

Cambridge University Press

978-1-108-08457-4 - A History of the Mathematical Theories of Attraction and the  
Figure of the Earth: From the Time of Newton to that of Laplace: Volume 1

Isaac Todhunter

Table of Contents

[More information](#)

## CONTENTS.

## VOLUME I.

	PAGE
CHAPTER I. NEWTON . . . . .	1
Publication of the Principia, 1. Attractions, 2. Spherical shell on an internal particle, 3. Spherical shell on an external particle, 4. Lemma, 6. Zone of an indefinitely thin spherical shell, 7. Sphere on an external particle, 8. Sphere on an internal particle, 10. Solid of revolution on a particle at any point of the axis, 13. Infinite plane lamina, 15. Jesuits' edition, 16. Density of the Earth, 17. Figure of the Earth, 18. Polar and equatorial canals of fluid, 19. Approximate estimate of attractions, 20. Oblatum and oblongum, 21. Remarks on the approximate estimate, 22. Equilibrium of the canals, 23. Attraction, gravity, weight, 25. Newton's value of the ellipticity, 26. Jupiter's figure, 29. Influence of the Sun's heat, 30. Measured lengths of degrees, 32. Weight resolved along the radius, 33. Increment of gravity, 34. Pendulums, 36. Newton's error, 37. Table of the lengths of a degree, 39. Pendulums, 40. The Cassinian hypothesis untrue, 43. Summary of results, 44. Laplace's opinion, 45. Halley's opinion, 46.	
CHAPTER II. HUYGENS . . . . .	28
Publication of the Discourse, 47. Date of composition, 49. Vortex, 50. Air pump, and weight in a mine, 51. Pendulum at Cayenne, 52. Principle in Hydrostatics, 53. Value of the ellipticity, 54. Equation, 55. Result extended, 56. Remarks on the Principia, 59. The Sun's distance from the Earth, 61. Resisting medium, 62. Huygens's problem, 64. Mistake as to priority, 65.	
CHAPTER III. MISCELLANEOUS INVESTIGATIONS UP TO THE YEAR 1720 . . . . .	37
Norwood, 68. Pendulums, 69. The Arabian measure, 70. Norwood, 71. Halley, 72. Burnet, Whiston, and Keill, 73. Incidental mistakes, 75.	

Cambridge University Press

978-1-108-08457-4 - A History of the Mathematical Theories of Attraction and the Figure of the Earth: From the Time of Newton to that of Laplace: Volume 1

Isaac Todhunter

Table of Contents

[More information](#)

xx

CONTENTS.

	PAGE
Keill's error, 76. Keill and Halley, 77. Keill and Bentley, 78. Keill and Whiston, 79. Pendulums, 80. D. Cassini adopts Keill's error, 81. Snell, 82. De La Hire, 83. D. Gregory, 84. Keill, 85. Quotations from Keill and Arago, 87. Numerical result, 89. Pendulums, 90. Freind, 91. J. Cassini, 92. Hermann, 93. Hermann's problem, 95. Quotations from Hermann and Boscovich, 97. J. Cassini, 99. French arc, 100. Mairan, 109. Invents a law of attraction, 113. Summary of results, 115.	
 <b>CHAPTER IV. MAUPERTUIS</b>	 <b>63</b>
Saturn's ring, 117. First problem, 118. Second problem, 119. Figure des Astres, 122. A difficulty, 124; see 725. Variable stars and nebulae, 127. On the laws of attraction, 128. Incidental statements, 130. Figure de la Terre, 131. Figures des Corps Célestes, 132. Hydrostatical principles of Newton and Huygens, 133. Mistake, 137. View of an obscure passage in Newton, 138. Figure de la Terre, 140, 141, 142. Examen désintéressé, 143.	
 <b>CHAPTER V. STIRLING</b>	 <b>77</b>
Newton's postulate, 151. Sir J. Lubbock, 152. Resultant action at the surface of an oblatum, 153. Pendulum, 155. Remarks on the merits of Clairaut and Stirling, 156. Theory compared with fact, 157. Estimates of Stirling, 158.	
 <b>CHAPTER VI. CLAIRAUT</b>	 <b>83</b>
Geodesic curve, 160. Proposition in solid Geometry, 161. Arcs of meridian, 162. Newton's postulate, 163. Approximate attraction of an oblatum at the pole, 165. Fluid with variable density, 167. Unsatisfactory with respect to Hydrostatics, 170. Clairaut's Fraction, and Clairaut's Theorem, 171. Huygens's problem, 173. Hydrostatical principles, 174. Geodesic curve, 177.	
 <b>CHAPTER VII. ARC OF THE MERIDIAN MEASURED IN LAPLAND</b>	 <b>93</b>
Peruvian company, 178. Lapland company, 179. Maupertuis's book, 181. Outhier's book, 182. Selection of places, 184. Difference of latitude, 185. Measurement of the base, 186. Difference of latitude redetermined, 189. Hardships, 192. Incidental matters, 194. Ellipticity, 196. Svanberg, 197. Celsius, 198. Reference for further information, 199.	
 <b>CHAPTER VIII. MISCELLANEOUS INVESTIGATIONS BETWEEN THE YEARS 1721 AND 1740</b>	 <b>103</b>
Desaguliers, 200. Criticises the conclusions of J. Cassini, 201. Admits Keill's error, 202. Criticises Mairan's memoir, 203. Considers the French arc, 204.	

Cambridge University Press

978-1-108-08457-4 - A History of the Mathematical Theories of Attraction and the Figure of the Earth: From the Time of Newton to that of Laplace: Volume 1

Isaac Todhunter

Table of Contents

[More information](#)

## CONTENTS.

xxi

PAGE

An experiment, 205. Incidental matters, 206. Poleni, 209. *De la Croyere*, 210. J. Cassini, 211. Godin, 213. La Condamine, 214. J. Cassini, 215. Pendulum, 217. Manfredi, 218. Bouguer on Hydrostatics, 219. J. Cassini, 220. Prize Essay by John Bernoulli, 221. Modes of determining the form of the Earth, 222, 223. Cassini de Thury, 224. Pendulums, 225. Cassini de Thury, 226. Bouguer, 227. Delisle, 228. Euler, 229. D. Bernoulli's *Hydrodynamica*, 230. Cassini de Thury, 231. The Tides, 232. D. Bernoulli's Essay, 233. Euler's Essay, 234. Arc between Paris and Amiens, 235. French arc, 236. Remark on the error in Picard's base, 238. Jesuits' edition of the *Principia*, 239. Winsheim, 240.

## CHAPTER IX. MACLAURIN . . . . . 133

Treatise of Fluxions, 241. Attractions, 242. Ellipsoid of revolution, 244. Newton's Postulate, 245. Extends Newton's Hydrostatical principle, 246. Value of gravity, 247. Level surfaces, 248. Newton's Postulate, 249. Attraction of a sphere, 251. Of an ellipsoid of revolution at the equator and pole, 252. Reference to Newton, Cotes, and Stirling, 253. Attraction of confocal ellipsoids, 254. Maclaurin's demonstration, 255. Confocal shells, 256. External particle in the plane of the equator, 257. Maclaurin's results, 259. His investigations under-estimated, 260. Attraction of an oblatum, 261. Application to rotating fluid, 262. Variable density, 264. Attraction of oblatum of varying density, 266. Application of Newton's Hydrostatical principle, 267. Objections, 269. Polar and equatorial columns, 270. Jupiter, 272. Essay on the Tides, 275. Maclaurin's death, 276.

## CHAPTER X. THOMAS SIMPSON . . . . . 176

Mathematical Dissertations, 277. Reference to Maclaurin, 278. Attraction of an oblatum, 279. Value of a definite integral, 280. Values of two definite integrals, 281. Relation between excentricity and the angular velocity, 283. Limit of angular velocity, 284. Two solutions, 285. Conservation of Areas, 286. Oblatum not homogeneous, 289. Attraction of Spheroids, 290. Length of a Degree, 292. Fluxions, 293. Simpson's eminence as a mathematician, 294.

## CHAPTER XI. CLAIRAUT . . . . . 189

Figure de la Terre, 295. Fluid equilibrium, 296. Cartesians and Newtonians, 297. Principle of Canals and Principle of Level Surfaces, 298. Points of interest in the Introduction, 299. Clairaut's first part, 300. Comparison between Clairaut and others, 301. Principle of Canals, 302.

T. M. A.

c

Cambridge University Press

978-1-108-08457-4 - A History of the Mathematical Theories of Attraction and the Figure of the Earth: From the Time of Newton to that of Laplace: Volume 1

Isaac Todhunter

Table of Contents

[More information](#)

xxii

CONTENTS.

	PAGE
General reasoning, 303. Cases where equilibrium is impossible, 304. Principles of Newton and Huygens, 305. Rotating fluid, 306. Complete differential, 307. Principle of Level Surfaces, 308. Examples of fluid equilibrium, 309. Bouguer's problem, 310. Polar Coordinates, 312. Space of three dimensions, 313. Capillary attraction, 314. Heterogeneous fluid, 315. Law of attraction, 316. Clairaut's second part, 317. Homogeneous Figure of the Earth, 318. Jupiter, 319. Attraction of a circular lamina, 320. Attraction of an oblatum, 321. Ellipticity, 322. Fluid about a solid nucleus, 323. Particular cases, 324. Criticisms on Newton, 325. Oblongum may be a possible form, 326. Case in which the thickness of fluid is small, 327. Objection by D'Alembert and Cousin, 328. The ellipticity is in general less when the fluid is heterogeneous than when it is homogeneous, 329. Variation of gravity, 330. Attraction of a circular lamina, 331. Of an oval lamina, 332. Proposition as to the attraction of an ellipsoid of revolution, 333. Particle on the prolongation of the axis of revolution, 334. Analytical verification of a result obtained by Clairaut, 335. Gravity at the surface of the Earth: Clairaut's Theorem, 336. Inaccurate statements which have been made, 337. Applications of Clairaut's theorem, 338. Strata of varying density, 339. Attraction of a shell, 340. Primary Equation, 341. Derived Equation, 343. Another form of the derived equation, 344. Example of a particular law of density, 345. Clairaut's derived equation, 346. Case in which the mass consists of fluid of two densities, 347. Limits for the ellipticity of a planet, 348. Comparison of theory with observation, 349. Laplace's opinion, 350.	

## CHAPTER XII. ARC OF THE MERIDIAN MEASURED IN PERU

231

The Peruvian expedition started before the Lapland, 351. Literature of the subject, 352. Course of the operations, 353. Difficulties encountered, 354. Base of verification, 355. The astronomical part, 356. Discordance of the observations, 357. Final result, 358. The Spanish operations, 359. Return to Paris, 360. Miscellaneous points, 361. The Spanish account, 362. Bouguer's Figure de la Terre, 363. Summary, 364.

## CHAPTER XIII. D'ALEMBERT

249

Treatise on Fluids, 365. Historical sketch, 366. Comparison between theory and actual measurement, 367. Second edition, 368. Matters of interest, 369. Treatise on the Winds, 370. Two sentences quoted, 371. Companion to Huygens's problem, 372. Attraction of a homogeneous oblatum at its surface, 373. Spherical nucleus, 374. Results which D'Alembert considers strange, 375. Oblate nucleus, 376. Particular case of a formula given by Clairaut, 377. Criticism on Clairaut, 378.

Cambridge University Press

978-1-108-08457-4 - A History of the Mathematical Theories of Attraction and the Figure of the Earth: From the Time of Newton to that of Laplace: Volume 1

Isaac Todhunter

Table of Contents

[More information](#)

CONTENTS.	xxiii
	PAGE
Incorrect statement, 379. Attraction at the surface of an ellipsoid, 380. Error, 381. Contradiction and error, 382. Ellipsoidal nucleus, 383. Precession of the Equinoxes, 384. Important numerical relation, 385. Combination of D'Alembert's result with Clairaut's, 386. Example in the Integral Calculus, 387. Hypothesis as to the structure of the Earth, 390. Inequality, 391. Treatise on the Resistance of Fluids, 392. His hydrostatical principle, 394. Failure in attempt at generalization, 397. Surfaces of equal density not necessarily level surfaces, 400. Failure in attempt to generalize Clairaut's theory, 404. Researches on the System of the World, 409. Criticism on Euler, 411. Difference between theory and observation, 416. Results of integration, 423. General formula for the attraction of a spheroid, 424. Spherical nucleus, 426. Oblate nucleus, 429. Criticism on Clairaut, 431. Attraction of spherical shell, 434. Results of integration, 436. Attraction of a spheroid of revolution, 437. Three general equations, 444. General estimate of his researches, 452.	
<b>CHAPTER XIV. BOSCOVICH AND STAY</b>	<b>305</b>
The work by Boscovich and Le Maire, 454. Boscovich's dissertations, 457. Follows Maclaurin's solution, 461. Criticises Hermann, 466. Criticises D. Bernoulli, 469. Pendulums, 472. Suggestion, 476. Measures of a degree of the meridian, 481. Criticises Euler, 483. Abstract of the treatise, 489. Stay's poem, 490. Dugald Stewart's opinion, 491. Specimens of Stay's verses, 493. Supplementary dissertations, 494. Newton's error, 499. D. Bernoulli's opinion of Newton, 501. Measures of arcs, 508. Criticises Maupertuis, 510. Method for discordant observations, 511. French translation, 513.	
<b>CHAPTER XV. MISCELLANEOUS INVESTIGATIONS BETWEEN THE YEARS 1741 AND 1760</b>	<b>335</b>
Pendulums, 515. Murdoch, 516. Bremond's translation, 521. Addition by Murdoch, 522. Mairan, 526. Cassini de Thury, 527. Clairaut and Buffon, 528. E. Zanottus, 529. La Condamine, 530. Silvabelle, 531. Frisi and Short, 532. Defence of Newton, 534. Clairaut's reply to Frisi, 535. Bouguer, 538. La Caille's voyage, 539. La Caille's arc, 541. Maclear's arc, 542. La Lande, 543. Euler, 545. La Caille's reply to Euler, 546. Hollmannus, 547. Euler, 548. La Caille, 549. La Condamine, 550. Picard's base, 551. Walmesley, 552. La Caille, 553. D'Arcy, 554. Clairaut's Prize Essay, 555. Edition of the Principia, by Madame du Chastellet, 558. Lagrange on D'Alembert's paradox, 561.	
<b>CHAPTER XVI. D'ALEMBERT</b>	<b>365</b>
Articles in the Encyclopédie, 564. Reply to Boscovich, 567. Paradox, 569. Corrects former errors, 571. Fluid equilibrium, 574. Attempts to solve	
	c 2

Cambridge University Press

978-1-108-08457-4 - A History of the Mathematical Theories of Attraction and the Figure of the Earth: From the Time of Newton to that of Laplace: Volume 1

Isaac Todhunter

Table of Contents

[More information](#)

xxiv

CONTENTS.

	PAGE
Legendre's problem, 575. Failure, 576. Sixth volume of the <i>Opuscules Mathématiques</i> , 579. Two solutions of the problem of rotating fluid, 580. The case of a very small angular velocity, 584. Only two solutions, 586. Replies to the translator of Boscovich, 590. Attraction of mountains, 592. Discussion of a problem, 596. An oblongum not an admissible form, 601. Two analytical matters, 611. Generalisation of a former problem, 613. Correction of a recent error, 618. Attraction of an ellipsoid, 625. Extension of former problem, 629. Error, 630. Criticises Clairaut, 634. Considers Maclaurin's theorem, 636. Atmosphere, 637. Equation to the surface, 639. Letters to Lagrange, 643. Demonstrates Maclaurin's theorem, 645. Rejects an important formula, 651. Investigates a theorem given by Laplace, 652. Fluid equilibrium, 654. Unsound demonstration, 657. Summary of his contributions, 658.	
 <b>CHAPTER XVII. FRISI</b>	<b>424</b>
De Gravitate, 660. Measurements hitherto made, 661. Extends a result of Newton's, 662. Criticism on Newton, 663. Newton's error, 664. Cosmographia, 668. Suggestion as to La Caille's arc, 671. Mistake corrected, 673. Newton's error, 676. Stability of equilibrium, 678. Fontana, 679. Opera, 680. Silvabelle's problem, 682.	
 <b>CHAPTER XVIII. MISCELLANEOUS INVESTIGATIONS BETWEEN THE YEARS 1761 AND 1780</b>	<b>439</b>
Frisoni, 686. Krafft, 687. Error, 689. Sum of a series, 694. Osterwald, 696. Michell, 697. Canterzanus, 698. E. Zanottus, 699. J. A. Euler, 700. Lambert, 701. Liesganig, 702. Mason and Dixon, Maskelyne, 703. Liesganig, 704. Cavendish, 705. La Condamine, 706. Lagrange, 707. Transformation of variables, 710. Ellipsoid, 712. Beccaria, 717. Cassini de Thury, 718. Lagrange, 720. Demonstrates Maclaurin's theorem, 721. Cassini de Thury, 723. Schehallien experiment, 724. Statement by Newton, 725. Dionis du Séjour, 728. Whitehurst, 729. Hutton's calculations for Schehallien, 730. Playfair's survey of the mountain, 731. Hutton, 732. Density of the Earth, 733. Cousin, 734. Euler, 735. Hutton, 736. Titles of works, 738. Additional remarks, 740.	

Cambridge University Press

978-1-108-08457-4 - A History of the Mathematical Theories of Attraction and the Figure of the Earth: From the Time of Newton to that of Laplace: Volume 1

Isaac Todhunter

Table of Contents

[More information](#)

## CONTENTS.

XXV

## VOLUME II.

	PAGE
<b>CHAPTER XIX. LAPLACE'S FIRST THREE MEMOIRS</b>	<b>1</b>
Five divisions in Laplace's writings, 741. First Memoir, 742. Legendre's problem, 744. Second memoir, 751. Law of variation of gravity, 753. Laplace's equation, 755. Third memoir, 764. Mistake, 769. Laplace's equation, 771. Order of the writings of Laplace and Legendre, 778.	
<b>CHAPTER XX. LEGENDRE'S FIRST MEMOIR</b>	<b>20</b>
Treatment of Maclaurin's theorem, 781. Extension of it, 782. Laplace's coefficients, 783. Heine's work, 784. General expression for a coefficient, 786. Potential, 789. Green and Gauss, 790. Legendre's theorem, 791. Criticism on the demonstration, 792. Extension of Maclaurin's theorem, 793. Character of the memoir, 794.	
<b>CHAPTER XXI. LAPLACE'S TREATISE.</b>	<b>29</b>
Its scarcity, 796. Publication, 797. Number of sections, 800. Equation to an ellipsoid, 801. Potential, 802. Polar coordinates, 803. Laplace's theorem, 804. Attraction of an ellipsoid, 805. History of the problem, 806. Rotating fluid mass, 807. Treatment of D'Alembert's problem, 808. Case of the Moon, 809. Case of the Earth, 810. Two and only two solutions, 811. Conservation of areas, 813. Laplace's equation, 814. General problem of the form of fluid, 815. Law of gravity at the surface of a fluid mass, 816. Spherical shell attracting external particle, 817.	
<b>CHAPTER XXII. LEGENDRE'S SECOND MEMOIR</b>	<b>43</b>
Object of the memoir to solve Legendre's problem, 820. Conditions of the demonstration, 821. Seven theorems as to Laplace's coefficients, 822. Equation to be solved, 831. Mode of treatment, 837. Remarks on the demonstration, 842. Legendre's own opinion respecting it, 844. Quotation from Laplace, 845. Quotation from Ivory, 846. Quotation from Jacobi, 847.	
<b>CHAPTER XXIII. LAPLACE'S FOURTH, FIFTH, AND SIXTH MEMOIRS</b>	<b>55</b>
Fourth memoir, 848. Laplace's theorem, 850. Partial Differential Equation for the Potential, 851. Laplace's Equation, 852. Error, 854. Property of Laplace's functions, 857. Fifth memoir, 859. Degrees and Pendulums, 860. Numerical example, 862. Oversight, 863. Sixth memoir, 864. Saturn's ring, 865. Partial Differential Equation for the Potential, 866. Division of Saturn's ring, 867. Form of the ring, 868. Plana's criticisms on Laplace, 871. Instability of a ring, 872.	

Cambridge University Press

978-1-108-08457-4 - A History of the Mathematical Theories of Attraction and the Figure of the Earth: From the Time of Newton to that of Laplace: Volume 1

Isaac Todhunter

Table of Contents

[More information](#)

xxvi

CONTENTS.

	PAGE
<b>CHAPTER XXIV. LEGENDRE'S THIRD MEMOIR</b>	<b>74</b>
Object of the memoir to demonstrate Laplace's Theorem, 875. Transformation of multiple integrals, 877. Case in which the attracted particle is in a principal plane of the ellipsoid, 880. General problem, 881. Point of the Integral Calculus, 882. Legendre's theorem, 883. Limitation in Legendre's process, 886. Opinions of the method, 887.	
<b>CHAPTER XXV. LEGENDRE'S FOURTH MEMOIR</b>	<b>87</b>
Object of the memoir, 892. Error, 896. Laplace's coefficients, 897. First hypothesis, 902. Result of first approximation, 909. Second approximation, 912. Ellipticity, 917. Length of degree, 918. General theorem, 920. Force of gravity, 921. Clairaut's theorem, 923. Second hypothesis, 925. Clairaut's equation generalised, 929. Vanishing of terms, 933. Examples of laws of density, 939. First example; density constant, 940. Second example, 941. Third example, 942. Second approximation, 943. Third hypothesis, 944. Numerical values, 945. Case in which the figure is not assumed to be one of revolution, 948. Laplace's functions, 949. Correction of an oversight in Laplace, 953. The form of a planet must be an oblatum, 955. General estimate, 957.	
<b>CHAPTER XXVI. LAPLACE'S SEVENTH MEMOIR</b>	<b>131</b>
Mode of treating measured lengths of degrees, 960. Another method due to Boscovich, 962. Lengths of seconds pendulum, 964. Two methods of treatment, 966. Clairaut's Theorem, 967.	
<b>CHAPTER XXVII. MISCELLANEOUS INVESTIGATIONS BETWEEN THE YEARS 1781 AND 1800</b>	<b>138</b>
Euler, 970. Kraft, 971. Error of B. St Pierre, 972. Cousin's Elementary Treatise, 973. Error, 976. Approximate formulæ, 978. Attempts to generalise Clairaut's theory, 980. Error, 982. Roy, 984. La Lande, 985. Roy, 986. Embarrassed by an error of Bouguer's, 987. Williams, 988. La Lande on Fernel's measure, 989. Legendre's Theorem in Spherical Trigonometry, 990. Monge, 991. Borda and others, 992. Coulomb's theorem, 993. Lagrange's Mécanique Analytique, 994. Waring, 995. Triesnecker, 996. Pictet, 997. Waring, 998. Dalby, 999. Cassini IV. and others, 1000. Cagnoli and Baily, 1001. Topping, 1002. La Lande's Astronomie, 1003. Lagrange, 1004. Partially investigates Laplace's theorem, 1008. Rumovsky, 1012. Prony, 1014. Cavendish, 1015. Result as to the mean density of the Earth, 1018. Trembley, 1019. Algebraical identity, 1024. Error, 1027. Legendre's coefficients, 1028. Laplace's equation, 1030. Fontana, 1034. Van-Swinden, 1035. Survey of England and Wales, 1036. General Roy's rule, 1037.	

Cambridge University Press

978-1-108-08457-4 - A History of the Mathematical Theories of Attraction and the Figure of the Earth: From the Time of Newton to that of Laplace: Volume 1

Isaac Todhunter

Table of Contents

[More information](#)

## CONTENTS.

xxvii

PAGE

176

CHAPTER XXVIII. LAPLACE, MÉCANIQUE CÉLESTE,  
FIRST AND SECOND VOLUMES

First volume, 1041. Potential, 1042. Case of a spherical shell, 1043. Objection, 1044. Spherical shell and external particle, 1045. Laplace's enunciation extended, 1046. Spherical shell and internal particle, 1047. Cylinder, 1048. Cylindrical shell, 1050. Second volume, 1052. First Chapter, Spheroid, 1053. Attraction of an ellipsoid, 1054. Laplace's theorem, 1060. Professor Cayley's paper, 1061. Second Chapter, Laplace's functions, 1064. Various names for them, 1066. Any function may be expanded in a series of them, 1067. Form of the expansion, 1069. Legendre's theorem extended, 1076. Third Chapter, Oblatum, 1078. His results, 1079. Sources of them, 1080. Oblongum inadmissible, 1082. One oblatum for given moment of rotation, 1085. Fourth Chapter, 1088. Bouguer's hypothesis rejected, 1094. Approximation extended, 1098. Fifth Chapter, 1100. Misprints, 1102. Fifth Chapter, 1103. First method of calculation, 1104. Second method, 1105. Jupiter, 1109. Lapland arc, 1111. Arc perpendicular to the meridian, 1112. Sixth Chapter, Saturn's ring, 1116. Attraction of an elliptic cylinder, 1119. The resultant constant at the surface, 1123. Seventh Chapter, 'Atmosphere, 1126. Summary of results, 1127.

216

CHAPTER XXIX. LAPLACE'S THEOREM

History of the Theorem, 1129. Biot's investigation, 1130. General theorem, 1133. Particular case, 1136. Biot's historical statement questioned, 1138. Ivory, 1140. His enunciation, 1143. Opinions of his merit, 1146. Plana, 1147. Biot's appendix, 1148. Legendre, 1149. History of the theorem, 1150. Expressions for the attraction of an ellipsoid, 1152. Approximation for a nearly spherical body, 1155. Elliptic integrals, 1156. Algebraical relation, 1157. Another relation, 1158. Poisson extends Ivory's theorem, 1160. This does not apply to Laplace's theorem, 1161. Gauss, 1162. His fifth theorem, 1169. His fourth theorem, 1170. His third theorem, 1171. His sixth theorem, 1172. Application to the ellipsoid, 1173. Gauss's reference to Ivory, 1175. Rodrigues, 1176. Relative Potential, 1179. Symmetrical expression for the Potential of an ellipsoid, 1184. Formula for Legendre's coefficient, 1187. General property of the coefficients, 1189. Summary of results, 1194.

253

CHAPTER XXX. LAPLACE'S EQUATION

Particular form, 1196. Lagrange, 1197. The difficulty which he explains, 1200. Ivory's objections, 1203. History of the equation, 1204. The point of difficulty, 1207. Expansion in a series of Laplace's functions, 1213. General form of the equation, 1214. Ivory's objections, 1215. Ivory's notice of Lagrange's memoir, 1216. Second memoir by Ivory, 1219. Laplace's later investigation, 1220. Poisson, 1223. Ivory returns to the sub-

Cambridge University Press

978-1-108-08457-4 - A History of the Mathematical Theories of Attraction and the Figure of the Earth: From the Time of Newton to that of Laplace: Volume 1

Isaac Todhunter

Table of Contents

[More information](#)

xxviii

CONTENTS.

	PAGE
ject, 1224. Laplace's opinion of Ivory, 1226. Airy, 1227. His treatment of the equation, 1228. Expansion in a series of Laplace's functions, 1230. Mc Cullagh, 1233. Plana, 1235.	
<b>CHAPTER XXXI. PARTIAL DIFFERENTIAL EQUATION FOR <math>V</math></b>	<b>274</b>
Laplace's equation, 1236. Poisson's correction, 1237. Applications, 1239. Rodrigues, 1240. Poisson's later method, 1241. The three cases, 1244. Objection, 1245. Ostrogradsky, 1247. Extends the unsatisfactory case given by Poisson, 1248. Sturm, 1251. Bowditch, 1252. Gauss's criticism, 1253.	
<b>CHAPTER XXXII. LAPLACE'S SECOND METHOD OF TREATING LEGENDRE'S PROBLEM</b>	<b>284</b>
History of the problem, 1254. Liouville, 1255. Laplace's process, 1256. Limits of the integration, 1261. Liouville's objection to Laplace's process, 1263. Poisson's process, 1265. Remarks on it, 1267. Laplace's supplementary investigation, 1270. Wantzel, 1272. Transformation of a double integral, 1273.	
<b>CHAPTER XXXIII. LAPLACE'S MEMOIRS</b>	<b>305</b>
Saturn's ring, 1275. Rotation of the Earth, 1276. Figure of the Earth, 1277. Rotation of the Earth, 1278. Theorem as to a principal axis, 1281. Law of gravity, 1283. Figure of the Earth, 1284. Cooling of the Earth, 1287. Increase of temperature with increase of depth, 1290. Mean density of the Earth, 1291. Extracts on Hutton, Cavendish and Newton, 1292.	
<b>CHAPTER XXXIV. FIFTH VOLUME OF THE MÉCANIQUE CÉLESTE</b>	<b>315</b>
Eleventh Book, 1294. First Chapter, Historical sketch, 1295. Elephant preserved in the ice, 1296. Second Chapter, Results obtained by Analysis, 1301. Value of gravity, 1305. Expression for the depth of the sea, 1310. Value of gravity, 1313. Approximate values of Legendre's functions, 1314. Variations of the lengths of degrees and of the value of gravity, 1316. Expression for gravity at the surface of a supposed atmosphere, 1318. Comparison of the analysis with observations, 1320. Lunar Theory, 1322. Measures of degrees, 1323. Precession and Nutation, 1324. Hypothetical law connecting the pressure and the density, 1325. Legendre's law of density, 1326. Numerical results, 1328. Young and D. Bernoulli, 1330. Attraction of a mountain, 1332. Third Chapter, 1336. Theorem as to the three principal axes, 1338. Transformation of angular coordinates, 1340. Fourth Chapter, 1345. Fourier and Poisson, 1346. Analytical results, 1350. Correction, 1351. General summary, 1354.	

Cambridge University Press

978-1-108-08457-4 - A History of the Mathematical Theories of Attraction and the Figure of the Earth: From the Time of Newton to that of Laplace: Volume 1

Isaac Todhunter

Table of Contents

[More information](#)

## CONTENTS.

xxix

## CHAPTER XXXV. POISSON

PAGE

349

List of his writings, 1356. Memoir on electricity, 1357. Attraction of spheroids, 1358. Laplace's functions, 1360. Value of the potential, 1363. Accurate distinction of cases, 1364. Partial differential equation for the potential, 1365. Spheroids which differ little from spheres, 1367. A point hitherto neglected now examined, 1368. Investigation carried to a certain order, 1369. May be extended, 1371. Two forms coincide at the surface, 1372. A transformation, 1373. Laplace's equation extended, 1374. Application to rotating fluid, 1375. Novelties in the process, 1376. Approximation to the second order, 1378. Coulomb's theorem, 1380. Heterogeneous fluid, 1381. Partial differential equation for the potential, 1382. Ivory's criticism, 1384. Addition to the memoir, 1385. Double integral, 1386. Argument against Ivory, 1388. Convergence of series, 1389. Poisson's Mechanics, 1390. Attraction of ellipsoids, 1391. Remarkable result, 1394. New forms for the component attractions, 1395. Elliptic integrals, 1398. Note on a result given by Jacobi, 1401. Controversy between Poisson and Poincot, 1404. Poisson's remarks on Poincot's report, 1405. Poincot's reply, 1408. Poisson's Addition, 1410. Poincot's reply, 1411. Remarks on the controversy, 1412. Note on the general formulæ of attractions, 1413. Liouville's process of investigation, 1414. General summary, 1415.

## CHAPTER XXXVI. IVORY

391

List of his writings, 1416. Attraction of an extensive class of spheroids, 1417. Expansion in a series, 1420. Equilibrium of a fluid, 1421. New principle assumed, 1422. Theorem on the Potential, 1424. Inconclusive reasoning, 1425. True proposition, 1426. Proposition which is not necessarily true, 1427. Error, 1428. Converse of a known result, 1429. Supposed solution of Legendre's problem, 1430. Good treatment of a standard equation, 1432. Unsupported assertion, 1433. Unintelligible reason, 1434. Article on Attraction, 1435. Figure of the Earth, 1436. Fluid attracted to a fixed centre, 1441. Proposed second approximation, 1442. Opinion on the Theory of the Figure of the Earth, 1444. Criticisms on Professor Airy and Poisson, 1445. Pronounces a certain theorem inaccurate, 1448. Demonstrates the theorem, 1449. Criticises a remark made by Biot, 1453. Erroneous statements, 1456. Ivory's assumed principle, 1459. Discusses erroneously Jacobi's theorem, 1460. Remarks on Poisson, 1461. General summary, 1464.

## CHAPTER XXXVII. PLANA

413

List of his writings, 1465. Commentary on Lagrange, 1466. Problems in Attraction, 1468. Erroneous result, 1472. Solution of a problem, 1474. Neglect of the principle of dimensions, 1476. Saturn's ring, 1479. Solution of a problem, 1480. Two definite integrals, 1484. Law of

Cambridge University Press

978-1-108-08457-4 - A History of the Mathematical Theories of Attraction and the Figure of the Earth: From the Time of Newton to that of Laplace: Volume 1

Isaac Todhunter

Table of Contents

[More information](#)

xxx

CONTENTS.

PAGE

density and pressure, 1486. Formula given by Gauss, 1489. Law of density, 1491. Attraction of ellipsoid, 1492. Opinion of Legendre's process, 1495. Criticises Legendre and Pontécoulant, 1498. Remark on Rodrigues, 1502. Unsupported statement, 1503. Remark on Legendre, 1505. Refers to the example of Euler, 1507. Poisson and Legendre, 1508. The potential of an ellipsoid, 1509. Appendix to the memoir on the attraction of an ellipsoid, 1513. Notes on Newton, 1515. Opinion on Newton's method, 1519. Density of the superficial stratum, 1520. Difficulties, 1524. Density of a mountain, 1527. Force of gravity, 1528. Hypothesis of Huygens, 1529. Criticises Laplace, 1530. Attributes an error to Newton, 1534. Criticises Calandrini, 1535. Approximate solution, 1538. Oblongum is not an admissible figure, 1539. Jacobi's Theorem, 1540. Potential of an ellipsoid, 1543. Criticises Poisson, 1544. Hypothesis of uniformly increasing density, 1546. Formula of the Integral Calculus, 1550. Gravity at the surface of the sea, 1551. Laplace's equation, 1553. Remark on D'Alembert, 1556. Tides, 1558. Remark on Newton, 1559. General summary, 1560.

CHAPTER XXXVIII. MISCELLANEOUS INVESTIGATIONS  
BETWEEN THE YEARS 1801 AND 1825

453

Clay, 1562. Benzenberg, 1563. Burckhardt's translation of Laplace, 1564. Other works of the same nature, 1565. Playfair, 1566. Length of arc of the meridian, 1568. The Survey of England, 1572. Svanberg, 1575. Von Zach, 1576. Base du Système Métrique, 1577. Dr Young, 1578. De Zach, 1579. Lagrange, 1580. Example of Fluid equilibrium, 1581. Error, 1582. Playfair, 1583. Silvabelle's problem, 1584. Error, 1587. General result, 1589. Knight, 1593. Attraction of a right-angled triangle, 1595. Extension of Silvabelle's problem, 1599. Modification of the problem, 1601. Formula of the Integral Calculus, 1603. Solution of an example, 1604. De Zach on the Attraction of mountains, 1605. Quotations, 1607. Tadino, 1608. Cauchy, 1609. Dissertation on pendulums, 1610. Luckcock, 1611. Adrain, 1612. Lambton's Indian arc, 1614. Nobili, 1615. Dr Young, 1616. The mean density of the Earth, 1617. Dr Young's Rule, 1618. Ellipticity calculated, 1619. Erroneous solution of a problem, 1621. General remarks, 1623. Wronski, 1624. Airy, 1625. Saturn's ring, 1627. Different numerical calculations, 1628. Form of Saturn, 1630. Beccaria's arc, 1631. Bowditch, 1632.