Cellular Flows

A cell, whose spatial extent is small compared with a surrounding low, can develop inside a vortex. Such cells, often referred to as vortex breakdown bubbles, provide stable and clean flame in combustion chambers; they also reduce the lift force of delta wings. This book analyzes cells in slow and fast, one- and two-fluid flows and describes the mechanisms of cell generation: (a) minimal energy dissipation, (b) competing forces, (c) jet entrainment, and (d) swirl decay. This book explains the vortex breakdown appearance, discusses its features, and indicates means of its control. Written in acceptable, non-math-heavy format, it stands to be a useful learning tool for engineers working with combustion chambers, chemical and biological reactors, and delta-wing designs.

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Cellular Flows

Topological Metamorphoses in Fluid Mechanics

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