

Index

- π , Monte Carlo estimation, 319
- σ -algebra, 28
- σ -field, 28

- affine function, 116, 190
- annulus, 94
- area, 215

- Bayes' formula, 49, 50
 - prior and posterior probability, 50, 362
- Bayesian statistics, 362
- Bernoulli distribution, 59
 - expectation, 105
 - moment generating function, 184
 - variance, 114
- Berry–Esseen theorem, 169, 322
- beta distribution, 244, 361
 - beta-gamma algebra, 244
- beta integral, 264
- beta-gamma algebra, 244
- binomial coefficient, 390
- binomial distribution, 60
 - expectation, 104, 273
 - law of rare events, 156
 - Poisson limit, 157
 - variance, 115, 278
- binomial theorem, 402
- birthday problem, 66
- Box–Muller transformation, 234
- Buffon's needle problem, 40

- cardinality of a set, 385
- Cartesian product, 4, 387
- Cauchy distribution, 133, 270
- central limit theorem, 315
 - error bound, 169, 322
 - for binomial, 143
- characteristic function, 196

- chi distribution, 265
- chi-squared distribution, 264
- classical statistics, 362
- complement of a set, 381
- conditional density function, 338
- conditional expectation
 - given a random variable, 332
 - with respect to an event, 330
- conditional independence, 62, 360
- conditional probability, 44
 - for equally likely outcomes, 45
- conditional probability mass function
 - with respect to a random variable, 332
 - with respect to an event, 330
- conditional variance, 376
- confidence interval, 151
- constant function, 23
- continuity correction, 145
- continuous random variable, 92
- convolution
 - probability density functions, 248
 - probability mass functions, 247
- correlation, 291
- countably infinite, 13
- coupon collector's problem, 280, 307
 - law of large numbers, 327
- covariance, 284
- covariance matrix, 296
- cumulative distribution function, 95
 - and the probability density function, 97
 - and the probability mass function, 96
 - joint, 227
 - piecewise constant, 96
 - properties, 100
 - step function, 96

- de Moivre–Laplace theorem, 143
- de Morgan's laws, 382

- decision tree, 387
- deck of cards, 393
- degenerate random variable, 23, 117
- density function
 - see probability density function, 90
- descending factorial, 7, 389
- difference of sets, 381
- discrete random variable, 24
 - possible value, 24
- discrete sample space, 13
- disjoint events, 2
- disjoint sets, 381
- disk, 12, 102
- distribution
 - see probability distribution, 1
- distribution function
 - see cumulative distribution function, 95
- double factorial, 185

- equality in distribution, 186
- estimator
 - Bayesian, 362
 - confidence interval, 150
 - maximum likelihood, 151
- exchangeable random variables, 256
- expectation
 - continuous random variable, 106
 - discrete random variable, 103
 - infinite, 107
 - linear function, 116
 - multivariate continuous, 212
 - multivariate discrete, 206
 - nonexistent, 108
 - well-defined, 108
- exponential distribution, 161
 - mean and variance, 161
 - minimum of independent exponentials, 226, 244
 - moment generating function, 183, 184
- exponential function, 400
- exponential Markov inequality, 320
- extreme value distribution, 328

- factorial, 389
- factorial moment, 175
- financial crisis, 70
- floor function, 164
- Fourier transform, 196
- fractional part of a real number, 202, 375

- frequency, 148
- function
 - bijection, 385
 - decreasing (nonincreasing), 403
 - k -to-one, 385

- Gambler's ruin, 87, 377
- gamma distribution, 168, 190
 - beta-gamma algebra, 244
 - sum of exponentials, 263
- gamma function, 168, 362
- Gaussian distribution, 122
 - standard, 119
- generating function
 - characteristic function, 196
 - moment generating function, 181
 - probability generating function, 197
- geometric distribution, 61
 - expectation, 105, 134, 135
 - minimum, 223, 241
 - shifted, 61, 355
 - variance, 116
- geometric series, 400
- Gumbel distribution, 328

- hypergeometric distribution, 65
 - binomial approximation, 153
 - expectation, 275
 - sampling without replacement, 65
 - variance, 287

- inclusion-exclusion formula, 18
 - proof with indicator variables, 301
- independence
 - complements, 53
 - conditional, 62, 360
 - discrete random variables, 56, 220
 - functions of random variables, 223
 - in terms of the joint cumulative distribution function, 229
 - information, 53
 - jointly continuous random variables, 220
 - multiple events, 53
 - of polar coordinates, 229, 233
 - pairwise, 54
 - random variables, 56
 - two events, 51
- independent and identically distributed (i.i.d.), 258

- indicator function, 296
- indicator method, 273
- indicator random variable, 105, 114, 276, 301
 - expectation, 105
 - variance, 114
- induction, proof by, 394
- infinitesimal method for density function, 93, 134
 - conditional density function, 345
 - joint density function, 230, 262
- information
 - independence, 53
 - represented by σ -algebras, 28
- integral test for convergence, 403
- intersection of sets, 381
 - alternative notation, 17

- Jacobian, 232
- joint cumulative distribution function, 227
- joint distribution, 205
- joint moment generating function, 236
- joint probability density function, 212
 - infinitesimal method, 230, 262
 - nonexistence, 219
- joint probability mass function, 205

- kittens, 66

- Laplace's law of succession, 363
- law of large numbers, 148, 169, 313
 - strong, 169, 321
 - weak, 148, 169, 314
- law of rare events, 156
- law of total probability, 47
- Lebesgue integral, 125
- limit in distribution
 - binomial limit of the hypergeometric, 153
 - exponential limit of the geometric, 164
 - normal limit of the binomial, 143
 - normal limit of the Poisson, 316
 - Poisson limit for matched hats, 159
 - Poisson limit of the binomial, 157
- linear function, 116, 190
- log-normal distribution, 200

- marginal distribution, 205
- marking a Poisson random variable, 336
- Markov chain, 364
- maximum likelihood estimator, 151

- mean
 - continuous random variable, 106
 - discrete random variable, 103
 - infinite, 107
- mean square, 110
- measure theory, 28
- median, 112, 126
- medical test example, 49, 63
- memoryless property
 - exponential distribution, 163
 - uniqueness, 169
 - geometric distribution, 83
- minimum of independent exponentials, 226, 244
- moment generating function, 181
 - exponential distribution, 183
 - identifying a distribution, 187
 - normal distribution, 182
 - Poisson distribution, 182
- monkey
 - typing monkey problem, 38
- Monte Carlo integration, 320
- Monte Carlo method, 318
- Monty Hall problem, 33, 86
- mortgage-backed securities, 70
- multinomial coefficient, 392
- multinomial distribution, 210
 - correlation, 293
 - covariance, 290
- multinomial theorem, 403
- multiplication principle of counting, 386
- multiplication rule of probabilities, 45
- mutually exclusive events, 2

- negative binomial distribution, 251
 - expectation, 278
 - variance, 278
- normal approximation
 - central limit theorem, 315
 - of the binomial, 143
 - versus Poisson approximation, 159
 - of the hypergeometric, 152
 - of the Poisson, 316
- normal distribution, 122
 - bivariate normal, 294
 - moment generating function, 182, 185
 - multivariate, 296
 - standard, 119
 - standard bivariate normal, 229, 293

428 Index

- or, 381
- ordered pair, 3
- pairwise disjoint events, 2
- pairwise independent events, 54
- partition, 47
- permutation, 256, 389
- point probabilities, 100
- Poisson approximation of the binomial
 - versus normal approximation, 159
- Poisson distribution, 156
 - law of rare events, 156
 - limit of binomials, 157
 - mean and variance, 156
 - moment generating function, 182
 - normal limit, 316
- Poisson point process, 166
- Poisson process, 165, 261
 - spatial Poisson process, 170
 - thinning, 337
- poker hand, 394, 396
- polling, 152
- possible value of a discrete random variable, 24
- possible value of a random variable, 124
- posterior probability, 50, 362
- power set, 27
- prior probability, 50, 362
- prisoner's paradox, 80
- probability density function, 90
 - and the cumulative distribution function, 97
 - infinitesimal method, 93, 134, 230, 262
 - marginal, 213
 - of the maximum, 244
 - of the minimum, 226, 244
- probability distribution
 - Bernoulli, 59
 - beta, 244, 361
 - binomial, 60
 - Cauchy, 133
 - exponential, 161
 - gamma, 168
 - Gaussian, 122
 - geometric, 61
 - Gumbel, 328
 - hypergeometric, 65
 - joint, 205
 - marginal, 205
 - multinomial, 210
 - negative binomial, 251
 - normal, 122
 - bivariate, 295
 - multivariate, 296
 - of a random variable, 24
 - Poisson, 156
 - shifted geometric, 61
 - standard normal, 119
 - uniform, 12, 92
 - higher dimensional, 216
- probability generating function, 197
- probability mass function, 24, 89
 - and the cumulative distribution function, 96
 - joint, 205
 - marginal, 207
- product symbol, 399
- proof by induction, 394
- prosecutor's fallacy, 69
- quantile, 113
- quantile function, 125
- quartile, 113
- quiz show problem, 33, 86
- random graph, 276
- random network, 276
- random variable, 21
 - absolutely integrable, 125
 - continuous, 92
 - expectation, 106
 - cumulative distribution function, 95
 - degenerate, 23, 38, 117
 - discrete, 24, 89
 - expectation, 103
 - possible value, 24, 124
 - first moment, 104
 - infinite expectation, 107
 - mean square, 110
 - moment, 110
 - neither discrete nor continuous, 102
 - probability distribution, 24
 - probability mass function, 24
 - range, 124
 - second moment, 110, 115
 - standard deviation, 113
 - standardized, 142
 - summary table of properties, 118

- support, 124
- symmetric, 137
- variance, 113
- random walk, 154, 175
- range of a random variable, 124
- rate
 - of a Poisson process, 165
 - of an exponential random variable, 161
- reasoning with uncertainty, 69
- Sally Clark case, 69
- sample mean, 279
- sample space
 - countably infinite, 13
 - discrete, 13
 - uncountable, 13
- sample variance, 280
- sampling with replacement, 5
- sampling without replacement, 5
 - covariance, 287
 - exchangeability, 260
 - lack of independence, 58
- semicircle law, 92
- sequence space, 71
- sets
 - cardinality, 385
 - de Morgan's laws, 382
 - measurable, 28
 - number of elements in a Cartesian product, 388
 - operations, 380
- shifted geometric distribution, 61, 355
- snake eyes, 173
- St. Petersburg paradox, 107
- standard bivariate normal distribution, 229, 293
 - correlation, 294
- standard deviation, 113
- standard normal distribution, 119
 - bivariate, 229
 - correlation, 294
 - mean and variance, 121
- statistic, 279
- Stieltjes integral, 125
- Stirling's formula, 147, 155
- stochastic process, 155, 363
- stopping time, 359
- summation symbol, 399
- sunrise problem, 363
- support of the distribution of a random variable, 124
- symmetric function, 256
- tail probability, 309
- three sigma rule, 145
- tuple, 385
- unbiased estimator, 279, 280
- uncorrelated random variables, 285, 289
- uniform distribution, 12, 92
 - expectation, 106
 - higher dimensional, 215
 - transformed into a discrete distribution, 195
 - transformed into a mixed distribution, 201
 - transformed into an exponential distribution, 189
 - transformed into another continuous distribution, 196
 - variance, 116
- union of sets, 381
- variance, 113, 115
 - linear function, 116
 - zero, 117
- Venn diagram, 381
- volume, 215
- Wald's identity, 358
- with probability one, 17, 297