

## AN INTRODUCTION TO THE ATOMIC AND RADIATION PHYSICS OF PLASMAS

Plasmas comprise more than 99% of the observable universe. They are important in many technologies and are potential sources for fusion power. Atomic and radiation physics is critical for the diagnosis, observation and simulation of astrophysical and laboratory plasmas, and plasma physicists working in a range of areas from astrophysics, magnetic fusion and inertial fusion utilise atomic and radiation physics to interpret measurements. This book develops the physics of emission, absorption and interaction of light in astrophysics and in laboratory plasmas from first principles, using the physics of various fields of study, including quantum mechanics, electricity and magnetism, and statistical physics. Linking undergraduate-level atomic and radiation physics with the advanced material required for postgraduate study and research, the text adopts a highly pedagogical approach and includes numerous exercises within each chapter to reinforce students' understanding of key concepts.

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# AN INTRODUCTION TO THE ATOMIC AND RADIATION PHYSICS OF PLASMAS

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#### **Preface**

This book provides an introduction to the physics of emission, absorption and interaction of light in astrophysics and in laboratory plasmas. Such study necessarily requires a wide range of modern physics understanding involving electricity and magnetism, relativity, atomic structure, quantum mechanics, particle collision theory, statistical physics and more. Indeed, the analysis of light emission and collisional processes relevant to plasmas has provided much of the experimental evidence for quantum mechanics. The atomic and radiation physics of plasmas is, consequently, an ideal subject for study as an extension to material taught to physics undergraduates. The book combines undergraduate-level studies of the quantum mechanics of ions/atoms with the atomic and radiation physics of plasmas, though non-quantum models are used extensively. Atomic and radiation physics is presented at a level aimed at undergraduates in their final two years through to graduate students and researchers. Material needed for research in plasma physics and astrophysics is derived.

Plasma physicists working in a range of areas from astrophysics, magnetic fusion and inertial fusion to low-temperature plasmas of technological significance utilise atomic and radiation physics to interpret measurements. Plasma physics is a growing research area with the construction of the ITER tokamak, new laser-plasma facilities and the development of new methods of creating plasma, such as with free-electron lasers. Atomic and radiation physics is also an essential component in the theoretical development and simulation of astrophysical and laboratory plasmas. One aim of this book is to emphasise the overlap of atomic/radiation physics between astrophysical and laboratory plasmas, an imbrication exploited in the expanding field of laboratory astrophysics where physical scenarios relevant to astrophysics are simulated in the laboratory.

Due to the range of understanding required for research in the atomic and radiation physics of plasmas, the underlying physics is often not developed in research publications in astrophysics and plasma spectroscopy. An aim of this book has been

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to start with the knowledge obtained by physics graduates before they begin to specialise and to develop formulae and explain techniques used in plasma spectroscopy. The areas of plasma research utilising aspects of atomic and radiation physics are briefly introduced before spectroscopic applications are covered, but this book concentrates on the underlying atomic and radiation physics.

As this is a textbook, rather than a monograph, some presented treatments are not the most comprehensively complete available, but illustrate the way to standard formulae and techniqes. Similarly, the citations presented are representative and do not give a full coverage of the development of topics. I offer my apologies to those whose contribution to knowledge is described but not cited.

Exercises are included at the end of each chapter and form an integral component of the text. Where a numerical answer is required, this is added in brackets, sometimes along with comment indicating, for example, wider implications of the exercise. Material is presented using the International System of Units (SI) unless explicitly defined otherwise. The convention common in laboratory plasma work to define temperatures (T) in units of energy ( $k_BT$ ) using electron volts (eV) is widely used in the text. Here  $k_B$  is Boltzmann's constant. In SI units,  $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$  and corresponds to  $11605^{\circ}\text{K}$ . While formulas are developed in SI units, research areas often use centimeter-gram-second (CGS) units, so some expressions are converted from SI where numerical values are presented.

Much of the content presented here has been developed for courses taught at the University of York. The treatment of the atomic physics of the hydrogen and multi-electron atoms has been taught to third-year students for several years, while other material has featured in lecture courses presented to MSc and PhD students of fusion energy. I am grateful to the University of York for the opportunity to develop some of these lecture courses into the present book and also thank many students for their questions, comments and corrections. I am grateful to Professor Geoff Pert FRS for his comments on a draft of the manuscript and to Dr Erik Wagenaars for providing lecture material.