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Yiannis N. Moschovakis
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Abstract Recursion and Intrinsic Complexity

This book presents and applies a framework for studying the complexity of algorithms. It is aimed at logicians, computer scientists, mathematicians and philosophers interested in the theory of computation and its foundations, and it is written at a level suitable for non-specialists.

Part I provides an accessible introduction to abstract recursion theory and its connection with computability and complexity. This part is suitable for use as a textbook for an advanced undergraduate or graduate course: all the necessary elementary facts from logic, recursion theory, arithmetic and algebra are included. Part II develops and applies an extension of the homomorphism method due jointly to the author and Lou van den Dries for deriving lower complexity bounds for problems in number theory and algebra which (provably or plausibly) restrict all elementary algorithms from specified primitives.

The book includes over 250 problems, from simple checks of the reader's understanding, to current open problems.

YIANNIS N. MOSCHOVAKIS is Professor Emeritus and Distinguished Research Professor of Mathematics at the University of California, Los Angeles, and Professor Emeritus at the University of Athens. Among his many professional commendations he is a Fellow of the AMS and a corresponding member of the Academy of Athens. He has received Guggenheim and Sloan Fellowships and has been an invited speaker at the International Congress of Mathematicians. Professor Moschovakis has worked primarily in the theory of recursion, descriptive set theory and the foundations of the theory of algorithms and computation.

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ASSOCIATION FOR SYMBOLIC LOGIC

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