1 Introduction: Flow Cells and Mechanisms of Their Formation

This book discusses flow cells, their appearance, transformations, and physical reasons of their metamorphoses. A flow cell is a compact region with no inflow and outflow at any part of the region boundary. A cellular flow must have at least one cell. A well-known driving mechanism of cellular motion is buoyancy. For example, it generates cellular thermal convection in a horizontal layer of a fluid heated from below. The heating reduces the fluid density, and the buoyancy force pushes the warm fluid upward from the hot bottom toward the cold free surface, where the fluid spreads and becomes cold. The gravity force pushes the cooled fluid downward. These upward and downward motions constitute fluid circulation in the thermal (Bénard) cells. Since warm particles can be cooled down by thermal diffusion before they reach the surface, the cell generation only occurs if the temperature difference, characterized by the Rayleigh number, Ra, exceeds some threshold, Ra_{cr}. Heating from below produces the thermal cells via the Rayleigh instability (Chandrasekhar 1961). No Bénard cell develops if the diffusion dominates convection, i.e., for $Ra < Ra_{cr}$.

In contrast, the lateral heating can cause very elongated circulation, like the Gulf Stream and its backflow in the ocean depths. This circulation develops with no instability and can be stable up to very large Ra. New cells can emerge within the global circulation also with no instability. These striking and practically important features are discussed in Chapter 4 of this book.

Another well-known cellular motion is the Taylor vortices (Chandrasekhar 1961). They are observed in the gap between coaxial cylinders; for example, if the outer cylinder is still while the inner cylinder rotates. The Taylor vortices develop via the centrifugal instability as the rotation strength, characterized by the Taylor number, Ta, exceeds its critical value, Ta_{cr}. The centrifugal force pushes fast-rotating fluid to the outer cylinder and moves slowly rotating fluid to the inner cylinder, thus developing circulation rings. No Taylor vortex develops if the viscous diffusion dominates convection and consumes the angular momentum of a rotating fluid particle before it reaches the outer cylinder. Thus, the Taylor eddies also emerge via instability for Ta exceeding Ta_{cr}.

In contrast, a cell can develop with no instability in a sealed cylindrical container filled with a fluid whose circulation is driven by a rotation lid. Moreover, multiple cells emerge as the rotation intensifies. These counterintuitive features are important for bioreactor and combustion applications, as Chapters 6–8 of this book discuss.

The Bénard and Taylor cells have been explored rather in detail. This book discusses different and recently revealed cellular motions that emerge with no instability. The described and explained scenarios of cell appearance and their metamorphoses include: (a) the eddy formation in a very slow motion (Chapter 2), (b) the eddy generation in two-fluid flows by competing forces (Chapter 3), (c) velocity reversals via bifurcations in the thermal gravitational and centrifugal convections (Chapter 4), (d) the axial velocity reversal in a swirling flow, which is often referred to as vortex breakdown (Chapters 5–8), (e) the radial velocity reversal in vortex-sink near-wall flows used in vortex burners and chemical reactors (Chapter 9), and (f) the cell emergence due to entrainment (Chapters 4 and 9). These mechanisms have important technological applications, are of fundamental interest, and have not been well understood until recent times. Hence, they deserve a detailed discussion, as addressed in this book.

The cell emergence and transformations change the flow topology. Therefore, this book is a contribution to the topological fluid mechanics. This discipline has been mostly mathematical for many years. Its origin can be traced back to the paper by Arnold (1966). The notions, foundations, and typical features of topological fluid dynamics are discussed in the book by Arnold & Khesin (1998) and in the proceedings of two IUTAM symposiums edited by Moffatt & Tsinober (1989) and Moffatt, Bajer & Kimura (2013). The current book focuses on the flow physics by discussing and explaining mechanisms of vortex breakdown, eddy formation in the centrifugal convection, in creeping and two-fluid cellular flows, at the interface, separating fluids; and discusses physical reasons of topological metamorphoses related to cell appearance, multiplication, and disappearance. The following review in Sections 1.1–1.8 briefly describes cellular motions and their physical mechanisms addressed in this book.

1.1 Vortex Breakdown

Vortex breakdown was first recognized and studied due to its occurrence above aircraft wings (Peckham & Atkinson 1957). The lift force of a delta-wing aircraft (Figure 1.1) is enlarged compared with that of a conventional aircraft due to vortices that develop on the upper surface of delta wings. The air flow separates from the leading edge and rolls inward and above the wings, forming a pair of counter-rotating vortices. The white threads in Figure 1.2 visualize air particles going from the wing leading edge (Werle 1963). The black curves in Figure 1.2 illustrate how air trajectories spiral inward, forming a rapidly rotating vortex core (Shtern et al. 1997). The emerging centrifugal force tends to push air away from the core and thus reduces pressure below its atmospheric value. The difference between the significantly reduced pressure above and nearly atmospheric pressure below the wings creates the enlarged lift force – the main advantage of delta-wing design.

The vortex arises at the leading edge point, where the wing and fuselage meet. The swirl and longitudinal velocities increase downstream, reach their maxima near

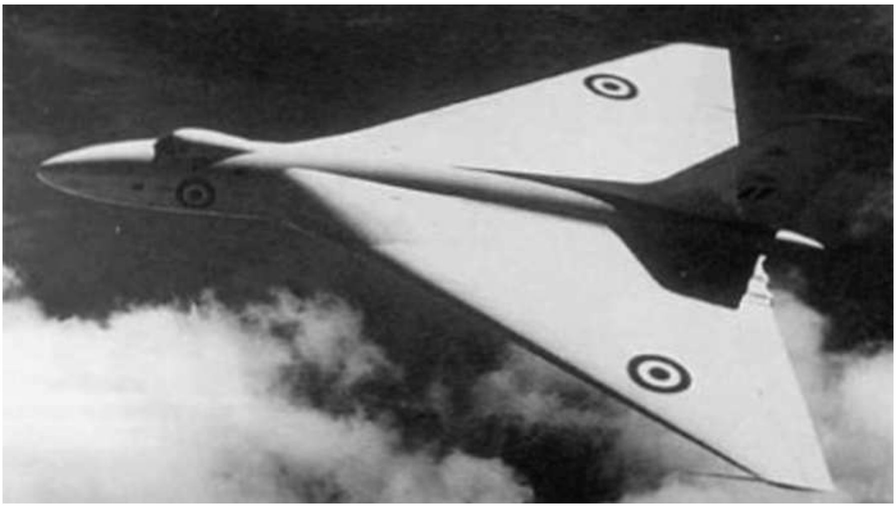

Figure 1.1 Delta-wing aircraft.
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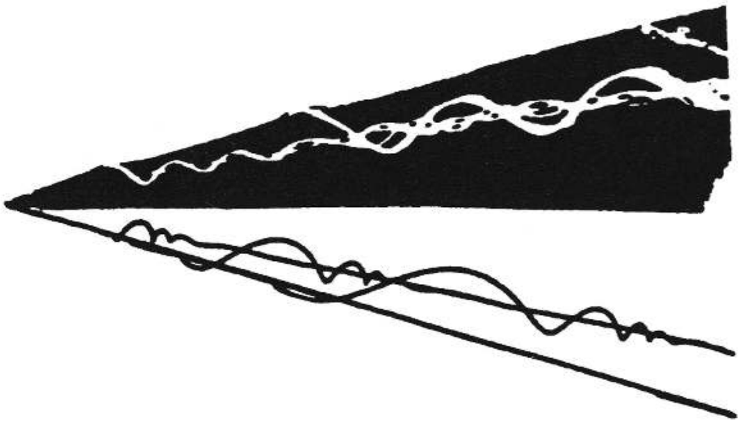

Figure 1.2 Visualization (above) and mechanism (below) of delta-wing vortex.

the wing middle, then decay, and vanish far away from aircraft. At the location of maximal swirl velocity, pressure reaches its minimal value. This drop in pressure sucks ambient air, decelerates the downstream flow, and can reverse it.

This reversal is a sign of vortex breakdown (Leibovich 1978), and results in the vortex core, visualized by the white threads in Figure 1.3, expanding into either a bubble-like circulation region, shown in the lower part of Figure 1.3, or a helical pattern, shown in the upper part of Figure 1.3 (Lambourne & Brayer 1961).

The vortex breakdown reduces the lift force. The closer to the leading edge the vortex breakdown occurs, the smaller is the lift force. As the angle of attack

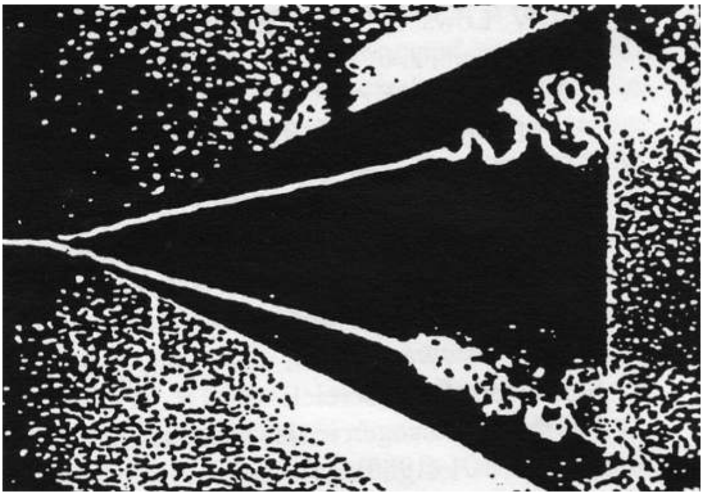

Figure 1.3 Spiral (above) and bubble-like (below) vortex breakdown over a delta wing.

α (between the wing plane and the flow direction, see the left inset in Figure 1.4) increases, the lift force initially also increases, but at some threshold value of $\alpha = \alpha_2$ abruptly drops down. Now, if α decreases, the lift force initially decreases, but at another threshold value of $\alpha = \alpha_1$ abruptly jumps up. Accordingly, the lift and drag forces also jump. Figure 1.4 shows the dependence of α_1 and α_2 on the aircraft speed characterized by the Mach number (*Ma*). The right inset in Figure 1.4 depicts the abrupt changes in the lift force as α varies – *hysteresis* – and two stable states existing in the range, $\alpha_1 < \alpha < \alpha_2$ (Muylaert 1980). The sudden changes in lift and drag are dangerous because they can cause the loss of flight control.

While vortex breakdown is problematic for aircraft control, it can be beneficial for other applications. One important application is combustion (Gupta et al. 1984). A flame front propagates via diffusion with a speed around 1 m/s. For turbines, the front must be stationary. Therefore, a flow is required that moves slowly against the flame propagation. A circulatory motion, induced by vortex breakdown, has such necessary pattern including stagnation points, near which the flow velocity is small. The reversed flow transports the combustion heat back to a fuel source and warms up a fuel and an oxidizer that makes combustion stable and clean. As an example, Figure 1.5 shows a photo of vortex combustion chamber where swirl generates a double-reversed flow having stagnation points in the chamber depth (Borissov et al. 2010).

Pressurized air tangentially enters the chamber through the inlet, which is visible in the lower-right corner above the time record in Figure 1.5. The air does not go outward, but paradoxically spirals inward near the sidewall to the dead end, turns around (this is the first flow reversal), and meets an injected fuel (here, propane), which combusts due to air oxygen. Flue gases, consisting of combustion

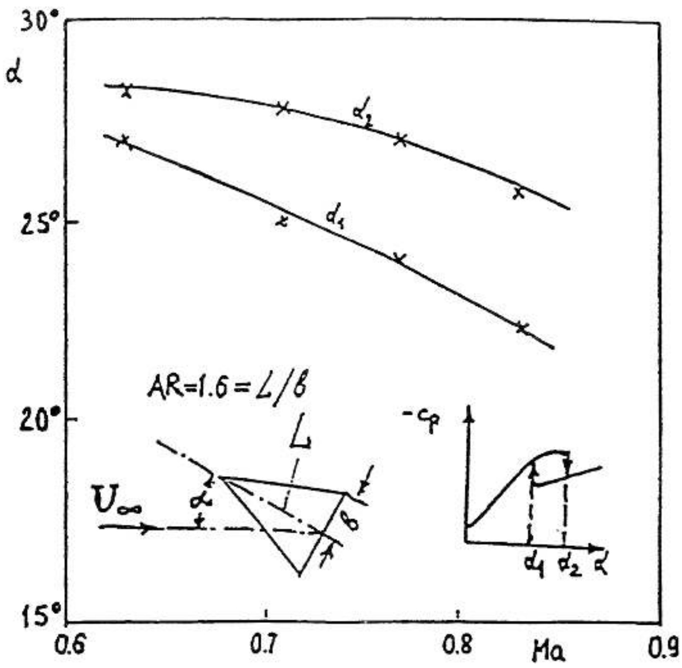

Figure 1.4 Hysteretic transitions in the vortex breakdown location as the angle of attack α varies.

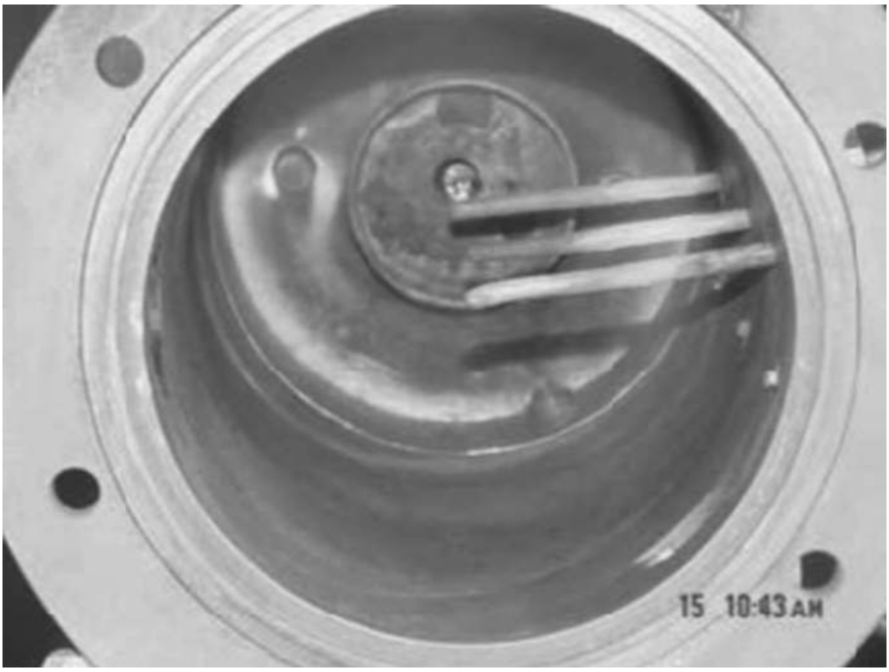

Figure 1.5 (Color online) Combustion with double counterflow.

Figure 1.6 Tornado funnel with flow reversal.

products, exhaust through an annular region. The ambient air is sucked inward near the chamber axis, turns around (this is the second flow reversal), and mixes with the flue gases.

The thermocouple rods, located at the right-hand side of the chamber, visualize the temperature distribution: the rods are dark where the flow is cold (in the near-wall and near-axis inflows), and bright where the flow is hot (the annular exhaust motion of flue gases). This double counterflow provides the transparent combustion observed in Figure 1.5, and significantly reduces harmful emissions as Section 5.4.11 describes in more detail.

In addition to the mentioned technological devices, vortex breakdown sometimes occurs in a tornado (Figure 1.6), which is a kind of atmospheric swirling jet. In a swirling motion of air away from the ground, the centrifugal force is balanced by the radial gradient of pressure. Where the spinning air touches the ground, the centrifugal force vanishes, since the swirl velocity drops down to zero on the ground. In contrast, the radial gradient of pressure does not vanish and, being unbalanced by the centrifugal force, pushes air toward the axis of rotation. Therefore, a converging, swirling near-ground flow develops. This flow turns upward near the rotation axis and forms a jet spiraling along the axis. The upper part of converging flow transports the angular momentum to the axis vicinity, thus enhancing the air swirl velocity and, therefore, the centrifugal force. This results in a deep minimum of pressure located near the ground-axis intersection. The reduced pressure sucks ambient air and thus can reverse its near-axis motion, forming a flow pattern, schematically shown by the arrows in Figure 1.6. There are the annular ascending flow and near-axis downflow. Such vortex breakdown pattern widens the tornado funnel, making the twister less destructive.

This scenario likely explains the two tornado patterns and the transitions between them, observed in Jordan, Iowa on June 13, 1976. One pattern had a narrow funnel and the other pattern had a wide funnel. These patterns were recorded by detailed photographs reported by Burggraf & Foster (1977). The flow can switch between these two states, which are stable for small disturbances but unstable for large disturbances. Such behavior is quite similar to the hysteretic transitions between the two stable flow states above a delta wing (the right inset in Figure 1.4).

To summarize, vortex breakdown has three striking and important features: (i) the flow reversal, which is obligatory, and two optional: (ii) the emergence of helical patterns and (iii) hysteretic transitions. Chapters 5 and 6 of this book discuss these features, explain their physical nature, and illustrate their mechanisms with the help of analytical and numerical solutions describing appropriate model flows.

The flow reversal, related to vortex breakdown, typically occurs near the axis of rotation. Another important phenomenon of swirling flows is the reversal of radial velocity occurring remote from the axis in a disk-like vortex chamber (DVC). DVCs attracted the attention of researchers due to applications in a rocket nuclear engine (Savino & Keshock 1965), which was conceived starting from 1957. DVCs are also used in chemical technology (Kovacevic et al. 2014, 2015).

A fluid nearly tangentially enters a DVC through slots in the disk sidewall, and develops the centrifugal force that presses nuclear-fuel or catalyst particles to the periphery, balancing the particle-fluid drag. This DVC feature is beneficial for achieving a high power-to-volume ratio of propulsion that is especially important for space applications.

Unfortunately, strong jets develop near the disk end walls. The jets entrain particles and can result in their loss, which is absolutely unacceptable for nuclear devices. Moreover, experiments revealed that the radial velocity reverses in the middle part of the disk. The reversal, being counterintuitive, was initially interpreted as an artifact of the measurement inaccuracy (Donaldson & Williamson 1964). However, the precise and detailed experimental study by Savino & Keshock (1965) definitely established that the reversal does occur, and results in a flow cell that occupies a rather large portion of DVC disk. The reversal was later confirmed numerically (Vatistas et al. 2008). This phenomenon is problematic for nuclear reactors, but can be beneficial for other applications, e.g., combustion. A toroidal circulation region of swirling gas can provide stable and clean flame in a disk-like combustion chamber. To this end, the reversal nature should be well understood. This book explains in Section 9.3 that the radial velocity reversal develops via the jet-entrainment mechanism.

1.2 Centrifugal Convection

Thermal convection in rotating systems has been extensively studied for astrophysical and geophysical applications, e.g., for large-scale circulations in the atmosphere and oceans caused by the temperature difference between equatorial and polar regions (Herrmann & Busse 1997; Hart 2000). The centrifugal convection can also have important applications in vortex tubes (Secchiaroli et al. 2009).

The centrifugal-to-gravity acceleration ratio, g_c/g, is small for planets and stars, but is very large (up to 10^6) in vortex tubes, as is the length-to-radius ratio. These features can be utilized for the development of efficient heat exchangers. One more important factor is that the steady flow can be stable up to very large value of the Rayleigh number *Ra*, which characterizes the convection strength. The steady laminar convection is experimentally observed in a horizontal layer with the horizontal gradient of temperature up to *Ra* around 10^9 (Bejan et al. 1981; Kirdyashkin 1984). The physical reason of this striking flow stability is that the fluid circulation develops the stable vertical stratification of density that suppresses disturbances. The effect is even stronger for the centrifugal convection due to large g_c/g and the stable stratification of angular momentum.

Chapter 4 of this book discusses these paradoxical and important features of thermal convection. In particular, it is shown that the stable thermal convection, in a rotating container for large *Ra*, concentrates in a thin near-wall jet adjacent to the entire container boundary. In the rest domain, the flow is comparatively slow and multicellular. The cells emerge near the container center via bifurcations. This book explains the bifurcation mechanism and the physical reason for the cell formation and the nature of multicell flow stability. Two instability mechanisms could work here. One is shear-layer instability occurring for a small Prandtl number *Pr*. As *Pr* exceeds its threshold value, the shear-layer instability is suppressed by the emerging density stratification. The other is thermal instability developing for large *Pr*, if sidewall temperature is prescribed. This instability disappears if the sidewalls are adiabatic. The density stratification is stabilizing in this case. These features and mechanisms are generic, also being observed in the thermo-gravitational convection of one and two fluids in a horizontal layer with lateral heating (Chapter 4 of this book).

1.3 Creeping Eddies

Moffatt (1964) revealed a counterintuitive fluid mechanics phenomenon: the existence of eddies in a creeping flow that is dominated by viscous diffusion. Moffatt considered a two-dimensional flow, driven by some source – e.g., by a rotating cylinder between inclined walls – and showed that there is an unbounded set of eddies whose dimension and intensity decrease down to zero as the wall-intersection edge is approached. Figure 1.7 depicts a chain of the Moffatt eddies in the 20° corner.

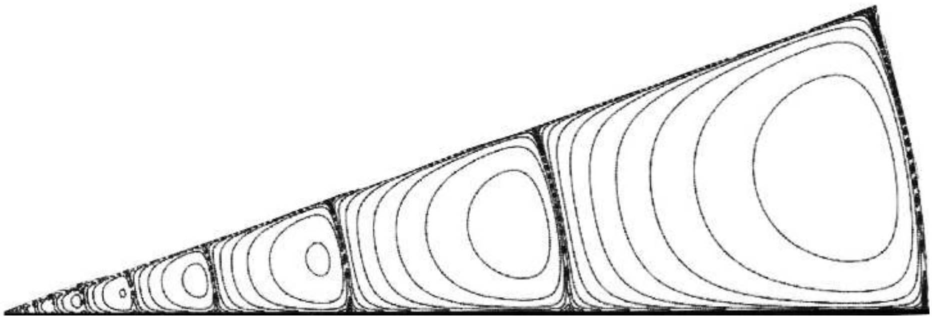

Figure 1.7 Moffatt eddies in the 20° corner.

The existence of unbounded number of cells in a compact domain, where a slow fluid motion occurs, seems contradicting to the common sense. Motivated by Moffatt's discovery, following numerous studies revealed similar eddies in a plane cavity (Moffatt 1964, Shankar & Deshpande 2000), cone (Wakiya 1976), cylinder (Blake 1979, Hills 2001), in cavities with oppositely moving walls (Gürcan et al. 2003, Wilson et al. 2005), and between concentric cones (Hall et al. 2007) and coaxial cylinders (Shtern 2012b).

This book systematically discusses cellular creeping flows of one (Chapter 2) and two (Chapter 3 of this book) fluids; considers analytical and numerical solutions, describing their paradoxical features; and explains the physical mechanism of cell occurrence.

1.4 Two-Fluid Cellular Flows

Air-water circulatory flows have recently attracted the attention of researchers due to their applications in aerial vortex biological reactors (Ramazanov et al. 2007). These bioreactors provide the gentle and fine mixing of ingredients required for growth of proteins, enzymes, vitamins, antibiotics, sensitive embryonic, hybrid and other medical cells, ferments and supplements for food industry, and other tissue cultures.

For a proper mixing, a rotating disk is typically used to induce both swirl and the meridional motions of air and water in bioreactors. Stimulated by bioreactor applications, Lo Jacono et al. (2009) experimentally, and Liow et al. (2008, Liow et al. 2009) numerically, explored air-water flows in cylindrical containers. Lo Jacono et al. (2009) addressed a whirlpool-like flow where the rotating bottom disk drives the meridional water flow and pushes the interface down near the axis and up near the sidewall. However, such driving can result in large shear stresses, which can be harmful for the tissue culture. A more careful driving of the water flow can be provided by an air flow driven above the interface (Ramazanov et al. 2007).

Liow et al. (2008) and Liow et al. (2009) investigated a model aerial bioreactor, where the mixing occurs due to the rotating top disk. The induced centrifugal force pushes the air to the periphery near the top disk and thus drives the meridional circulation. The circulation transports air oxygen, required for the efficient growth of tissue culture, to the interface. The air flow converges to the axis near the interface and smoothly drives a slow counter-circulation of the water. This circulation transports the oxygen from the interface in the water depth, mixes the oxygen with other ingredients and thus helps the culture growth. Such aerial bioreactors have a number of advantages (Ramazanov et al. 2007), but their control parameter must be carefully chosen to avoid undesirable effects such as the tearing of culture by shear stresses.

As this book discusses for whirlpool (Chapter 7) and water-spout (Chapter 8) flows, the air and water motions in bioreactors can be rather complicated, including a number of eddies depending on the reactor geometry. Cylindrical, semispherical, conical, and truncated-conical geometries are particularly addressed. The flow pattern includes a number of air and water eddies, depending on the fluid fractions and the device geometry. It is paradoxical that two-fluid eddies can develop even in a very slow – creeping – motion. As the water fraction varies, numerous changes in the flow topology occur. These striking features of two-fluid creeping flows are discussed in detail in Chapter 3 of this book. Some physical mechanisms behind the above-mentioned cellular motions and their metamorphoses are briefly discussed next.

1.5 Eddy Generation by Swirl Decay

The nature of bubble-like vortex breakdown was a subject of numerous studies during more than a half-century. The proposed conjectures include (a) inertial wave roll-up (Benjamin 1962), (b) collapse of the near-axis boundary layer (Hall 1972), (c) flow separation (Leibovich 1978, 1984), (d) fold catastrophe (Trigub 1985), and (e) transition from convective to absolute instability (Olendraru et al. 1996). However, no consensus has been achieved on the vortex breakdown nature. Chapter 5 of this book focuses on a recently proposed swirl decay mechanism that explains vortex breakdown features and means of its control (Shtern et al. 2012).

In a few words, the swirl decay mechanism is the following. In rapidly rotating flows, the centrifugal force induces the radial gradient of pressure p – according to the cyclostrophic balance $\partial p/\partial r = \rho v^2/r$ – where ρ is the fluid density, v is the swirl velocity, and r is the distance from the rotation axis. The reduction of pressure near the axis, compared with its peripheral value, is larger in the vicinity of a swirl source than that away from the source because swirl decays, e.g., due to friction at the sidewall in a vortex device. Therefore, the near-axis pressure is smaller in the vicinity of the swirl source than that away from it. As this pressure difference increases, it