Contents of Volume 2

Contents of Volume 1 *page* ix
Preface xv

16 The action of $\mathsf{GL}(n)$ on flags 1
16.0 Introduction 1
16.1 Permutation matrices 2
16.2 The Bruhat decomposition of $\mathsf{GL}(n)$ 8
16.3 The flag module $\mathsf{FL}(n)$ 12
16.4 Partial flags and parabolic subgroups 14
16.5 The direct sum decomposition of $\mathsf{FL}(n)$ 20
16.6 The structure of $\mathsf{FL}(3)$ 23
16.7 Remarks 27

17 Irreducible $\mathbb{F}_2\mathsf{GL}(n)$-modules 29
17.0 Introduction 29
17.1 The Hecke algebra $\mathsf{H}_0(n)$ 31
17.2 The Schubert monoid $\mathsf{Sch}(n)$ 33
17.3 The permutation matrices W_0^I 36
17.4 Generators of the summands of $\mathsf{FL}(n)$ 38
17.5 The submodule $s_I(\mathsf{FL}(n))$ of $\mathsf{FL}_I(n)$ 43
17.6 Irreducible submodules of $\mathsf{FL}(n)$ 45
17.7 Remarks 49

18 Idempotents and characters 53
18.0 Introduction 53
18.1 Idempotents and direct sums 55
18.2 Splitting fields and conjugacy classes 58
18.3 The Steinberg idempotents $e(n)$ and $e'(n)$ 60

18.4 Embedding $\mathsf{H}_0(n)$ in $\mathbb{F}_2\mathsf{GL}(n)$ 63
18.5 Brauer characters 67
18.6 The representation ring $R_2(\mathsf{GL}(n))$ 70
18.7 Remarks 72

19 Splitting $\mathsf{P}(n)$ as an A_2-module 75
19.0 Introduction 75
19.1 The A_2-modules $\mathsf{P}(n,\lambda)$ 76
19.2 Poincaré series of $\mathsf{P}(n,\lambda)$ 80
19.3 The coinvariant algebra $\mathsf{C}(n)$ 84
19.4 $\mathsf{B}(n)$-invariants and irreducible modules 87
19.5 Irreducible submodules of $\mathsf{P}(n)$ 91
19.6 Remarks 93

20 The algebraic group $\overline{\mathsf{G}}(n)$ 95
20.0 Introduction 95
20.1 Polynomial representations of $\overline{\mathsf{G}}(n)$ 96
20.2 The $\overline{\mathsf{G}}(n)$-module $\overline{\mathsf{P}}(n)$ 100
20.3 Weyl modules 104
20.4 Weyl modules and flag modules 106
20.5 Weyl modules and the hit problem 109
20.6 First occurrences of irreducibles in $\mathsf{P}(n)$ 111
20.7 Remarks 113

21 Endomorphisms of $\mathsf{P}(n)$ over A_2 114
21.0 Introduction 114
21.1 Embedding $\mathbb{F}_2\mathsf{M}(m,n)$ in $\mathrm{Hom}_{\mathsf{A}_2}(\mathsf{P}(m),\mathsf{P}(n))$ 116
21.2 Detecting squares and Dickson invariants 116
21.3 Kernels of squaring operations 119
21.4 Root generation 121
21.5 Key monomials 124
21.6 Values on key monomials 127
21.7 Examples 132
21.8 Remarks 137

22 The Steinberg summands of $\mathsf{P}(n)$ 138
22.0 Introduction 138
22.1 Steenrod operations on rational functions 140
22.2 The cyclic A_2-module $\mathsf{T}(n)$ 143
22.3 The Mitchell–Priddy module $\mathsf{MP}(n)$ 146
22.4 The hit problem for $\mathsf{MP}(n)$ 150
22.5 The symmetric Steinberg summand 152

22.6 The $\mathsf{B}(n)$-invariant Steinberg summand 153
22.7 Remarks 156

23 The d-spike module $\mathsf{J}(n)$ 158
23.0 Introduction 158
23.1 Flag d-polynomials 160
23.2 Zip d-monomials 163
23.3 Permutation tableaux 167
23.4 $\mathsf{J}^d(n)$ for 1-dominant degrees 171
23.5 Zip monomials and $\mathsf{Q}^d(n)$ 175
23.6 Remarks 179

24 Partial flags and $\mathsf{J}(n)$ 180
24.0 Introduction 180
24.1 d-spikes and d-duplication 182
24.2 The Grassmannian case 184
24.3 Zip d-monomials for $\mathsf{FL}_J(n)$ 187
24.4 Lower bounds for $\mathsf{J}^\omega(4)$ 194
24.5 Counterexamples to Kameko's conjecture 202
24.6 Remarks 203

25 The symmetric hit problem 204
25.0 Introduction 204
25.1 The hit problem for permutation groups 205
25.2 The hit problem for $\mathsf{S}(2)$ 209
25.3 The hit problem for $\mathsf{S}(3)$ 211
25.4 The case $n = 3$, $\mu(d) = 1$ 214
25.5 The Wu formula 217
25.6 Remarks 218

26 The dual of the symmetric hit problem 220
26.0 Introduction 220
26.1 The dual algebra DS 221
26.2 The symmetric Steenrod kernel $\mathsf{K}(\mathsf{DS}(3))$ 227
26.3 The case $n = 3$, $\mu(d) = 1$ 231
26.4 The bigraded Steenrod algebra $\widetilde{\mathsf{A}}_2$ 235
26.5 Remarks 239

27 The cyclic splitting of $\mathsf{P}(n)$ 240
27.0 Introduction 240
27.1 The twisted A_2-module $\widetilde{\mathsf{P}}(n)$ 242
27.2 The cyclic splitting and blocks 246

27.3 The Kameko map for $\widetilde{\mathsf{P}}(n)$ 249
27.4 Twisted tails and heads 252
27.5 The twisted cohits $\widetilde{\mathsf{Q}}^{\omega}(3)$ 255
27.6 Remarks 263

28 The cyclic splitting of $\mathsf{DP}(n)$ 264
28.0 Introduction 264
28.1 The twisted A_2-module $\widetilde{\mathsf{DP}}(n)$ 264
28.2 Relations in $\widetilde{\mathsf{J}}(n)$ 269
28.3 The Kameko map for $\widetilde{\mathsf{DP}}(n)$ 275
28.4 Calculation of $\widetilde{\mathsf{K}}(2)$ 280
28.5 Calculation of $\widetilde{\mathsf{K}}(3)$ 281
28.6 Remarks 285

29 The 4-variable hit problem, I 286
29.0 Introduction 286
29.1 $\mathsf{Q}^d(4)$ and $\mathsf{K}^d(4)$ for $d = 7$, 8 and 16 286
29.2 Non-decreasing ω-sequences 293
29.3 A basis for $\mathsf{Q}^{\omega}(4)$, $\omega = (2, \ldots, 2)$ 297
29.4 Remarks 306

30 The 4-variable hit problem, II 307
30.0 Introduction 307
30.1 The large body case $t \geq 4$ 308
30.2 Strongly spike-free polynomials 315
30.3 The spike-free modules $\mathsf{SF}^{\omega}(4)$ 316
30.4 $\mathsf{Q}^{\omega}(4)$ and $\mathsf{K}^{\omega}(4)$ for decreasing ω 329
30.5 Remarks 332

Bibliography 335
Index of Notation for Volume 2 347
Index for Volume 2 349
Index of Notation for Volume 1 351
Index for Volume 1 353