

Dynamics of Flexible Aircraft

Coupled Flight Mechanics, Aeroelasticity, and Control

Explore the connections among aeroelasticity, flight dynamics, and control with an up-to-date multidisciplinary approach. New insights into the interaction between these fields, which is a distinctive feature of many modern aircraft designed for very high aerodynamic efficiency, are fully illustrated in this one-of-a-kind book.

Presenting basic concepts in a systematic and rigorous, yet accessible way, this book builds up to state-of-the-art models through an intuitive step-by-step approach. Both linear and nonlinear attributes are covered and, by revisiting classical solutions using modern analysis methods, this book provides a unique perspective to bridge the gap between disciplines.

Numerous original numerical examples, including online source codes, help to build intuition through hands-on activities. This book will empower the reader to design better and more environmentally friendly aircraft, and is an ideal resource for graduate students, researchers, and aerospace engineers.

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Dynamics of Flexible Aircraft

Coupled Flight Mechanics, Aeroelasticity,
and Control

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**For
Nelya and Sasha
and
Cibele and Hannah**

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Somewhat as a voice in the wilderness,
I would like to pass on a few thoughts.

Letter to Robert Seamans, NASA
associate administrator
John Houbolt, 1961.

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Preface

No other issue is more pressing for mankind to maintain, not to mention improve, its standard of living in the second half of this century than climate change. At the time of writing, aviation contributes only a relatively small fraction to global carbon (or more generally, carbon-equivalent) emissions, yet a clear pathway for decarbonization of air transportation does not yet exist. This is a collective challenge for the aerospace community, and it is our responsibility to identify solutions that will enable today's children to continue enjoying into their adulthood the joy and opportunities provided by air travel, and affordably so.

In response to those renewed, and urgent, environmental concerns, aircraft manufacturers are currently in a race to identify emission-free energy carriers. Hydrogen, electrochemical storage, and various synthetic fuels are all being considered at the time of writing. Success on this quest will likely need radical changes on how aircraft both look like and operate, not only to seek additional efficiencies that make the cost and performance of any new solutions comparable to the current ones, but also to accommodate the design and operational requirements of those new propulsion systems. A direct implication of this is that traditional methods for design and analysis, which so successfully steered us into the current golden age of air travel, will be insufficient to propel aviation into a sustainable future. This book is written to support the search for those additional efficiencies in the synergistic interactions among structural sizing, aerodynamic design, and flight control systems. Consequently, it takes a multi-disciplinary view of the aircraft, which blends traditional concepts of flight dynamics, control, and aeroelasticity to describe the rich and possibly nonlinear dynamics of more flexible vehicles and their interactions with the atmosphere.

Anthropogenic climate change also manifests itself in a different manner in the pages of this book. The warming of the atmosphere is already a sad reality, and this brings changes to the environment in which aircraft operate. Not only do we need to design more efficient vehicles, but we also need them to fly in more adverse conditions as high turbulence events become more frequent. The implication of this is a need for a better understanding of the non-stationary atmospheric conditions, which would enable aerospace engineers to challenge the current design paradigms, should they be necessary. We take in this book a broad view in our exploration of the atmospheric conditions, which is not restricted by current certification processes, to try to equip our readers with a basic understanding of the atmospheric physics and its impact on aviation.

The book draws from over 50 years of combined research efforts in these matters by the authors. It also builds on our teaching and research experiences bringing those concepts into the graduate curriculum. The result is a synthesis of the current disciplinary knowledge in flight dynamics, aeroelasticity, atmospheric turbulence, and controls, from which we will systematically build increasingly complex models to explain and explore the stronger interactions that appear among those individual constituents. The range of topics is vast, and we have concentrated on the tools and methods that we consider to be more suitable for multidisciplinary analysis of aircraft dynamics, while giving pointers to other important solutions in the literature, including some promising ones that still have not achieved a similar level of maturity. We have also made an effort to ease in the transition from linear to nonlinear thinking in aircraft design, which is one of the major undercurrents of twenty-first-century aerospace engineering. We need to acknowledge, however, a bias toward the problems that we are more familiar with, but we can only write about what we know.

We cannot obviate the conditions in which the manuscript took the final form. The roots of this book should be found in the early work of the second author with Prof. Dewey Hodges. Sadly, Dewey has recently left us, but his outstanding intellectual contributions stay behind and many of his ideas will be found in the pages in this book. The final push has occurred during the coronavirus pandemic that has overwhelmed us since early 2020. These terrible events have highlighted once again the precariousness of our relationship with the natural environment but also the enormous power of science for good. It has made us optimists about our ability to overcome the climate crisis. Eventually.

The material in this book should be accessible to final-year and postgraduate students in aerospace engineering, as well as practicing engineers. It builds on the core knowledge in an aerospace engineering degree and expects good command of classical mechanics, aerodynamics, and linear systems. We have included mathematical derivations whenever we believe they are key to grasp concepts and point to references in the literature otherwise. Often, we present the mathematical apparatus together with its application to tackle a problem, so that we can directly draw from relevant examples while establishing a strong link between a general theory and the particular problem of interest.

As a final point, we would like to thank many colleagues for their help and support while preparing the manuscript. The first draft of Chapter 12 was prepared by Dr. Hector Climent, from Airbus Defense and Space. Professor Eli Livne, from the University of Washington, has been a constant source of wisdom in all things aeroelastic. The sections on nonlinear control build upon a long and fruitful collaboration of the first author with Dr. Andrew Wynn, from Imperial College. Many of the figures have been meticulously prepared by Kelvin Cheng. Dr. Alessandra Vizzacaro, from Imperial College, kindly produced the backbone curves in Chapter 8. We would like to thank, for reading parts of the manuscript: Stefanie Duessler, Drs. Marc Artola, Alvaro Cea, and Norberto Goizueta, while at the LoCALab, Prof. Cristina Riso, Dr. Mateus Pereira, Bilal Sharqi, Prof. Leandro Lustosa, while at A²SRL. Dr. Arturo Munoz for his help in setting up the aerodynamic simulations examples. Dr. Jessica Jones generated the

X-56A results of Chapter 6, Dr. Salvatore Maraniello computed the T-Tail results of Chapter 7, Drs. Alfonso del Carre and Patricia Texeira built the models and computed the results on the X-HALE UAV (Chapter 8), Prof. Weihua Su generated the blended wing body results (Chapter 9), Dr. Alvaro Cea produced the sample results on a commercial transport aircraft, Dr. Norberto Goizueta computed the Pazy wing results, and Drs. Marc Artola and Robert Simpson computed the nonlinear control results. Our students have greatly inspired us and they make our job exciting and, in many ways, unique.

Online material

We have developed numerous numerical examples for this book. The captions to many of the figures include, in square brackets, the name of the file with the software used to generate them. All the scripts have been made available as open source with a permissive license and can be found at

<https://github.com/ImperialCollegeLondon/flexibleaircraftbook>

Symbols

A	Aerodynamic influence coefficient matrix, generalized aerodynamic forces
\mathcal{I}	Unit or identity matrix
C	Beam cross-sectional compliance matrix
\mathcal{M}	Beam cross-sectional mass matrix
A	State matrix
B	Input matrix
C	Output matrix
D	Damping matrix; feedthrough matrix
e_i	Unit vector along axis i , for $i = 1, 2, 3$
f_B	Components of the applied force vector in frame B
f_q	Generalized modal forces; state weighting matrix
\bar{f}_S	Components of the applied distributed force vector in frame S
f_S	Components of the beam internal force vector in frame S
h_B	Components of the angular momentum vector in frame B
I_B	Inertia tensor in body-fixed axes
K	Stiffness matrix; feedback gain matrix
k_S	Components of the local curvature vector of a beam in frame S
M	Mass matrix
m_B	Components of the applied moment vector in frame B
\bar{m}_S	Components of the applied distributed moment vector in frame S
m_S	Components of the beam internal moment vector in frame S
n	Uncorrelated white noise signals
p_B	Components of the linear momentum vector in frame B
p_E	Components of the absolute position vector in Earth frame
q	Modal degrees of freedom
R	Control weighting matrix
r_B	Components of the position vector in frame B
R_{BA}	Coordinate transformation matrix from frame A to B
T	Tangential operator
u	Input vector
u_B	Components of the displacement vector in frame B
v_B	Components of the linear velocity vector in frame B

\mathbf{v}_{gB}	Components of the gust velocity vector in frame B
$\bar{\mathbf{v}}_S$	Components of the beam linear velocity vector in frame S
\mathbf{w}	Disturbance vector, wind velocity
\mathbf{W}_c	Controllability Gramian
\mathbf{W}_o	Observability Gramian
\mathbf{x}	State vector
\mathbf{y}	Output vector
$\bar{\omega}_S$	Components of the beam angular velocity vector in frame S
χ	Generalized forces associated with the nodal elastic degrees of freedom
χ_e	Internal loads
δ_c	Control surface inputs
γ	Beam force strains
κ	Beam moment strains
Λ	Matrix of eigenvalues
Ω	Diagonal matrix of natural frequencies in vibration analysis
ω_B	Components of the angular velocity vector in frame B
ω_{gB}	Components of the angular gust velocity (or rotary gust) vector in frame B
Φ	Matrix of eigenmodes; modal matrix in vibration analysis
$\Phi_w(\omega)$	Spectral density of the random signal \mathbf{w}
$\phi_w(\tau)$	Correlation function of the random signal \mathbf{w}
ψ	Cartesian rotation vector
Σ_w	Autocorrelation matrix of the random signal \mathbf{w}
ξ	(Nodal) elastic degrees of freedom
ζ	Amplitude of the elastic modes
α	Angle of attack
η	Curvilinear coordinate along beam axis
β	Sideslip angle
δ_a	Aileron input
δ_e	Elevator input
δ_r	Rudder input
δ_T	Throttle input
ℓ	Turbulence length scale
γ	Climb angle
\mathcal{J}	Jacobian operator originating from linearization
κ	Wavenumber or spatial frequency
$\mathcal{C}(ik)$	Theodorsen's lift deficiency function
\mathcal{H}	Heaviside or unit step function
\mathcal{S}_x	Sears' function at location x/c from airfoil leading edge
\mathcal{T}	Kinetic energy
\mathcal{U}	Internal or strain energy
$\hat{\mathcal{K}}$	Kernel function or matrix in potential flow aerodynamics

η	Beam cross-sectional coordinates
μ	Mass parameter
ω	Angular frequency
ω_k	Natural frequency of the k th vibration mode
ϕ	Bank (roll) Euler angle
ψ	Heading (azimuth) Euler angle
ρ	Air density
σ_w^2	Mean square value of signal \mathbf{w}
θ	Pitch (elevation) Euler angle; twist angle of a flexible beam
A	Beam cross-sectional area
b	Wing semispan
c	Wing chord
C_D	Drag Coefficient
c_d	Sectional drag coefficient
C_L	Lift coefficient
c_l	Sectional lift coefficient
C_M	Aerodynamic moment coefficient
c_m	Sectional moment coefficient about the aerodynamic center
g	Magnitude of the Gravitational acceleration
i	imaginary unit
I_{xx}	Moment of inertia about axis x
I_{xy}	Product of inertia about axes x and y
k	Reduced frequency
K_g	Critical gust alleviation factor
m	Mass
n	Load factor
N_m	Number of normal modes
N_u	Number of inputs
N_x	Number of states
N_y	Number of outputs
q	Dynamic pressure ($q = \frac{1}{2}\rho V^2$)
S	Wing reference area
s	Dimensionless time (reduced time)
s	Laplace variable
T	Magnitude of the total thrust force
V	Airspeed
w_b	Relative normal flow velocity or upwash (of airfoil or wing)
w_g	Vertical gust velocity
CM	Center of mass
Ma	Mach number
Re	Reynolds number
\bullet^*	Transpose conjugate of a complex matrix
\bullet^T	Transpose of a matrix

\bullet_B	Components in body-attached frame of reference, stability axes
\bullet_E	Components in Earth (inertial) frame of reference
\bullet_S	Components in local material frame of reference
$E[\bullet]$	Expectation operator
$\text{Im}(\bullet)$	Imaginary part of a complex number
$\text{Re}(\bullet)$	Real part of a complex number
$\text{tr}(\bullet)$	Trace of a matrix
$\tilde{\bullet}$	Cross-product (or skew-symmetric) operator