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New Directions in Locally Compact Groups

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Contents

Foreword by George Willis

Totally disconnected locally compact (tdlc) groups are of interest for two reasons: on one hand, important classes of tdlc groups arise in combinatorial geometry, number theory and algebra and, on the other, an essential part of the task of describing the structure of general locally compact groups is understanding the totally disconnected case. Interest in these groups is currently very high because of the rapid progress being made with the general theory.

Advances in our understanding of the structure of tdlc groups are being made through three loosely related approaches:

- the *scale*, a positive integer-valued function defined on automorphisms of tdlc groups that relates to eigenvalues in algebraic representations of these groups and to translation distance in geometric representations;
- the *structure lattice* of locally normal subgroups of a tdlc group, which gives rise to a local theory underpinning a typology of simple tdlc groups; and
- a *decomposition theory* for tdlc groups that exploits methods for gauging their size and breaks a given group into smaller, and often simple, pieces.

These approaches are developing a conceptual framework that promises to support a comprehensive description of tdlc groups. At the same time, more examples filling out this framework are being found. There is still some way to go before this description could be regarded as complete however.

The current interest perhaps prompted the Oberwolfach Forschungsinstitut to ask P.-E. Caprace and N. Monod to organise the Arbeitsgemeinschaft *Totally Disconnected Groups* in October, 2014, with the aim of bringing together these approaches and the researchers and students involved. Lectures surveyed the background for the study of tdlc groups and introduced the main ideas and most recent developments in the three approaches described above. These notes, which cover the lectures as well as including a couple of other invited surveys, thus provide a valuable review of the current state of knowledge. It is to be

hoped that they will serve as a reference for further work that goes toward completing the description of totally disconnected, locally compact groups.