An Introduction to Formal Logic

Formal logic provides us with a powerful set of techniques for criticizing some arguments and showing others to be valid. These techniques are relevant to all of us with an interest in being skilful and accurate reasoners. In this very accessible book, extensively revised and rewritten for the second edition, Peter Smith presents a guide to the fundamental aims and basic elements of formal logic. He introduces the reader to the languages of propositional and predicate logic, and develops natural deduction systems for evaluating arguments translated into these languages. His discussion is richly illustrated with worked examples and exercises, and alongside the formal work there is illuminating philosophical commentary. This book will make an ideal text for a first logic course and will provide a firm basis for further work in formal and philosophical logic.

Peter Smith was formerly Senior Lecturer in Philosophy at the University of Cambridge. His books include *Explaining Chaos* (Cambridge, 1998) and *An Introduction to Gödel's Theorems* (Cambridge, 2007; 2013).

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Second edition

Peter Smith

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Preface

The world is not short of introductions to logic aimed at philosophy students. They differ widely in pace, style, coverage, and the ratio of formal work to philosophical commentary. Like other authors, my initial excuse for writing yet another text was that I did not find one that offered quite the mix that I wanted for my own students (first-year undergraduates doing a compulsory logic course).

Logicians are an argumentative lot and get quite heated when discussing the best route into our subject for beginners. But, standing back from our differences, we can all agree on this: in one form or another, the logical theory eventually arrived at in this book – *classical first-order quantification theory*, to give it its trade name – is a wonderful intellectual achievement by formally-minded philosophers and philosophically-minded mathematicians. It is a beautiful theory of permanent value, a starting point for all other modern work in logic. So we care greatly about passing on this body of logical knowledge. And we write our logic texts – like this one – in the hope that you too will come to appreciate some of the elements of our subject, and even want to go on to learn more.

This book starts from scratch, and initially goes pretty slowly; I make no apology for working hard at the outset to nail down some basic ideas. The pace gradually picks up as we proceed and as the idea of a formal approach to logic becomes more and more familiar. But almost everything should remain quite accessible even to those who start off as symbol-phobic – especially if you make the effort to read slowly and carefully.

You should check your understanding by tackling at least some of the routine exercises at the ends of chapters. There are worked answers available at the book's online home, www.logicmatters.net. These are often quite detailed. For example, while the book's chapters on formal proofs aim to make the basic principles really clear, the online answers provide extended 'examples classes' exploring proof-strategies, noting mistakes to avoid, etc. So do make good use of these further resources.

As well as exercises which test understanding, there are also a few starred exercises which introduce additional ideas you really ought to know about, or which otherwise ask you to go just a little beyond what is in the main text.

The first edition of this book concentrated on *logic by trees*. Many looking for a course text complained about this. This second edition, as well as significantly revising all the other chapters, replaces the chapters on trees with chapters on a *natural deduction* proof system, done Fitch-style. Which again won't please everyone! So the chapters on trees are still available, in a revised form. But to keep the length of the printed book under control, you will find these chapters – together with a lot of other relevant material including additional exercises – at the book's website.

Many more people have helped me at various stages in the writing and rewriting of this book than I can now remember.

Generations of Cambridge students more or less willingly road-tested versions of the lecture handouts which became the first edition of this book, and I learnt a lot from them. Then Dominic Gregory and Alexander Paseau read and commented on full drafts of the book, and the late Laurence Goldstein did very much more than it was reasonable to expect of a publisher's reader. After publication, many people then sent further corrections which made their way into later reprintings, in particular Joseph Jedwab. Thanks to all of them.

While writing the second edition, I have greatly profited from comments from, among many others, Mauro Allegranza, David Auerbach, Roman Borschel, David Furcy, Anton Jumelet, Jan von Plato, and especially Norman Birkett and Rowsety Moid. Exchanges on logical Twitter, a surprisingly supportive resource, suggested some memorable examples and nice turns of phrase – particularly from Joel D. Hamkins. Scott Weller was extremely generous in sending many pages of corrections. Matthew Manning was equally eagle-eyed, and also made a particularly helpful suggestion about how best to arrange the treatment of metatheory. At a late stage, David Makinson rightly pressed me hard to be clearer in distinguishing between types of rules of inference. And my daughter Zoë gave especially appreciated feedback in the last phases of writing. I am again very grateful to all of them.

This book owes its existence to two people in particular, and they require very special thanks. It was Hilary Gaskin who initially encouraged me to turn lecture handouts into a book, and then insisted that I didn't keep revising drafts of the first edition for ever. She then offered me the very welcome chance to write a second edition, and again was very patient when the re-writing took much longer than I'd predicted. I am not the only Cambridge University Press author who owes her a great deal.

But most of all, I must thank my wife Patsy without whose love and support neither version of this book would ever have been finished.