A PHILOSOPHICAL APPROACH TO QUANTUM FIELD THEORY

This text presents an intuitive and robust mathematical image of fundamental particle physics based on a novel approach to quantum field theory, which is guided by four carefully motivated metaphysical postulates. In particular, the book explores a dissipative approach to quantum field theory, which is illustrated for scalar field theory and quantum electrodynamics and proposes an attractive explanation of the Planck scale in quantum gravity. Offering a radically new perspective on this topic, the book focuses on the conceptual foundations of quantum field theory and ontological questions. It also suggests a new stochastic simulation technique in quantum field theory, which is complementary to existing ones. Encouraging rigor in a field containing many mathematical subtleties and pitfalls, this text is a helpful companion for students of physics and philosophers interested in quantum field theory, and it allows readers to gain an intuitive rather than a formal understanding.

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A PHILOSOPHICAL APPROACH TO
QUANTUM FIELD THEORY

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Swiss Federal Institute of Technology
To Ricarda,
who means the world to me
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Preface

“All men by nature desire to know,” states Aristotle in the famous first sentence of his *Metaphysics*. Knowledge about fundamental particles and interactions, that is, knowledge about the deepest aspects of matter, is certainly high if not top on the priority list, not only for physicists and philosophers. The goal of this book is to contribute to this knowledge by going beyond the usual presentations of quantum field theory in physics textbooks, both in mathematical approach and by critical reflections inspired by epistemology, that is, by the branch of philosophy also referred to as the theory of knowledge.

This book is particularly influenced by the epistemological ideas of Ludwig Boltzmann: “…it cannot be our task to find an absolutely correct theory but rather a picture that is as simple as possible and that represents phenomena as accurately as possible” (see p. 91 of [1]). This book is an attempt to construct an intuitive and elegant image of the real world of fundamental particles and their interactions. To clarify the word *picture* or *image*, the goal could be rephrased as the construction of a genuine *mathematical representation* of the real world.

Consciously or unconsciously, the construction of any image of the real world relies on personal beliefs. I hence try to identify and justify my own personal beliefs thoroughly and in various ways. Sometimes I rely on philosophical ideas, for example, about space, time, infinity, or irreversibility; as a theoretical physicist, I have a limited understanding of philosophy, but that should not keep me from trying my best to benefit from philosophical ideas. Often I rely on successful physical theories, principles or methods, such as special relativity, quantum theory, gauge invariance, or renormalization. Typically I need to do some heuristic mathematical steps to consolidate the various inputs adopted as my personal beliefs. All these efforts ultimately lead to an image of nature, in the sense of a mathematical representation,

1 W. D. Ross’s translation of this major work, which initiated an entire branch of philosophy, can be found on the Internet (classics.mit.edu/Aristotle/metaphysics.html); nowadays Aristotle would undoubtedly say, “All human beings by nature desire to know.”
but they are not part of this image. The final mathematical representation should convince by its intrinsic logical clarity, mathematical rigor, and natural beauty.

Emphasis on the importance of beliefs, even if they are justified by a variety of philosophical and physical ideas, may irritate the physicist. The philosopher, on the other hand, is used to the definition of knowledge as true justified belief. But how can one claim truth for one’s justified beliefs? In physics, this happens by confronting an image of nature with the real world.

According to Pierre Duhem [2], known to thermodynamicists from the Gibbs-Duhem relation, and the analytic philosopher Willard Van Orman Quine [3], only the whole image, rather than individual elements or hypotheses, should be tested. The confrontation of a fully developed image with the real world depends on all its background assumptions or an even wider web-of-belief, including the assumed logics (confirmation holism). Following Boltzmann’s approach of “deductive representation” (see p. 107 of [1]), this book makes an attempt to show how such a testable whole image of fundamental particle physics can be constructed within the framework of quantum field theory.

The focus of this book is on conceptual issues, on the clarification of the foundations of quantum field theory, and ultimately even on ontological questions. For our intuitive approach, we choose to go back to the historical origins of quantum field theory. In view of the severe problems that had to be overcome on the way to modern quantum field theory, that may seem to be naive to the experts. However, with the present-day knowledge and with philosophical guidance, the intuitive origins can nicely be developed into a perfectly consistent image of the real world. On the one hand, there is a price to pay for this: practical calculations, in particular perturbation methods, are less elegant and more laborious than in other approaches. Symbolic computation is the modern answer to this challenge. On the other hand, there is a promising reward: a new stochastic simulation methodology for quantum field theory emerges naturally from our approach.

Hopefully, this book motivates physicists to appreciate philosophical ideas. Epistemology and the philosophy of the evolution of science often seem to lag behind science and to describe the developments a posteriori. As philosophical arguments here have a profound influence on the actual shaping of an image of fundamental particles and their interactions, this book should stimulate the curiosity and imagination of physicists.

Students of physics can use this book as a reliable companion whenever their standard textbooks focus on pragmatic calculations and fail to clarify important conceptual issues; philosophers and physicists interested in the epistemological foundations of particle physics can use it as a thought-provoking monograph. The benefits of an approach resting on philosophical foundations is twofold: the reader is stimulated to critical thinking and the entire story flows very naturally, thus removing the mysteries from quantum field theory.
I am indebted to Martin Kröger for countless stimulating discussions and constructive comments during all stages of writing this book. Comments of Antony Beris and Jay Schieber helped to clarify the philosophical part at an early stage. The physical part was improved with the help of remarks by Pep Español, Bert Schroer, and Marco Schweizer. Discussions with Vlasis Mavrantzas and Alberto Montefusco helped to clarify a number of specific problems.

I would not have embarked on this book project without the inspiration from Suzann-Viola Renninger’s philosophy courses. For the first time in my life I got the impression that philosophical ideas can support me in doing more solid and more beautiful work in physics. Her comments and questions on the philosophical part of this book added depth and substance.

I am very grateful that several philosophers with an interest in quantum field theory looked critically at earlier versions of this manuscript and provided encouraging and constructive feedback. In particular, I would like to thank Michael Esfeld, Simon Friederich, Antonio Augusto Passos Videira, and Bryan W. Roberts for all their helpful comments.