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# Evolution Equations: Long Time Behavior and Control

*Edited by*

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## Preface

This volume constitutes the proceedings of the summer school MIS 2015, “Mathematics In Savoie 2015,” whose theme was: “Evolution Equations: long time behavior and control.”

This summer school was held at the University Savoie Mont Blanc, Chambéry in the period June 15–18, 2015 (see <http://lama.univ-savoie.fr/MIS2015> for details). It was organized by Kaïs Ammari, UR Analysis and Control of PDE, University of Monastir, Tunisia, and Stéphane Gerbi, Laboratoire de Mathématiques, University Savoie Mont Blanc, France.

The summer school consisted of two mini-courses in the morning while the afternoons were devoted to various contributions on the theme.

The first mini-course was held by Farid Ammar-Khodja, University of Franche-Comté, France. The topic was: “Controllability of parabolic systems: the moment method.” This recent point of view on the controllability of parabolic systems permits to overview the moment method for parabolic equations. This course constitutes the first part of this volume.

The second part of this volume is devoted to the second mini-course which was held by Emmanuel Trélat, UPMC, Paris. The topic was “Stabilization of semilinear PDEs, and uniform decay under discretization.” This course was devoted to the numerical stabilization and control of partial differential equations and more specifically it addresses the problem of the construction of numerical feedback control that will preserve the theoretical rate of decay.

Several of the speakers agreed to write review papers related to their contributions to the summer school, while others have written more traditional research papers, which constitute the last part of this volume.

We believe that this volume therefore provides an accessible summary of a wide range of active research topics, along with some exciting new results,

and we hope that it will prove a useful resource for both graduate students new to the area and to more established researchers.

The summer school brought together internationally leading researchers from the community of control theory and young researchers who came from all around the world. The organizers' intention was to provide a wide angle snapshot of this exciting and fast moving area and facilitate the exchange of ideas on recent advances in its various aspects. The numerous formal, informal, and sometimes lively discussions that resulted from this interaction were for us a sign that we achieved something in the direction of fulfilling this aim.

Our second aim was to ensure that the diffusion of these recent results was not limited to established researchers in the area who were present at the summer school, but also available to newcomers and more junior members of the research community. This was reflected by the presence of many unfamiliar and/or young faces in the audience. The present proceedings should hopefully complete the fulfillment of our second aim.

This summer school would not have materialized without the help and support of the following institutions.

We are very grateful to the CNRS (Centre National de la Recherche Scientifique), the University Savoie Mont Blanc; La Région Auvergne-Rhône-Alpes; the GDRI LEM2I: "Laboratoire Euro-Maghrébin de Mathématiques et leurs Interactions;" the GDR MACS: "Modélisation, Analyse et Conduite des Systèmes dynamiques;" the GDR EDP: "Equations aux dérivées partielles;" the GDRE CONEDP: "Control of Partial Differential Equations;" the MaiMoSine: "Maison de la Modélisation et de la Simulation, Nanosciences et Environnement;" and the PERSYVAL-lab: "PERvasive SYstems and ALgorithms" for their financial support without which this summer school would not be accessible without fees.

Finally we would like to thank all the participants of the summer school who have made this event a success, the contributors to these proceedings, and the reviewers for their hard work.

*Kaïs Ammari and Stéphane Gerbi*  
Chambéry, July 07, 2017



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