on quantum field theory.

Unlike mathematics, which has results of great generality, such as theorems and lemmas, one only needs to flip through the pages of a textbook on quantum mechanics or quantum field theory to see that there are no theorems in quantum physics; instead, what one has are leading models and important examples – with the mathematical analysis flowing naturally in interpreting, explaining and deriving the "physics" of these models. Quantum field theory is illustrated and elaborated on by analyzing a number of exemplary models, such as the scalar, vector and spinor fields. Each of these quantum fields is described by a specific Lagrangian and Hamiltonian – and has distinctive properties on which the book focuses. More advanced chapters such as the structure of the renormalization group are presented later, when the reader has a better grasp of the underlying ideas.

The methodology of the chapters on economics and finance is quite different from the chapters on quantum field theory. In my view, the only justification for the application of quantum mathematics to empirical disciplines outside quantum physics – including economics and finance – is that it must be supported by empirical evidence. In the absence of such evidence – and there are many papers and books that make conceptual connections between quantum mathematics and classical systems with little or no empirical evidence [Bagarello (2013)] – the application in my view is still not complete, and stands only as an interesting mathematical metaphor. For the metaphor to become a concrete mathematical model, empirical evidence is indispensable.

For this reason, topics from economics and finance have been chosen (for inclusion in the book) that have empirical support from market data. Furthermore, a detailed analysis is given on how these quantum mathematical models are adapted to the market – and subsequently calibrated and tested. In chapters on economics and finance, very specific and concrete theoretical models are analyzed – all based on path integrals and Hamiltonians. The introductory chapter on nonlinear interest rates concentrates on the formalism; the reason is that a quantum finance model of nonlinear interest rates, as realized by the London Interbank Offered Rate (Libor), has been calibrated and exhaustively tested using market data; hence, only certain key features of the formalism are discussed. Two chapters use numerical algorithms and simulations to study nonlinear interest rates; these chapters illustrate a key feature of nonlinear interest rates, which is that in most cases numerical techniques are necessary for obtaining a solution.

The models that have been proposed in economics and finance – all of which are based on work done by myself and collaborators – are quite distinct from those that appear in quantum physics. In particular, all the models in economics and finance have an "acceleration" kinetic term – a term forbidden in quantum mechanics (due to the violation of conservation of probability); it is this term that gives a flavor to all the results in economics and finance that is quite different from what one is familiar with in physics.

The derivations in this book are not tied down to the application of quantum fields to physics – as this would require concepts that are not necessary for understanding the mathematical formalism of quantum fields. Furthermore, topics that apparently have no connection with finance or economics – but have played a pivotal role in quantum field theory – have been included in the hope that these ideas may lay lead to ground-breaking theories and models in economics and finance.

Nonlinearities of quantum fields arise due to self-interactions or because of coupling to other fields – and require the procedure of renormalization for obtaining finite results. The canonical case of a self-interacting nonlinear scalar field is studied in great detail so as to illustrate and analyze the issues that arise in renormalization. The formalism of quantum field theory culminates in the concepts of renormalization, renormalizability and the renormalization group – and which are among the deepest ideas of quantum field theory. It has been shown by Sornette (2003) that ideas from the renormalization group can provide a mathematical framework for understanding, and even predicting, market meltdowns.

Many topics, such as fermions, spinors, ghost fields, bosonic strings and gauge-fixing, are discussed that may seem to have no connection with economics and finance. The reason for including these topics is intentional. The broad range of topics covered gives a flavor to the reader of the great variety and complexity of the models that are a part of quantum field theory. A major omission has been the study of Yang–Mills gauge fields and that of spacetime supersymmetry. These topics need a background far in advance to what has been assumed, and hence could not be covered.

It is impossible and unwise to try to second guess what future directions economics and finance will turn toward; furthermore, gearing the topics discussed toward what is known closes off many future applications. For this reason, the main thrust of this book is to make the reader aware of, and familiar with, a wide array of quantum mathematical models so that a researcher can make leading edge connections and create new pathways between the domains of quantum fields and economics and finance.

Acknowledgments

I would like to acknowledge and express my thanks to many outstanding teachers, scholars and researchers whose work motivated me to study quantum field theory and to grapple with its mathematical formalism.

I had the singular privilege of doing my PhD thesis under the guidance of Kenneth G. Wilson; his visionary conception of quantum mechanics and of quantum field theory greatly enlightened and inspired me, and continues to do so till today. As an undergraduate I had the honor of meeting and conversing a number of times with Richard P. Feynman, which left a permanent impression on me.

I have benefited from discussions with many physicists and field theorists, including but not limited to, in no particular order, Abdus Salam, John Strathdee, M. Ali Namazie, Jamal Nazrul Islam, John Kogut, Leonard Susskind, Michael Fisher, Kurt Gottfried, H. R. Krishnamurthy, Michael Peskin, Steve Shenker, Spenta Wadia, Ashoke Sen, Tan Meng Chwan, Ahmed Zewail, S. G. Rajeev, Cumrun Vafa, Edward Witten, David Gross, Steven Weinberg, Jack Ng, Lay Nam Chang, Frank Wilczek, Nee Pong Chang, Anthony J. Leggett, Sidney Drell, Marvin Weinstein, Phua Kok Khoo, Andrew Strominger, Jayant Narlikar, Juan Maldecena, Bambi Hu, Chen Ning Yang, Bertrand Roehner, Murray Gell-Mann, Gerard 't Hooft, James D. Bjorken, Gyan Bhanot, Nicolas Sourlas, Faheem Hussain, Avinash Dhar, Asghar Qadir, Robert C. Richardson, Francois Martin, Kerson Huang, Syed Wassim, Jean Philippe Bouchaud, Munawar Karim, Brian Miller, Obiyathulla Ismath Bacha, Oh Choo Hiap, Michael Spalinski, Andrew Strominger, Baharom Abdul Hamid, Lai Choy Heng, Mohamed Eskandar Shah Mohd Rasid, Thomas Osipowicz, Emmanuel Haven, Sandro Sozzo, Claudio Coriano, Abbas Mirakhor and many others.

I thank Ashok Das, Yim Kok Kean, Lee Jianwei, Pan Tang, Jufri Setianegara, Cui Liang, Xin Du, L.C. Kwek, Yu Miao and Cao Yang for many helpful discussions. I thank Mahbub Majumdar and Mazhar Kantakji for their input in the preparation of the book.

I owe a special vote of thanks to Wang Qinghai; our enjoyable and varied discussions over many years led to greater clarity about the key concepts of quantum fields. The presentation of many of the chapters is the result of our discussions.

I specially thank Frederick H. Willeboordse for his valuable advice on how to position this book, and for his consistent support. I would like to thank to Simon Capelin of Cambridge University Press for his valuable advice on how to present the material of this book.

I thank my wife, Najma, for being an indispensable presence, a wonderful companion and for her uplifting approach to family and professional life. I thank my precious family members Arzish, Farah and Tazkiah for their delightful company and warm encouragement. Their love, affection and support over many years has made this book possible.