

## The Student's Introduction to *Mathematica*<sup>®</sup> and the Wolfram Language<sup>™</sup>

Third edition

The unique feature of this compact student's introduction to *Mathematica* and the Wolfram Language is that it presents concepts in an order that closely follows a standard mathematics curriculum, rather than structure the book along features of the software. As a result, the book provides a brief introduction to those aspects of the *Mathematica* software program most useful to students. Used as a supplementary text, it will aid in bridging the gap between *Mathematica* and the mathematics in the course. In addition to its course use, this book will serve as an excellent tutorial for former students.

There have been significant changes to *Mathematica* since the publication of the second edition, and all chapters in this third edition have been updated to account for new features in the software, including natural language queries and the vast stores of real-world data that are now integrated through the cloud. This third edition includes many new exercises and a new chapter on 3D printing that utilizes concepts introduced in earlier chapters to showcase new computational geometry capabilities that will equip readers to design and print in 3D.

Bruce F. Torrence and Eve A. Torrence are both Professors in the Department of Mathematics at Randolph-Macon College, Virginia.

# The Student's Introduction to *Mathematica*<sup>®</sup> and the Wolfram Language<sup>™</sup>

Third edition

Bruce F. Torrence

Eve A. Torrence



CAMBRIDGE  
UNIVERSITY PRESS



Shaftesbury Road, Cambridge CB2 8EA, United Kingdom  
One Liberty Plaza, 20th Floor, New York, NY 10006, USA  
477 Williamstown Road, Port Melbourne, VIC 3207, Australia  
314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi – 110025, India  
103 Penang Road, #05–06/07, Visioncrest Commercial, Singapore 238467

Cambridge University Press is part of Cambridge University Press & Assessment, a department of the University of Cambridge.

We share the University's mission to contribute to society through the pursuit of education, learning and research at the highest international levels of excellence.

[www.cambridge.org](http://www.cambridge.org)  
Information on this title: [www.cambridge.org/9781108406369](http://www.cambridge.org/9781108406369)  
DOI:10.1017/9781108290937

First edition © Cambridge University Press & Assessment 1999  
Second and Third editions © Bruce F. Torrence and Eve A. Torrence 2009, 2019  
Page design and composition: Paul Wellin

This publication is in copyright. Subject to statutory exception and to the provisions of relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Cambridge University Press & Assessment.

First published 1999  
Second edition 2009  
Third edition 2019

Mathematica® and Wolfram Alpha® are registered trademarks of Wolfram Research, Inc. Wolfram Language and Wolfram One are trademarks of the same company.

*A catalogue record for this publication is available from the British Library*

ISBN 978-1-108-40636-9 Paperback

Additional resources for this publication at [www.cambridge.org/torrence3](http://www.cambridge.org/torrence3).

Cambridge University Press & Assessment has no responsibility for the persistence or accuracy of URLs for external or third-party internet websites referred to in this publication and does not guarantee that any content on such websites is, or will remain, accurate or appropriate.

Wolfram Research, Inc. is not affiliated with the authors nor associated with the development of the content of the book.

*For*

*Alexandra and Robert*

# Contents

## Preface · xi

## 1 Getting Started · 1

- 1.1 Launching *Mathematica* · 1
- 1.2 The Basic Technique for Using *Mathematica* · 1
- 1.3 The First Computation · 2
- 1.4 Commands for Basic Arithmetic · 2
- 1.5 Input and Output · 3
- 1.6 The Basic Math Assistant Palette · 4
- 1.7 Decimal In, Decimal Out · 6
- 1.8 Use Parentheses to Group Terms · 7
- 1.9 Three Well-Known Constants · 8
- 1.10 *Mathematica* Commands from Palettes · 9
- 1.11 Let *Mathematica* Do Your Typing · 20
- 1.12 Free-Form Input · 21
- 1.13 Computing with Real Data · 22
- 1.14 Saving Your Work and Quitting *Mathematica* · 23
- 1.15 Frequently Asked Questions About *Mathematica*'s Syntax · 24

## 2 Working with *Mathematica* · 27

- 2.1 Opening Saved Notebooks · 27
- 2.2 Adding Text to Notebooks · 27
- 2.3 Printing · 30
- 2.4 Creating Presentations · 31
- 2.5 Sharing a Notebook · 32
- 2.6 *Mathematica*'s Kernel · 32
- 2.7 Tips for Working Effectively · 34
- 2.8 Working with Entities and Units · 41
- 2.9 Getting Help from *Mathematica* · 45
- 2.10 Troubleshooting · 48

## 3 Functions and Their Graphs · 51

- 3.1 Defining a Function · 51
- 3.2 Plotting a Function · 53
- 3.3 Using *Mathematica*'s Plot Options · 59

3.4	Investigating Functions with Manipulate · 76
3.5	Producing a Table of Values · 85
3.6	Working with Piecewise Defined Functions · 93
3.7	Plotting Implicitly Defined Functions · 96
3.8	Combining Graphics · 100
3.9	Enhancing Your Graphics · 108
3.10	Working with Data · 116
3.11	Managing Data: An Introduction to Lists · 122
3.12	Importing Data · 126
3.13	Working with Difference Equations · 139
<b>4</b>	<b>Algebra · 145</b>
4.1	Factoring and Expanding Polynomials · 145
4.2	Finding Roots of Polynomials with Solve and NSolve · 147
4.3	Solving Equations and Inequalities with Reduce · 155
4.4	Understanding Complex Output · 160
4.5	Working with Rational Functions · 168
4.6	Working with Other Expressions · 173
4.7	Solving General Equations with FindRoot · 178
4.8	Solving Difference Equations with RSolve · 184
4.9	Solving Systems of Equations and Inequalities · 187
<b>5</b>	<b>Calculus · 191</b>
5.1	Computing Limits · 191
5.2	Working with Difference Quotients · 195
5.3	The Derivative · 198
5.4	Visualizing Derivatives · 201
5.5	Higher-Order Derivatives · 202
5.6	Maxima and Minima · 203
5.7	Inflection Points · 211
5.8	Implicit Differentiation · 212
5.9	Differential Equations · 215
5.10	Integration · 218
5.11	Definite and Improper Integrals · 224
5.12	Numerical Integration · 234
5.13	Surfaces of Revolution · 238
5.14	Sequences and Series · 242
<b>6</b>	<b>Multivariable Calculus · 249</b>
6.1	Vectors · 249
6.2	Real-Valued Functions of Two or More Variables · 255
6.3	Parametric Curves and Surfaces · 302

- 6.4 Other Coordinate Systems · 314
- 6.5 Vector Fields · 326
- 6.6 Line Integrals and Surface Integrals · 331

## 7 Linear Algebra · 335

- 7.1 Matrices · 335
- 7.2 Performing Gaussian Elimination · 345
- 7.3 Matrix Operations · 347
- 7.4 Minors and Cofactors · 351
- 7.5 Working with Large Matrices · 353
- 7.6 Solving Systems of Linear Equations · 357
- 7.7 Vector Spaces · 363
- 7.8 Eigenvalues and Eigenvectors · 370
- 7.9 Visualizing Linear Transformations · 375

## 8 Programming · 383

- 8.1 Introduction · 383
- 8.2 FullForm: What the Kernel Sees · 384
- 8.3 Numbers · 390
- 8.4 Map and Function · 400
- 8.5 Control Structures and Looping · 411
- 8.6 Scoping Constructs: With and Module · 421
- 8.7 Iterations: Nest and Fold · 429
- 8.8 Patterns · 440

## 9 3D Printing · 457

- 9.1 Introduction · 457
- 9.2 3D Printing Basics · 457
- 9.3 MeshRegions · 461
- 9.4 MeshRegions from the Wolfram Knowledgebase · 472
- 9.5 Mesh Assembly, Modification, and Diagnostics · 476
- 9.6 Extrusion · 494
- 9.7 Printing a Solid of Revolution · 504
- 9.8 Printing a 3D Terrain Map · 509

## Index · 523

Solutions to Exercises · [www.cambridge.org/torrence3](http://www.cambridge.org/torrence3)

# Preface

The mathematician and juggler Ronald L. Graham has likened the mastery of computer programming to the mastery of juggling. The problem with juggling is that the balls go exactly where you throw them. And the problem with computers is that they do exactly what you tell them.

This is a book about *Mathematica* and the Wolfram<sup>1</sup> Language™. *Mathematica* is a comprehensive technical computing environment that has long been the flagship product of Wolfram Research. With a functional programming language at its core, *Mathematica* is an ambitious project that has always taken the long view regarding structure and design. And with 30 years of continuous development, it has grown into something far beyond its original concept. Interactive mathematical typesetting arrived in 1996, and dynamically interactive computation was incorporated in 2007. The introduction of Wolfram Alpha® in 2009 brought natural language queries into the fold, and a decade of data curation has brought in the entire world. The underlying symbolic language, now called the Wolfram Language, has maintained its syntactical structure while permitting access to both natural language inputs and a huge wealth of curated data and algorithms. It is a truly unique and powerful platform.

As software programs go, *Mathematica* is big—really big. We said that back in 1999 in the preface to the first edition of this book. And it has gotten a good deal bigger since then. The printed user manual was abandoned after version 5 as it was approaching 1000 pages in length. Now in version 12, the program is many times larger. Yes, *Mathematica* will do exactly what you ask it to do, and it has the potential to amaze and delight—but you have to know how to ask.

That's where this book comes in. Originally intended as a supplementary text for high school and college students using *Mathematica* in their mathematics classes, it introduces the reader to the Wolfram Language in an order that roughly coincides with the usual mathematics curriculum. The idea is to provide a coherent introduction to *Mathematica* that does not get ahead of itself mathematically. Most of the available reference materials make the assumption that the reader is thoroughly familiar with the mathematical concepts underlying each Wolfram Language command and procedure, many of which are rather advanced. This book does not. It presents *Mathematica* as a means not only of solving mathematical problems, but of exploring and clarifying the underlying mathematical ideas. It also provides examples of procedures that students will need to master, showing not just individual commands, but sequences of commands that together accomplish a larger goal.

But a funny thing happened. While written primarily for students, the first editions were well-received by many non-students who just wanted to learn *Mathematica*. Because the book follows the standard mathematics curriculum, we were told, the presentation exudes a certain familiarity and coherence. What better way to learn a computer language than to rediscover the beautiful ideas from your foundational mathematics courses? In seeing how *Mathematica* deals with familiar concepts, its underlying principles become clearer and more transparent.

1. [www.wolfram.com](http://www.wolfram.com)



## What's New in this Edition?

The impetus for a third edition was driven by the software itself. Since the second edition we have seen the launch of Wolfram Alpha<sup>2</sup>, the development of the Wolfram Knowledgebase and the corresponding incorporation of Entities into the language, and the introduction of *Mathematica* Online and cloud computing. Just as the second edition was rewritten from the ground up, so too has the third edition been extensively revised to reflect these and other enhancements to the software.

In addition, a new chapter has been added (Chapter 9) on the topic of 3D printing. While *Mathematica* has never been a CAD program, its inherent capabilities have been enhanced with the active development of new tools for computational geometry, and the net result is impressive. The chapter provides a fitting showcase for the techniques introduced earlier.

## How to Use this Book

Of course, this is a printed book and as such is perfectly suitable for bedtime or beach reading. But in most cases you will want to be interacting with *Mathematica* as you read the book. You can mimic the inputs and then try variations.

The first chapter provides a brief tutorial for those just getting started with the software, be it a desktop or cloud platform. The second delves a bit deeper into the fundamental design principles and can be used as a reference for the rest of the book. Chapters 3 and 4 provide information on those *Mathematica* commands and procedures relevant to the material in a precalculus course, including some basic data science. Chapter 5 adds material relevant to single-variable calculus, and Chapter 6 deals with multivariable calculus. Chapter 7 introduces commands and procedures pertinent to the material in a linear algebra course, and Chapter 8 provides an introduction to programming in the Wolfram Language. Chapter 9 wraps things up with an introduction to 3D printing.

△ Some sections of the text carry this warning sign. These sections provide slightly more comprehensive information for the curious reader. They can be skipped by less hardy souls.

Beginning in Chapter 3, each section has exercises. Solutions to *every* exercise can be freely downloaded from the Cambridge University Press website at [www.cambridge.org/torrence3](http://www.cambridge.org/torrence3).

*Mathematica* runs on every major operating system, from Macs and PCs to machines running Linux<sup>®</sup>, and with cloud-based versions it is now accessible from mobile touchscreen devices. For the most part it works with admirable consistency across platforms. There are, however, a few procedures (such as certain keyboard shortcuts) that are platform-specific. In such cases we have provided specific information for both the Mac OS<sup>®</sup> and Microsoft<sup>®</sup> Windows<sup>®</sup> platforms, as well as for cloud-based environments. If you find yourself running *Mathematica* on some other platform, you can be assured that the procedure you need is virtually identical to one of these.

2. [www.wolframalpha.com](http://www.wolframalpha.com)

Acknowledgments

Time flies. When we wrote the first edition of this book our children were toddlers who wanted to sit on our laps while we worked. When we wrote the second edition they were teenagers who just wanted our laptops. Now they are young adults with laptops and lives of their own. We are so proud of the wonderful people they have become and love the creativity, energy, joy, and spark they bring to our lives.

We extend our thanks to Randolph-Macon College and the Chenery and Rashkind Family Endowments for the support we received throughout this project. We are especially grateful to Paul Wellin, who handled the page design and composition and who dealt tirelessly with countless other issues, both editorial and technical.