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978-1-108-08457-4 - A History of the Mathematical Theories of Attraction and the Figure of the Earth: From the Time of Newton to that of Laplace: Volume 1

Isaac Todhunter

Frontmatter

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A History of the Mathematical Theories of Attraction and the Figure of the Earth

Newton's *Principia* paints a picture of the earth as a spinning, gravitating ball. However, the earth is not completely rigid and the interplay of forces will modify the earth's shape in subtle ways. Newton predicted a flattening at the poles, yet others disagreed. Plenty of books have described the expeditions which sought to measure the shape of the earth, but very little has appeared on the mathematics of a problem, which remains of enduring interest even in an age of satellites. Published in 1874, this two-volume work by Isaac Todhunter (1820–84), perhaps the greatest of Victorian historians of mathematics, takes the mathematical story from Newton, through the expeditions which settled the matter in Newton's favour, to the investigations of Laplace which opened a new era in mathematical physics. Volume 1 traces developments from Newton up to 1780, including coverage of the work of Maupertuis, Clairaut and d'Alembert.

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Isaac Todhunter

Frontmatter

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Isaac Todhunter

Frontmatter

[More information](#)

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From the Time of Newton to that of Laplace

VOLUME 1

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Frontmatter

[More information](#)

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Isaac Todhunter

Frontmatter

[More information](#)

HISTORY OF
THE THEORIES OF ATTRACTION
AND
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Figure of the Earth: From the Time of Newton to that of Laplace: Volume 1

Isaac Todhunter

Frontmatter

[More information](#)

Cet admirable Ouvrage [Newton's *Principia*] contient les germes de toutes les grandes découvertes qui ont été faites depuis sur le système du monde: l'histoire de leur développement par les successeurs de ce grand géomètre serait à la fois le plus utile commentaire de son Ouvrage, et le meilleur guide pour arriver à de nouvelles découvertes.

LAPLACE. *Connaissance des Temps pour l'an 1823.*

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Figure of the Earth: From the Time of Newton to that of Laplace: Volume 1

Isaac Todhunter

Frontmatter

[More information](#)

A HISTORY
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AND
THE FIGURE OF THE EARTH,
*FROM THE TIME OF NEWTON TO THAT
OF LAPLACE.*
BY
I. TODHUNTER, M.A., F.R.S.
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Isaac Todhunter

Frontmatter

[More information](#)

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Isaac Todhunter

Frontmatter

[More information](#)

P R E F A C E.

IN the volumes now offered to students I have written the history of an important branch of science in the manner in which I formerly treated the Calculus of Variations and the mathematical theory of Probability; and in the present work, as in those, I undertake a task hitherto unattempted. For although much has been published on the History of Astronomy, yet the progress of the mathematical development of the principle of Attraction has been left almost untouched. The last of the six volumes which constitute the great work of Delambre is devoted to the Astronomy of the eighteenth century; but the Astronomy discussed is almost entirely that of observation, and the investigations of the eminent mathematicians who contributed to fill up the outline traced by Newton are scarcely noticed. There are indeed interesting and valuable works in which the results obtained by theory are stated in popular language for the benefit of general readers; such is the well-known history by Bailly in French, with its continuation by Voiron; and in English we have various excellent productions of the same kind, especially Narrien's *Historical Account of the Origin and Progress of Astronomy*, and Grant's *History of Physical Astronomy*. But the object of these works is quite distinct from that which I have kept in view in my contributions to scientific history. I desire not merely to record the results which may have been obtained but to trace the analysis which led to those results, to estimate its value, and to discriminate between its failure and its success, its error and its truth. So far as I know the only example of a mathematical treatise bearing on the history of Physical Astronomy is Gautier's *Essai Historique sur le problème des trois corps*: but as this treats of the Lunar and Planetary Theories, omitting the Figure of the Bodies, it has nothing in common with the present work.

In the fifth volume of the *Mécanique Céleste* Laplace arranges the whole subject of Physical Astronomy in six divisions, and gives brief sketches of the progress of the theory of all: in every case sound knowledge practically begins with Newton. Laplace's first division is devoted to the Figure and Rotation of the Earth; and this has suggested to me the subject of the present work. I

T. M. A.

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Isaac Todhunter

Frontmatter

[More information](#)

undertake accordingly to trace the history of the Theories of Attraction and of the Figure of the Earth from Newton to Laplace. The two subjects are necessarily associated in origin, and have been historically always united; they are discussed together by Laplace in the second volume of his great work. I have confined myself to a single division of the wide subject of Physical Astronomy, for the extent and difficulty of the whole might deter even a professional cultivator of the science; and the numerous unfinished fragments of works intended to bear on the *Mécanique Céleste* furnish an impressive warning against the rashness of any extravagant design.

I will now give an outline of the plan of my work. The first Chapter is necessarily occupied with Newton, the founder of Physical Astronomy. The power revealed in all his efforts is nowhere more conspicuous than in his treatment of our two subjects.

In the theory of attraction, among other important results, he shewed that the attraction of a spherical shell on an external particle is the same as if the shell were collected at its centre, and that the attraction on an internal particle is zero. These two propositions constitute a complete theory of the attraction of a sphere in which the density varies as the distance from the centre. Moreover the result with respect to an internal particle was extended by Newton to the case in which the bounding surfaces of the shell are similar, similarly situated, and concentric ellipsoids of revolution.

Newton originated the idea of investigating the Figure of the Earth on the supposition that it might be treated as a homogeneous fluid rotating with uniform angular velocity. He assumed as a postulate that there could be relative equilibrium in such a case if the form were that of an oblate ellipsoid of revolution; and he determined the ratio of the axes and the law of variation of gravity at the surface. The investigation, though not free from imperfection, is a rare example of success in the first discussion of a most difficult problem, and constitutes an enduring monument to the surpassing ability of its author.

The second Chapter is devoted to Huygens. To him we owe the important condition of fluid equilibrium, that the resultant force at any point of the free surface must be normal to the surface at that point; and this has indirectly promoted the knowledge of our subject. But Huygens never accepted the great principle of the mutual attraction of particles of matter; and thus he contributed explicitly only the solution of a theoretical problem, namely the investigation of the form of the surface of rotating fluid under the action of a force always directed to a fixed point.

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978-1-108-08457-4 - A History of the Mathematical Theories of Attraction and the Figure of the Earth: From the Time of Newton to that of Laplace: Volume 1

Isaac Todhunter

Frontmatter

[More information](#)

PREFACE.

vii

The third Chapter treats of various miscellaneous investigations connected with the subject in the course of one generation after the publication of the *Principia*. No real addition was made to Newton's theoretical results, while the measurements of arcs of the meridian in France led the Cassinis to adopt the hypothesis that the form of the Earth was not oblate but oblong.

The fourth Chapter relates to Maupertuis. He wrote various memoirs, among which were two in the form of commentaries on Newton's theories of Attraction and the Figure of the Earth. These theories were rendered more accessible by the translation from their original geometrical expression into the familiar analytical language of the epoch. By adhering to Newton's conclusions Maupertuis must have contributed much to maintain the truth among his countrymen, in opposition to the errors recommended by the authority of Des Cartes and the Cassinis.

The important postulate assumed by Newton was first considered by Stirling, a mathematician of great power: the fifth Chapter shews that he obtained, at least implicitly, an approximate demonstration of the required result.

In the sixth Chapter an account is given of various memoirs by Clairaut which preceded the publication of his important work on the Figure of the Earth. Clairaut explicitly demonstrated the truth of Newton's postulate approximately. He also gave the theorem, called Clairaut's theorem, which establishes a connection between the ellipticity of the earth and the coefficient of the term expressing the increase of gravity in passing from the equator to the pole.

The seventh Chapter narrates briefly the circumstances of the measurement of an arc of the meridian in Lapland. I have undertaken to develop the progress of the Mathematical Theories of Attraction and of the Figure of the Earth; but I do not profess to include the practical operations conducive to our knowledge of the exact dimensions of the Earth. These consist mainly of observations of pendulums, and measurements of arcs; and an account of them drawn from the original sources would form an interesting and instructive work. But the more difficult matters to which I have devoted the present volumes have furnished ample employment without any serious divergence into the department of practical application. I have therefore limited myself to short notices of the earlier pendulum experiments, and of the two great measurements in Lapland and Peru; these measurements deserve some attention on account of their historical interest and their decisive testimony to the oblate form of the Earth.

The eighth Chapter treats of various miscellaneous investigations between 1721 and 1740. Desaguliers maintained, with

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Isaac Todhunter

Frontmatter

[More information](#)

a zeal not uniformly discreet, the oblate form against the Cassinian hypothesis; on the other hand, the measurements in France were still held to be in favour of that hypothesis. Towards the end of the period the Academy of Paris proposed the Tides as the subject of a Prize Essay; and this led to the important researches of Maclaurin.

The ninth Chapter is devoted to Maclaurin. He completely solved the problem of the attraction of an ellipsoid of revolution on an internal or superficial particle; and his method and results admitted of obvious extension to the case of an ellipsoid not of revolution. The extent to which he proceeded for the case of an external particle requires to be stated with accuracy, in order to correct errors of opposite kinds which are current. The most general result yet attained may be stated thus: the potentials of two confocal ellipsoids at a given point external to both are as their masses. This theorem was first established by Laplace, but Maclaurin demonstrated it for the particular case in which the external point is on the prolongation of an axis of the ellipsoids. In the theory of the Figure of the Earth, Maclaurin's main achievement was an exact demonstration of Newton's postulate, of which hitherto only approximate investigations had been given.

In the tenth Chapter the contributions of Thomas Simpson are noticed. This eminent mathematician explicitly shewed that if the angular velocity of rotation exceeds a certain value, the oblatum is not a possible form of relative equilibrium for a fluid mass; and it followed implicitly from his results that for any value of the angular velocity less than the limit, more than one figure for relative equilibrium would exist. Simpson also gave a remarkable investigation of the attraction at the surface of a very extensive class of nearly spherical bodies.

The eleventh Chapter consists of an analysis of the celebrated work by Clairaut. The first part of the work treats on the principles of fluid equilibrium; here Clairaut far surpassed his predecessors in extent and accuracy, and left the theory in the form which it still retains, with the single exception of the improvement effected by Euler, who introduced the notion of the pressure at any point of the fluid, together with the appropriate symbol by which it is denoted. The second part of the work treats on the Figure of the Earth. For the case of a homogeneous fluid Clairaut closely followed Maclaurin. The case of a heterogeneous fluid had been hitherto practically untouched, and Clairaut invented for it a beautiful process which has remained substantially unchanged to the present time; the chief result is a certain equation connecting the ellipticity of the strata with their density,

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Isaac Todhunter

Frontmatter

[More information](#)

PREFACE.

ix

which appears in two forms: these I have called respectively Clairaut's primary equation, and Clairaut's derived equation.

The twelfth Chapter narrates briefly the circumstances of the measurement of an arc of the meridian in Peru. I have carefully examined the extensive literature, much of which is controversial, arising from this memorable expedition; and by means of exact references I have afforded assistance to any student who wishes to render himself familiar with all the circumstances.

The thirteenth Chapter is devoted to the earlier half of the writings of D'Alembert which bear on our subjects. They are extensive in amount, and may have served indirectly to diffuse the interest in such investigations which the writer must have felt himself; but on account of errors in principle and inaccuracy of detail their direct value is small. In various attempts which D'Alembert made to criticise the work of Clairaut he was I believe almost uniformly wrong, so far as regards the Figure of the Earth, and barely right on some unimportant points of Hydrostatics. It is stated in the life of D'Alembert published in the *Biographical Dictionary of the Society for the Diffusion of Useful Knowledge* that "He and Clairaut were rivals, and no work of either appeared without finding a severe critic in the other; but D'Alembert, the more cautious and profound of the two, was generally on the right side of the question:..." The judgment is pronounced by a most eminent authority to which I usually bow with reverence; but so far as the subjects of the present work extend, I should venture to reverse it.

The fourteenth Chapter is devoted principally to Boscovich, whose writings furnish elementary accounts of the most important results which had been obtained up to their date. I have also given a brief notice of the poem by Stay, for which Boscovich supplied notes and supplementary dissertations.

The fifteenth Chapter treats of various miscellaneous investigations between the years 1741 and 1760. It includes a brief notice of a Prize Essay on the Figure of the Earth, published by Clairaut, some years after his treatise.

The sixteenth Chapter is occupied with the later half of the writings of D'Alembert. The general character is the same as of the earlier half; the investigations themselves are disfigured by serious errors, but they serve to suggest interesting and important matter.

The works of Frisi are noticed in the seventeenth Chapter: they resemble those of Boscovich in the fact that they served to teach the subject rather than to promote its progress.

The eighteenth Chapter treats of various miscellaneous investigations between the years 1761 and 1780. The first three

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Isaac Todhunter

Frontmatter

[More information](#)

x

PREFACE.

of Laplace's memoirs belong to this period, but for convenience the consideration of them is postponed. The Chapter includes an account of a memoir by Lagrange in which he proceeded by analysis to the point Maclaurin had reached by geometry. The operations carried on at Schellien for ascertaining the density of the Earth are noticed, and references are supplied to the subsequent labours on the same subject. Here the first volume ends, which contains the history of our subjects during the century, which followed the publication of Newton's *Principia*.

The nineteenth Chapter takes the first three memoirs of Laplace. The principal object of these memoirs may be said to be the solution of a problem which is an extension of Newton's postulate. Newton assumed that an oblatum was a possible form of relative equilibrium for rotating fluid; the present problem is to shew that an oblatum is the *only* possible form, at least under certain restrictions. I call the problem Legendre's, because he was the first who solved it with tolerable success. D'Alembert attempted the investigation, but failed. Laplace did not solve the problem completely; but he shewed that for a very large class of nearly spherical figures, the relative equilibrium was impossible. He also obtained the expression for the law of gravity which would hold universally.

The twentieth Chapter is devoted to a memoir which is conspicuous in the history of the Theory of Attraction, namely the earliest of Legendre's. The limit reached by Maclaurin is now for the first time left behind; Legendre shews that the theorem with respect to confocal ellipsoids is true for *any position* of the external point when the ellipsoids are solids of revolution. Legendre introduces here the memorable expressions, hitherto unknown, which are now usually called *Laplace's coefficients*; and also, at the suggestion of Laplace, the function now called the Potential function takes its place in the subject.

The twenty-first Chapter brings before us a scarce treatise by Laplace, and gives an analysis of that half of it which relates to Attraction and the Figure of the Earth. Here was published for the first time, the demonstration of the theorem relating to the action of confocal ellipsoids at an external point which I call by Laplace's name. The subjects of the Attraction of Ellipsoids and of the homogeneous Figure of the Earth appear in this treatise in nearly the same form as in the *Mécanique Céleste*.

The twenty-second Chapter relates to Legendre's second memoir. Here Legendre solves the problem which I call by his name. He assumes that the fluid is in the form of a figure of revolution, and that it does not deviate widely from the spherical form.

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Isaac Todhunter

Frontmatter

[More information](#)

PREFACE.

xi

The twenty-third Chapter notices Laplace's fourth, fifth, and sixth memoirs. The fourth and fifth memoirs contain the theory of the attraction of spheroids, and the theory of Laplace's functions, in the form they assume in the *Mécanique Céleste*. The sixth memoir relates to Saturn's ring.

The twenty-fourth Chapter is devoted to Legendre's third memoir. The object of this memoir is to demonstrate Laplace's theorem respecting confocal ellipsoids by a more direct process than Laplace himself had employed. Legendre does demonstrate the theorem, without expanding his expressions in series, but the process is excessively long and complicated.

The twenty-fifth Chapter analyses Legendre's fourth memoir. Here we have a great development of Clairaut's process for the case of heterogeneous fluid. A general equation is obtained analogous to Clairaut's primary equation; and from this it is shewn that the strata must be ellipsoidal.

The twenty-sixth Chapter is devoted to Laplace's seventh memoir. This contains some numerical discussion of the lengths of degrees, and of the lengths of the seconds pendulum; there is also a theory of the heterogeneous figure of the Earth, which substantially agrees with that in Legendre's fourth memoir.

The twenty-seventh Chapter treats of miscellaneous investigations between the years 1781 and 1800. Among other matters we have here to notice Cousin's Introduction to the study of Physical Astronomy, a memoir by Lagrange, and a memoir by Trembley; the last is of the same unsatisfactory character as various memoirs by the same writer which I have examined in my *History of the Mathematical Theory of Probability*.

The twenty-eighth Chapter gives an account of the first two volumes of the *Mécanique Céleste*, so far as they relate to our subjects. Laplace in effect reproduced with small change the last four of his seven memoirs; and the result is a treatise not yet superseded.

The twenty-ninth Chapter traces the history of investigation with respect to Laplace's Theorem. Ivory, Legendre, Gauss and Rodrigues all gave complete discussions of the attraction of ellipsoids; while Biot and Plana also commented on parts of the theory. The method of Ivory is the simplest of all, and has obtained a permanent position in our elementary works; insomuch that it is usual to speak of *Ivory's theorem*, although the more correct phrase would be *Ivory's demonstration of Laplace's theorem*.

The thirtieth Chapter treats on an equation which Laplace seems to have regarded with peculiar favour, and which occurs often in his writings. The equation however did not satisfy Ivory, and he criticised it with severity. The result of the discussion

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Isaac Todhunter

Frontmatter

[More information](#)

may be said to have established the accuracy of Laplace's equation when used, as he himself used it, with due caution. But at the same time the objects which Laplace sought by the aid of his equation are now generally obtained without it; so that practically the equation is at present rarely employed.

The thirty-first Chapter elucidates the partial differential equation for the symbol which denotes the potential function. Laplace had originally assumed that a certain equation held both for an external particle, and for a component particle of the body considered; but Poisson shewed that the two cases required different forms of the equation.

The thirty-second Chapter discusses a method which Laplace gave for solving Legendre's problem, with the objection brought against it by Liouville, and the treatment which Poisson substituted in place of Laplace's.

The thirty-third Chapter passes in review various memoirs which Laplace published during the first quarter of the present century.

The thirty-fourth Chapter is devoted to that part of the fifth volume of the *Mécanique Céleste* which relates to our subjects; it consists chiefly of a republication of the memoirs noticed in the thirty-third Chapter.

Strictly speaking the period of history which I proposed to describe closes here; but it seemed convenient to include within my range all the writings of three mathematicians who had already been prominent in my work, and who may be naturally associated with their predecessors, especially with Laplace. These writers are Poisson, Ivory and Plana.

The thirty-fifth Chapter contains an account of all Poisson's contributions which had not been previously examined. The most important of these are an elaborate memoir on the Attraction of Spheroids, and a memoir giving a new investigation of Laplace's theorem respecting confocal ellipsoids.

The thirty-sixth Chapter gives a brief sketch of the numerous articles and memoirs which Ivory produced, mainly in support of opinions of his own which were both peculiar and erroneous. The great promise which his early success held out was not followed by any corresponding merit in the essays of his later years.

The thirty-seventh Chapter is devoted to Plana, who wrote several papers chiefly in the form of comments on Lagrange, Legendre and Laplace.

The last Chapter treats of various miscellaneous investigations during the first quarter of the present century. It is by accident the history finishes with a paragraph relating to Bowditch; but on account of his moral and intellectual eminence, and of his unselfish devotion to science, the name of one of the most dis-

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Isaac Todhunter

Frontmatter

[More information](#)

PREFACE.

xiii

tinguished mathematicians beyond the Atlantic may justly close a roll which commences with that of Newton.

The period of time which I have traversed will be found to correspond with some accuracy to a distinct boundary line in the subject. The labours of more recent date present to us many indications of what may be more appropriately called new methods rather than mere developments of those already discussed. Among them we may mention the investigations respecting the Potential by Green and Gauss, and the numerous researches on the attraction of Ellipsoids by Chasles; all these writers will occupy conspicuous places in any future record of the subjects. Sir John Herschel spoke of my *History of Probability* as embracing the series of the *Pleiocene analysts* in distinction from the Post-Pleiocene; and the illustration might be similarly applied in the present case.

Such then is the outline of the history which the present volumes contain. The principles on which I have executed my task are the same as those adopted in my former works; and I may refer especially to the preface to my *History of Probability* for an account of them. I will only state here that I have not thought it necessary to preserve the exact notation of the original authors; that notation frequently varies much in various places, and it is really advantageous for the sake of brevity and clearness to use the same symbols throughout. For example the ratio of the centrifugal force at the equator to the gravity there is denoted in some English books by the letter m ; Clairaut uses ϕ ; D'Alembert in the sixth volume of his *Opuscules Mathématiques* uses ω ; Laplace in the *Mécanique Céleste*, Vol. v. page 7, uses ϕ , and in Vol. v. page 23 he uses $a\phi$. For the ratio of the centrifugal force at the equator to the attraction there, which is very approximately the same thing as the preceding ratio, the letter j is used throughout the present work.

I have been very sparing in the introduction of new terms, for this practice seems carried to an embarrassing extent in some modern mathematical works. I have however found it necessary to have short designations for two things which occur perpetually in these investigations. The body formed by the revolution of an ellipse round its minor axis I call an *oblatum*, and the body formed by the revolution of an ellipse round its major axis I call an *oblongum*. In English books the former has usually been called an *oblate spheroid*; and the latter a *prolate spheroid*. Something is gained in conciseness by using one word instead of two for a name which is frequently required; but the chief reason of the change arises from the fact that the word *spheroid* has been much used in a different sense, namely to denote a body which differs but little from a sphere. It would be very convenient if this sense

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978-1-108-08457-4 - A History of the Mathematical Theories of Attraction and the Figure of the Earth: From the Time of Newton to that of Laplace: Volume 1

Isaac Todhunter

Frontmatter

[More information](#)

xiv

PREFACE.

of the word *spheroid* could be so established as to render superfluous the formal enunciation of the condition of resemblance to a sphere. Perhaps the use of a word to express a form only approximately determined is felt to be somewhat unlike the ordinary precision of mathematical language; and this may account for the frequent repetition of the condition even after it has been explicitly adopted. Moreover the great French writers have often employed the word *spheroid* in a sense so wide as to render it practically equivalent to *body*; an example will be found in the title of a memoir by Poisson on page 388 of the second volume.

I have found it convenient to give a name to a certain ratio which is of importance in our subject, namely the ratio of the difference of gravity at the equator and at the pole to gravity at the equator. This ratio is one of the elements connected by Clairaut's theorem, and I have accordingly called it *Clairaut's fraction*.

There is one term, perhaps the most objectionable of all that have become permanent in mixed mathematics, which is used throughout the work, namely *centrifugal force*. It is with great reluctance that I have felt myself constrained to yield to universal authority and to employ language which experience shews to be most perplexing and misleading. The well-trained student will however have learned that the so-called centrifugal force is a fiction; the simple fact is that a dynamical problem relating to a body which is rotating uniformly, can be reduced to a statical problem by supposing the rotation to cease and a certain force to be introduced.

This History assumes on the part of the reader some elementary acquaintance with the subjects on which it treats. For the Theory of Attractions the Chapter in my work on *Statics*, to which I have occasionally referred, will be sufficient. For the Figure of the Earth the student may consult three well-known English treatises, namely one in Airy's *Mathematical Tracts*, one in O'Brien's *Mathematical Tracts*, and Pratt's Chapter on the subject in his *Mechanical Philosophy*, afterwards enlarged and published separately in a Treatise on *Attractions*, *Laplace's Functions and the Figure of the Earth*: Pratt's Treatise is the most comprehensive of these English treatises, and the easiest to procure. An interesting work was published at Paris in 1865, entitled *Traité Élémentaire de Mécanique Céleste*. *Par H. Resal*. About a third of this volume is devoted to our subjects; and it gives a very instructive account of them: but the extreme inaccuracy of the printing is a serious diminution of the value of the work.

The mathematical expressions which are called *Laplace's coefficients* and *Laplace's functions* play a very important part in

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Isaac Todhunter

Frontmatter

[More information](#)

PREFACE.

xv

the higher investigations of our subjects. The treatises of O'Brien, Pratt, and Resal, which have just been cited contain a sufficient account of these expressions for elementary purposes. The student who wishes to become intimately acquainted with them will have recourse to the work by Heine which is named on page 24 of the second volume; this is an admirable volume enriched with numerous references to the original authorities.

It may be naturally expected that a person who has devoted much time to the study of the history of science will feel disposed to attribute considerable value to the pursuit. The interest which attaches to the struggle of the human mind with serious difficulties, to its gradual progress and final triumph, may be at least as great as that which is excited by an account of the vicissitudes of civil history. An acquaintance with the origin and the course of any science will often give great assistance in the comprehension of its present state, and may even point out the most promising direction for future efforts. Moreover a familiarity with what has been already accomplished or attempted in any subject is conducive to a wise economy of labour; for it may often prevent a writer from investigating afresh what has been already settled, or it may warn him by the failure of his predecessors, that he should not too lightly undertake a labour of well-recognised difficulty. The opinions of Laplace and Arago, which are quoted in my title-pages, are justly entitled to great weight on these points.

That the subjects here treated historically are of no common importance and influence may be easily seen. A knowledge of the figure and dimensions of the Earth is the basis of all the numerical results of Astronomy, and therefore of the greatest practical value. Moreover the researches into the theories of Attraction and of the Figure of the Earth have been fertile in yielding new resources for mathematicians; it will be sufficient to point to the Transformation of Multiple Integrals, the theory of the Potential, and the elaborate doctrine of Laplace's functions, which have all sprung up in the cultivation of this field of Physical Astronomy. Humboldt has drawn attention to this circumstance in his *Cosmos*; the following passage occurs on pages 156 and 157 of the fifth edition of Sabine's translation of the first Volume: "Except the investigations concerning the parallax of the fixed stars, which led to the discovery of aberration and nutation, the history of science presents no problem in which the object obtained,—the knowledge of the mean compression of the Earth, and the certainty that its figure is not a regular one,—is so far surpassed in importance by the incidental gain which, in the course of its long and arduous pursuit, has accrued in the general cultivation and advancement of mathematical and astronomical knowledge."

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Isaac Todhunter

Frontmatter

[More information](#)

xvi

PREFACE.

It may appear that some apology is due for the extent to which the work has grown; this must be found in the extent and intricacy of the materials which had to be analysed. Indeed Ivory, who devoted much attention to the subject of the Figure of the Earth, asserts that it has been attended with greater difficulty and has occasioned a greater number of memoirs than any other branch of the system of the world. I have had some trouble in keeping within the limits of two volumes, and have been compelled to omit many developments which I should gladly have printed. I have also published separately various papers which have grown out of my historical studies; to these I refer in the appropriate places, but it may be convenient to give a list of them here. They are the following:

On Jacobi's Theorem respecting the relative equilibrium of a revolving ellipsoid of fluid, and on Ivory's discussion of the Theorem. *Proceedings of the Royal Society*, Vol. XIX.

Note relating to the Attraction of Spheroids. *Proceedings of the Royal Society*, Vol. XX.

Note on an erroneous extension of Jacobi's Theorem. *Proceedings of the Royal Society*, Vol. XXI.

On the Arc of the Meridian measured in Lapland. *Transactions of the Cambridge Philosophical Society*, Vol. XII.

On the equation which determines the form of the strata in Legendre's and Laplace's theory of the Figure of the Earth. *Transactions of the Cambridge Philosophical Society*, Vol. XII.

On the Proposition 38 of the Third Book of Newton's *Principia*. *Monthly Notices of the Royal Astronomical Society*, Vol. XXXII.

On the Arc of the Meridian measured in South Africa. *Monthly Notices of the Royal Astronomical Society*, Vol. XXXIII.

The account which is given of the memoirs and treatises will be found ample enough in most cases to supply all that a student will ever want to read of them; but this does not apply to the *Mécanique Céleste*, which I desire to illustrate not to supersede. In other words all that I say relative to that great work is intended as a commentary for the use of those who are consulting the original. I have usually cited it by *sections*, but in some cases, which occur almost exclusively in the fifth volume, I have for greater distinctness cited it by *pages*. The pages meant are those of Laplace's own edition; but the student who uses the national edition will be able to adjust the references by observing that in the fifth volume the 85 pages with which we are concerned correspond to 103 pages in the national edition.

It is well known that Laplace does not give any specific

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Isaac Todhunter

Frontmatter

[More information](#)

PREFACE.

xvii

references to the labours of his predecessors and contemporaries; in his great treatises on Physical Astronomy and Probability he embodied with his own results much that he derived from others, and as these treatises have become the standards of authority for the subjects to which they relate, it has followed that with uncritical readers Laplace has not unfrequently obtained credit for what was not distinctively his own production. A student of the course of science will often discover that important investigations which first came under his notice in the works of Laplace, are really due to other mathematicians; and by a natural reaction the conjecture will arise that further research will lead to the restitution of much more to the rightful owners; and thus there may be a recoil from an undue admiration to a suspicious depreciation. But a complete evolution of the history will restore the reputation of Laplace to its just eminence. The advance of mathematical science is on the whole remarkably gradual, for with the single exception of Newton there is very little exhibition of great and sudden developments; but the possessions of one generation are received, augmented, and transmitted by the next. It may be confidently maintained that no single person has contributed more to the general stock than Laplace.

In the life of Laplace in the *English Cyclopædia*, which we may safely attribute to the late Professor De Morgan, there are some valuable remarks suggested by the want of specific information in the writings of Laplace as to what was done by himself and what was done by others; and it is stated that no one has yet supplied the deficiency. With respect to Laplace it is said: "Had he consulted his own glory, he would have taken care always to note exactly that part of his own work in which he had a forerunner; and it is not until this shall have been well and precisely done, that his labours will receive their proper appreciation." In the present history and in that of Probability I have gone over a third part of the collected mathematical works of Laplace; and to that extent the evidence of his great power and achievements is I hope fully and fairly manifested.

I have not hesitated to criticise all that has come before me; and there is scarcely any memoir or treatise of importance left without the suggestion of corrections or additions. I cannot venture to hope that I have uniformly escaped without any obscurity or error. My readers will I trust excuse such blemishes, arising partly from the nature of the task and partly from the circumstance that only such leisure could be found for it as remained amidst continuous occupation in elementary teaching and writing. The work has thus furnished ample employment for seven years of labour, with the exception of a necessary digres-

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Isaac Todhunter

Frontmatter

[More information](#)

xviii

PREFACE.

sion in order to explain and illustrate some peculiarities in the Calculus of Variations. It was perhaps rash for a mere volunteer to undertake so extensive a task; but in spite of the imperfections with which it may have been accomplished, I am willing to hope that the result will be a permanent addition to the literature of Physical Astronomy.

It is not from any desire to challenge comparisons with illustrious men, but merely to justify my estimate of the labour involved, that I venture to quote the following opinion expressed by the late Professor James Forbes in his *Review of the Progress of Mathematical and Physical Science*, and to extend its application from pure to mixed mathematics: "Specimens of what a history of pure mathematics would be, and must be, are to be found in the able 'Reports' of Dr Peacock and Mr Leslie Ellis, in the Transactions of the British Association for 1833, and 1846. A glance at these profound and very technical essays will shew the impossibility of a popular mode of treatment, while the difficulty and labour of producing such summaries may be argued from their exceeding rarity in this or any other language."

I have to record my great obligations to the Rev. J. Sephton, Head Master of the Liverpool Institute, formerly Fellow of St John's College, for his most valuable assistance in conducting the work through the Press. To the Syndics of the University Press I am indebted for their liberality in defraying the expenses of the printing.

I. TODHUNTER.

ST JOHN'S COLLEGE, CAMBRIDGE,
July, 1873.

CONTENTS.

VOLUME I.

	PAGE
CHAPTER I. NEWTON	1
Publication of the Principia, 1. Attractions, 2. Spherical shell on an internal particle, 3. Spherical shell on an external particle, 4. Lemma, 6. Zone of an indefinitely thin spherical shell, 7. Sphere on an external particle, 8. Sphere on an internal particle, 10. Solid of revolution on a particle at any point of the axis, 13. Infinite plane lamina, 15. Jesuits' edition, 16. Density of the Earth, 17. Figure of the Earth, 18. Polar and equatorial canals of fluid, 19. Approximate estimate of attractions, 20. Oblatum and oblongum, 21. Remarks on the approximate estimate, 22. Equilibrium of the canals, 23. Attraction, gravity, weight, 25. Newton's value of the ellipticity, 26. Jupiter's figure, 29. Influence of the Sun's heat, 30. Measured lengths of degrees, 32. Weight resolved along the radius, 33. Increment of gravity, 34. Pendulums, 36. Newton's error, 37. Table of the lengths of a degree, 39. Pendulums, 40. The Cassinian hypothesis untrue, 43. Summary of results, 44. Laplace's opinion, 45. Halley's opinion, 46.	
CHAPTER II. HUYGENS	28
Publication of the Discourse, 47. Date of composition, 49. Vortex, 50. Air pump, and weight in a mine, 51. Pendulum at Cayenne, 52. Principle in Hydrostatics, 53. Value of the ellipticity, 54. Equation, 55. Result extended, 56. Remarks on the Principia, 59. The Sun's distance from the Earth, 61. Resisting medium, 62. Huygens's problem, 64. Mistake as to priority, 65.	
CHAPTER III. MISCELLANEOUS INVESTIGATIONS UP TO THE YEAR 1720	37
Norwood, 68. Pendulums, 69. The Arabian measure, 70. Norwood, 71. Halley, 72. Burnet, Whiston, and Keill, 73. Incidental mistakes, 75.	

xx	CONTENTS.	
		PAGE
	Keill's error, 76. Keill and Halley, 77. Keill and Bentley, 78. Keill and Whiston, 79. Pendulums, 80. D. Cassini adopts Keill's error, 81. Snell, 82. De La Hire, 83. D. Gregory, 84. Keill, 85. Quotations from Keill and Arago, 87. Numerical result, 89. Pendulums, 90. Freind, 91. J. Cassini, 92. Hermann, 93. Hermann's problem, 95. Quotations from Hermann and Boscovich, 97. J. Cassini, 99. French arc, 100. Mairan, 109. Invents a law of attraction, 113. Summary of results, 115.	
	CHAPTER IV. MAUPERTUIS	63
	Saturn's ring, 117. First problem, 118. Second problem, 119. Figure des Astres, 122. A difficulty, 124; see 725. Variable stars and nebulae, 127. On the laws of attraction, 128. Incidental statements, 130. Figure de la Terre, 131. Figures des Corps Célestes, 132. Hydrostatical principles of Newton and Huygens, 133. Mistake, 137. View of an obscure passage in Newton, 138. Figure de la Terre, 140, 141, 142. Examen désintéressé, 143.	
	CHAPTER V. STIRLING	77
	Newton's postulate, 151. Sir J. Lubbock, 152. Resultant action at the surface of an oblatum, 153. Pendulum, 155. Remarks on the merits of Clairaut and Stirling, 156. Theory compared with fact, 157. Estimates of Stirling, 158.	
	CHAPTER VI. CLAIRAUT	83
	Geodesic curve, 160. Proposition in solid Geometry, 161. Arcs of meridian, 162. Newton's postulate, 163. Approximate attraction of an oblatum at the pole, 165. Fluid with variable density, 167. Unsatisfactory with respect to Hydrostatics, 170. Clairaut's Fraction, and Clairaut's Theorem, 171. Huygens's problem, 173. Hydrostatical principles, 174. Geodesic curve, 177.	
	CHAPTER VII. ARC OF THE MERIDIAN MEASURED IN LAPLAND	93
	Peruvian company, 178. Lapland company, 179. Maupertuis's book, 181. Outhier's book, 182. Selection of places, 184. Difference of latitude, 185. Measurement of the base, 186. Difference of latitude redetermined, 189. Hardships, 192. Incidental matters, 194. Ellipticity, 196. Svanberg, 197. Celsius, 198. Reference for further information, 199.	
	CHAPTER VIII. MISCELLANEOUS INVESTIGATIONS BETWEEN THE YEARS 1721 AND 1740	103
	Desaguliers, 200. Criticises the conclusions of J. Cassini, 201. Admits Keill's error, 202. Criticises Mairan's memoir, 203. Considers the French arc, 204.	

CONTENTS. xxi
PAGE

An experiment, 205. Incidental matters, 206. Poleni, 209. *De la Croyere*, 210. J. Cassini, 211. Godin, 213. *La Condamine*, 214. J. Cassini, 215. *Pendulum*, 217. Manfredi, 218. Bouguer on Hydrostatics, 219. J. Cassini, 220. Prize Essay by John Bernoulli, 221. Modes of determining the form of the Earth, 222, 223. Cassini de Thury, 224. *Pendulums*, 225. Cassini de Thury, 226. Bouguer, 227. Delisle, 228. Euler, 229. D. Bernoulli's *Hydrodynamica*, 230. Cassini de Thury, 231. The Tides, 232. D. Bernoulli's Essay, 233. Euler's Essay, 234. Arc between Paris and Amiens, 235. French arc, 236. Remark on the error in Picard's base, 238. Jesuits' edition of the *Principia*, 239. Winsheim, 240.

CHAPTER IX. MACLAURIN 133

Treatise of Fluxions, 241. Attractions, 242. Ellipsoid of revolution, 244. Newton's Postulate, 245. Extends Newton's Hydrostatical principle, 246. Value of gravity, 247. Level surfaces, 248. Newton's Postulate, 249. Attraction of a sphere, 251. Of an ellipsoid of revolution at the equator and pole, 252. Reference to Newton, Cotes, and Stirling, 253. Attraction of confocal ellipsoids, 254. Maclaurin's demonstration, 255. Confocal shells, 256. External particle in the plane of the equator, 257. Maclaurin's results, 259. His investigations under-estimated, 260. Attraction of an oblatum, 261. Application to rotating fluid, 262. Variable density, 264. Attraction of oblatum of varying density, 266. Application of Newton's Hydrostatical principle, 267. Objections, 269. Polar and equatorial columns, 270. Jupiter, 272. Essay on the Tides, 275. Maclaurin's death, 276.

CHAPTER X. THOMAS SIMPSON 176

Mathematical Dissertations, 277. Reference to Maclaurin, 278. Attraction of an oblatum, 279. Value of a definite integral, 280. Values of two definite integrals, 281. Relation between excentricity and the angular velocity, 283. Limit of angular velocity, 284. Two solutions, 285. Conservation of Areas, 286. Oblatum not homogeneous, 289. Attraction of Spheroids, 290. Length of a Degree, 292. Fluxions, 293. Simpson's eminence as a mathematician, 294.

CHAPTER XI. CLAIRAUT 189

Figure de la Terre, 295. Fluid equilibrium, 296. Cartesians and Newtonians, 297. Principle of Canals and Principle of Level Surfaces, 298. Points of interest in the Introduction, 299. Clairaut's first part, 300. Comparison between Clairaut and others, 301. Principle of Canals, 302.
T. M. A. c

	PAGE
General reasoning, 303. Cases where equilibrium is impossible, 304. Principles of Newton and Huygens, 305. Rotating fluid, 306. Complete differential, 307. Principle of Level Surfaces, 308. Examples of fluid equilibrium, 309. Bouguer's problem, 310. Polar Coordinates, 312. Space of three dimensions, 313. Capillary attraction, 314. Heterogeneous fluid, 315. Law of attraction, 316. Clairaut's second part, 317. Homogeneous Figure of the Earth, 318. Jupiter, 319. Attraction of a circular lamina, 320. Attraction of an oblatum, 321. Ellipticity, 322. Fluid about a solid nucleus, 323. Particular cases, 324. Criticisms on Newton, 325. Oblongum may be a possible form, 326. Case in which the thickness of fluid is small, 327. Objection by D'Alembert and Cousin, 328. The ellipticity is in general less when the fluid is heterogeneous than when it is homogeneous, 329. Variation of gravity, 330. Attraction of a circular lamina, 331. Of an oval lamina, 332. Proposition as to the attraction of an ellipsoid of revolution, 333. Particle on the prolongation of the axis of revolution, 334. Analytical verification of a result obtained by Clairaut, 335. Gravity at the surface of the Earth: Clairaut's Theorem, 336. Inaccurate statements which have been made, 337. Applications of Clairaut's theorem, 338. Strata of varying density, 339. Attraction of a shell, 340. Primary Equation, 341. Derived Equation, 343. Another form of the derived equation, 344. Example of a particular law of density, 345. Clairaut's derived equation, 346. Case in which the mass consists of fluid of two densities, 347. Limits for the ellipticity of a planet, 348. Comparison of theory with observation, 349. Laplace's opinion, 350.	

CHAPTER XII. ARC OF THE MERIDIAN MEASURED IN PERU 231

The Peruvian expedition started before the Lapland, 351. Literature of the subject, 352. Course of the operations, 353. Difficulties encountered, 354. Base of verification, 355. The astronomical part, 356. Discordance of the observations, 357. Final result, 358. The Spanish operations, 359. Return to Paris, 360. Miscellaneous points, 361. The Spanish account, 362. Bouguer's Figure de la Terre, 363. Summary, 364.

CHAPTER XIII. D'ALEMBERT 249

Treatise on Fluids, 365. Historical sketch, 366. Comparison between theory and actual measurement, 367. Second edition, 368. Matters of interest, 369. Treatise on the Winds, 370. Two sentences quoted, 371. Companion to Huygens's problem, 372. Attraction of a homogeneous oblatum at its surface, 373. Spherical nucleus, 374. Results which D'Alembert considers strange, 375. Oblate nucleus, 376. Particular case of a formula given by Clairaut, 377. Criticism on Clairaut, 378.

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Isaac Todhunter

Frontmatter

[More information](#)

CONTENTS.

xxiii

PAGE

Incorrect statement, 379. Attraction at the surface of an ellipsoid, 380. Error, 381. Contradiction and error, 382. Ellipsoidal nucleus, 383. Precession of the Equinoxes, 384. Important numerical relation, 385. Combination of D'Alembert's result with Clairaut's, 386. Example in the Integral Calculus, 387. Hypothesis as to the structure of the Earth, 390. Inequality, 391. Treatise on the Resistance of Fluids, 392. His hydrostatic principle, 394. Failure in attempt at generalization, 397. Surfaces of equal density not necessarily level surfaces, 400. Failure in attempt to generalize Clairaut's theory, 404. Researches on the System of the World, 409. Criticism on Euler, 411. Difference between theory and observation, 416. Results of integration, 423. General formula for the attraction of a spheroid, 424. Spherical nucleus, 426. Oblate nucleus, 429. Criticism on Clairaut, 431. Attraction of spherical shell, 434. Results of integration, 436. Attraction of a spheroid of revolution, 437. Three general equations, 444. General estimate of his researches, 452.

CHAPTER XIV. BOSCOVICH AND STAY

305

The work by Boscovich and Le Maire, 454. Boscovich's dissertations, 457. Follows Maclaurin's solution, 461. Criticises Hermann, 466. Criticises D. Bernoulli, 469. Pendulums, 472. Suggestion, 476. Measures of a degree of the meridian, 481. Criticises Euler, 483. Abstract of the treatise, 489. Stay's poem, 490. Dugald Stewart's opinion, 491. Specimens of Stay's verses, 493. Supplementary dissertations, 494. Newton's error, 499. D. Bernoulli's opinion of Newton, 501. Measures of arcs, 508. Criticises Maupertuis, 510. Method for discordant observations, 511. French translation, 513.

CHAPTER XV. MISCELLANEOUS INVESTIGATIONS BETWEEN THE YEARS 1741 AND 1760

335

Pendulums, 515. Murdoch, 516. Bremond's translation, 521. Addition by Murdoch, 522. Mairan, 526. Cassini de Thury, 527. Clairaut and Buffon, 528. E. Zanottus, 529. La Condamine, 530. Silvabelle, 531. Frisi and Short, 532. Defence of Newton, 534. Clairaut's reply to Frisi, 535. Bouguer, 538. La Caille's voyage, 539. La Caille's arc, 541. Maclear's arc, 542. La Lande, 543. Euler, 545. La Caille's reply to Euler, 546. Hollmannus, 547. Euler, 548. La Caille, 549. La Condamine, 550. Picard's base, 551. Walmsley, 552. La Caille, 553. D'Arcy, 554. Clairaut's Prize Essay, 555. Edition of the Principia, by Madame du Chastellet, 558. Lagrange on D'Alembert's paradox, 561.

CHAPTER XVI. D'ALEMBERT

365

Articles in the Encyclopédie, 564. Reply to Boscovich, 567. Paradox, 569. Corrects former errors, 571. Fluid equilibrium, 574. Attempts to solve

c 2

xxiv

CONTENTS.

PAGE

Legendre's problem, 575. Failure, 576. Sixth volume of the *Opuscules Mathématiques*, 579. Two solutions of the problem of rotating fluid, 580. The case of a very small angular velocity, 584. Only two solutions, 586. Replies to the translator of Boscovich, 590. Attraction of mountains, 592. Discussion of a problem, 596. An oblongum not an admissible form, 601. Two analytical matters, 611. Generalisation of a former problem, 613. Correction of a recent error, 618. Attraction of an ellipsoid, 625. Extension of former problem, 629. Error, 630. Criticises Clairaut, 634. Considers Maclaurin's theorem, 636. Atmosphere, 637. Equation to the surface, 639. Letters to Lagrange, 643. Demonstrates Maclaurin's theorem, 645. Rejects an important formula, 651. Investigates a theorem given by Laplace, 652. Fluid equilibrium, 654. Unsound demonstration, 657. Summary of his contributions, 658.

CHAPTER XVII. FRISI

424

De Gravitate, 660. Measurements hitherto made, 661. Extends a result of Newton's, 662. Criticism on Newton, 663. Newton's error, 664. *Cosmographia*, 668. Suggestion as to La Caille's arc, 671. Mistake corrected, 673. Newton's error, 676. Stability of equilibrium, 678. Fontana, 679. Opera, 680. Silvabelle's problem, 682.

CHAPTER XVIII. MISCELLANEOUS INVESTIGATIONS BETWEEN THE YEARS 1761 AND 1780

439

Frise, 686. Krafft, 687. Error, 689. Sum of a series, 694. Osterwald, 696. Michell, 697. Canterzanus, 698. E. Zanottus, 699. J. A. Euler, 700. Lambert, 701. Liesganig, 702. Mason and Dixon, Maskelyne, 703. Liesganig, 704. Cavendish, 705. La Condamine, 706. Lagrange, 707. Transformation of variables, 710. Ellipsoid, 712. Beccaria, 717. Cassini de Thury, 718. Lagrange, 720. Demonstrates Maclaurin's theorem, 721. Cassini de Thury, 723. Schehallien experiment, 724. Statement by Newton, 725. Dionis du Séjour, 728. Whitehurst, 729. Hutton's calculations for Schehallien, 730. Playfair's survey of the mountain, 731. Hutton, 732. Density of the Earth, 733. Cousin, 734. Euler, 735. Hutton, 736. Titles of works, 738. Additional remarks, 740.

Cambridge University Press

978-1-108-08457-4 - A History of the Mathematical Theories of Attraction and the Figure of the Earth: From the Time of Newton to that of Laplace: Volume 1

Isaac Todhunter

Frontmatter

[More information](#)

CONTENTS.

xxv

VOLUME II.

	PAGE
CHAPTER XIX. LAPLACE'S FIRST THREE MEMOIRS	1
Five divisions in Laplace's writings, 741. First Memoir, 742. Legendre's problem, 744. Second memoir, 751. Law of variation of gravity, 753. Laplace's equation, 755. Third memoir, 764. Mistake, 769. Laplace's equation, 771. Order of the writings of Laplace and Legendre, 778.	
CHAPTER XX. LEGENDRE'S FIRST MEMOIR	20
Treatment of Maclaurin's theorem, 781. Extension of it, 782. Laplace's coefficients, 783. Heine's work, 784. General expression for a coefficient, 786. Potential, 789. Green and Gauss, 790. Legendre's theorem, 791. Criticism on the demonstration, 792. Extension of Maclaurin's theorem, 793. Character of the memoir, 794.	
CHAPTER XXI. LAPLACE'S TREATISE.	29
Its scarcity, 796. Publication, 797. Number of sections, 800. Equation to an ellipsoid, 801. Potential, 802. Polar coordinates, 803. Laplace's theorem, 804. Attraction of an ellipsoid, 805. History of the problem, 806. Rotating fluid mass, 807. Treatment of D'Alembert's problem, 808. Case of the Moon, 809. Case of the Earth, 810. Two and only two solutions, 811. Conservation of areas, 813. Laplace's equation, 814. General problem of the form of fluid, 815. Law of gravity at the surface of a fluid mass, 816. Spherical shell attracting external particle, 817.	
CHAPTER XXII. LEGENDRE'S SECOND MEMOIR	43
Object of the memoir to solve Legendre's problem, 820. Conditions of the demonstration, 821. Seven theorems as to Laplace's coefficients, 822. Equation to be solved, 831. Mode of treatment, 837. Remarks on the demonstration, 842. Legendre's own opinion respecting it, 844. Quotation from Laplace, 845. Quotation from Ivory, 846. Quotation from Jacobi, 847.	
CHAPTER XXIII. LAPLACE'S FOURTH, FIFTH, AND SIXTH MEMOIRS	55
Fourth memoir, 848. Laplace's theorem, 850. Partial Differential Equation for the Potential, 851. Laplace's Equation, 852. Error, 854. Property of Laplace's functions, 857. Fifth memoir, 859. Degrees and Pendulums, 860. Numerical example, 862. Oversight, 863. Sixth memoir, 864. Saturn's ring, 865. Partial Differential Equation for the Potential, 866. Division of Saturn's ring, 867. Form of the ring, 868. Plana's criticisms on Laplace, 871. Instability of a ring, 872.	

Cambridge University Press

978-1-108-08457-4 - A History of the Mathematical Theories of Attraction and the Figure of the Earth: From the Time of Newton to that of Laplace: Volume 1

Isaac Todhunter

Frontmatter

[More information](#)

xxvi

CONTENTS.

	PAGE
CHAPTER XXIV. LEGENDRE'S THIRD MEMOIR	74
Object of the memoir to demonstrate Laplace's Theorem, 875. Transformation of multiple integrals, 877. Case in which the attracted particle is in a principal plane of the ellipsoid, 880. General problem, 881. Point of the Integral Calculus, 882. Legendre's theorem, 883. Limitation in Legendre's process, 886. Opinions of the method, 887.	
CHAPTER XXV. LEGENDRE'S FOURTH MEMOIR	87
Object of the memoir, 892. Error, 896. Laplace's coefficients, 897. First hypothesis, 902. Result of first approximation, 909. Second approximation, 912. Ellipticity, 917. Length of degree, 918. General theorem, 920. Force of gravity, 921. Clairaut's theorem, 923. Second hypothesis, 925. Clairaut's equation generalised, 929. Vanishing of terms, 933. Examples of laws of density, 939. First example; density constant, 940. Second example, 941. Third example, 942. Second approximation, 943. Third hypothesis, 944. Numerical values, 945. Case in which the figure is not assumed to be one of revolution, 948. Laplace's functions, 949. Correction of an oversight in Laplace, 953. The form of a planet must be an oblatum, 955. General estimate, 957.	
CHAPTER XXVI. LAPLACE'S SEVENTH MEMOIR	131
Mode of treating measured lengths of degrees, 960. Another method due to Boscovich, 962. Lengths of seconds pendulum, 964. Two methods of treatment, 966. Clairaut's Theorem, 967.	
CHAPTER XXVII. MISCELLANEOUS INVESTIGATIONS BETWEEN THE YEARS 1781 AND 1800	138
Euler, 970. Kraft, 971. Error of B. St Pierre, 972. Cousin's Elementary Treatise, 973. Error, 976. Approximate formulæ, 978. Attempts to generalise Clairaut's theory, 980. Error, 982. Roy, 984. La Lande, 985. Roy, 986. Embarrassed by an error of Bouguer's, 987. Williams, 988. La Lande on Fernel's measure, 989. Legendre's Theorem in Spherical Trigonometry, 990. Monge, 991. Borda and others, 992. Coulomb's theorem, 993. Lagrange's Mécanique Analytique, 994. Waring, 995. Triesnecker, 996. Pictet, 997. Waring, 998. Dalby, 999. Cassini IV. and others, 1000. Cagnoli and Baily, 1001. Topping, 1002. La Lande's Astronomie, 1003. Lagrange, 1004. Partially investigates Laplace's theorem, 1008. Rumovsky, 1012. Prony, 1014. Cavendish, 1015. Result as to the mean density of the Earth, 1018. Trembley, 1019. Algebraical identity, 1024. Error, 1027. Legendre's coefficients, 1028. Laplace's equation, 1030. Fontana, 1034. Van-Swinden, 1035. Survey of England and Wales, 1036. General Roy's rule, 1037.	

CONTENTS. xxvii

	PAGE
CHAPTER XXVIII. LAPLACE, MÉCANIQUE CÉLESTE, FIRST AND SECOND VOLUMES	176
First volume, 1041. Potential, 1042. Case of a spherical shell, 1043. Objection, 1044. Spherical shell and external particle, 1045. Laplace's enunciation extended, 1046. Spherical shell and internal particle, 1047. Cylinder, 1048. Cylindrical shell, 1050. Second volume, 1052. First Chapter, Spheroid, 1053. Attraction of an ellipsoid, 1054. Laplace's theorem, 1060. Professor Cayley's paper, 1061. Second Chapter, Laplace's functions, 1064. Various names for them, 1066. Any function may be expanded in a series of them, 1067. Form of the expansion, 1069. Legendre's theorem extended, 1076. Third Chapter, Oblatum, 1078. His results, 1079. Sources of them, 1080. Oblongum inadmissible, 1082. One oblatum for given moment of rotation, 1085. Fourth Chapter, 1088. Bouguer's hypothesis rejected, 1094. Approximation extended, 1098. Fifth Chapter, 1100. Misprints, 1102. Fifth Chapter, 1103. First method of calculation, 1104. Second method, 1105. Jupiter, 1109. Lapland arc, 1111. Arc perpendicular to the meridian, 1112. Sixth Chapter, Saturn's ring, 1116. Attraction of an elliptic cylinder, 1119. The resultant constant at the surface, 1123. Seventh Chapter, 'Atmosphere, 1126. Summary of results, 1127.	
CHAPTER XXIX. LAPLACE'S THEOREM	216
History of the Theorem, 1129. Biot's investigation, 1130. General theorem, 1133. Particular case, 1136. Biot's historical statement questioned, 1138. Ivory, 1140. His enunciation, 1143. Opinions of his merit, 1146. Plana, 1147. Biot's appendix, 1148. Legendre, 1149. History of the theorem, 1150. Expressions for the attraction of an ellipsoid, 1152. Approximation for a nearly spherical body, 1155. Elliptic integrals, 1156. Algebraical relation, 1157. Another relation, 1158. Poisson extends Ivory's theorem, 1160. This does not apply to Laplace's theorem, 1161. Gauss, 1162. His fifth theorem, 1169. His fourth theorem, 1170. His third theorem, 1171. His sixth theorem, 1172. Application to the ellipsoid, 1173. Gauss's reference to Ivory, 1175. Rodrigues, 1176. Relative Potential, 1179. Symmetrical expression for the Potential of an ellipsoid, 1184. Formula for Legendre's coefficient, 1187. General property of the coefficients, 1189. Summary of results, 1194.	
CHAPTER XXX. LAPLACE'S EQUATION	253
Particular form, 1196. Lagrange, 1197. The difficulty which he explains, 1200. Ivory's objections, 1203. History of the equation, 1204. The point of difficulty, 1207. Expansion in a series of Laplace's functions, 1213. General form of the equation, 1214. Ivory's objections, 1215. Ivory's notice of Lagrange's memoir, 1216. Second memoir by Ivory, 1219. Laplace's later investigation, 1220. Poisson, 1223. Ivory returns to the sub-	

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978-1-108-08457-4 - A History of the Mathematical Theories of Attraction and the Figure of the Earth: From the Time of Newton to that of Laplace: Volume 1

Isaac Todhunter

Frontmatter

[More information](#)

xxviii

CONTENTS.

	PAGE
ject, 1224. Laplace's opinion of Ivory, 1226. Airy, 1227. His treatment of the equation, 1228. Expansion in a series of Laplace's functions, 1230. Mc Cullagh, 1233. Plana, 1235.	
CHAPTER XXXI. PARTIAL DIFFERENTIAL EQUATION FOR V	274
Laplace's equation, 1236. Poisson's correction, 1237. Applications, 1239. Rodrigues, 1240. Poisson's later method, 1241. The three cases, 1244. Objection, 1245. Ostrogradsky, 1247. Extends the unsatisfactory case given by Poisson, 1248. Sturm, 1251. Bowditch, 1252. Gauss's criticism, 1253.	
CHAPTER XXXII. LAPLACE'S SECOND METHOD OF TREATING LEGENDRE'S PROBLEM	284
History of the problem, 1254. Liouville, 1255. Laplace's process, 1256. Limits of the integration, 1261. Liouville's objection to Laplace's process, 1263. Poisson's process, 1265. Remarks on it, 1267. Laplace's supplementary investigation, 1270. Wantzel, 1272. Transformation of a double integral, 1273.	
CHAPTER XXXIII. LAPLACE'S MEMOIRS	305
Saturn's ring, 1275. Rotation of the Earth, 1276. Figure of the Earth, 1277. Rotation of the Earth, 1278. Theorem as to a principal axis, 1281. Law of gravity, 1283. Figure of the Earth, 1284. Cooling of the Earth, 1287. Increase of temperature with increase of depth, 1290. Mean density of the Earth, 1291. Extracts on Hutton, Cavendish and Newton, 1292.	
CHAPTER XXXIV. FIFTH VOLUME OF THE MÉCANIQUE CÉLESTE	315
Eleventh Book, 1294. First Chapter, Historical sketch, 1295. Elephant preserved in the ice, 1296. Second Chapter, Results obtained by Analysis, 1301. Value of gravity, 1305. Expression for the depth of the sea, 1310. Value of gravity, 1313. Approximate values of Legendre's functions, 1314. Variations of the lengths of degrees and of the value of gravity, 1316. Expression for gravity at the surface of a supposed atmosphere, 1318. Comparison of the analysis with observations, 1320. Lunar Theory, 1322. Measures of degrees, 1323. Precession and Nutation, 1324. Hypothetical law connecting the pressure and the density, 1325. Legendre's law of density, 1326. Numerical results, 1328. Young and D. Bernoulli, 1330. Attraction of a mountain, 1332. Third Chapter, 1336. Theorem as to the three principal axes, 1338. Transformation of angular coordinates, 1340. Fourth Chapter, 1345. Fourier and Poisson, 1346. Analytical results, 1350. Correction, 1351. General summary, 1354.	

Cambridge University Press

978-1-108-08457-4 - A History of the Mathematical Theories of Attraction and the Figure of the Earth: From the Time of Newton to that of Laplace: Volume 1

Isaac Todhunter

Frontmatter

[More information](#)

CONTENTS.

xxix

CHAPTER XXXV. POISSON

PAGE

349

List of his writings, 1356. Memoir on electricity, 1357. Attraction of spheroids, 1358. Laplace's functions, 1360. Value of the potential, 1363. Accurate distinction of cases, 1364. Partial differential equation for the potential, 1365. Spheroids which differ little from spheres, 1367. A point hitherto neglected now examined, 1368. Investigation carried to a certain order, 1369. May be extended, 1371. Two forms coincide at the surface, 1372. A transformation, 1373. Laplace's equation extended, 1374. Application to rotating fluid, 1375. Novelties in the process, 1376. Approximation to the second order, 1378. Coulomb's theorem, 1380. Heterogeneous fluid, 1381. Partial differential equation for the potential, 1382. Ivory's criticism, 1384. Addition to the memoir, 1385. Double integral, 1386. Argument against Ivory, 1388. Convergence of series, 1389. Poisson's Mechanics, 1390. Attraction of ellipsoids, 1391. Remarkable result, 1394. New forms for the component attractions, 1395. Elliptic integrals, 1398. Note on a result given by Jacobi, 1401. Controversy between Poisson and Poincot, 1404. Poisson's remarks on Poincot's report, 1405. Poincot's reply, 1408. Poisson's Addition, 1410. Poincot's reply, 1411. Remarks on the controversy, 1412. Note on the general formulæ of attractions, 1413. Liouville's process of investigation, 1414. General summary, 1415.

CHAPTER XXXVI. IVORY

391

List of his writings, 1416. Attraction of an extensive class of spheroids, 1417. Expansion in a series, 1420. Equilibrium of a fluid, 1421. New principle assumed, 1422. Theorem on the Potential, 1424. Inconclusive reasoning, 1425. True proposition, 1426. Proposition which is not necessarily true, 1427. Error, 1428. Converse of a known result, 1429. Supposed solution of Legendre's problem, 1430. Good treatment of a standard equation, 1432. Unsupported assertion, 1433. Unintelligible reason, 1434. Article on Attraction, 1435. Figure of the Earth, 1436. Fluid attracted to a fixed centre, 1441. Proposed second approximation, 1442. Opinion on the Theory of the Figure of the Earth, 1444. Criticisms on Professor Airy and Poisson, 1445. Pronounces a certain theorem inaccurate, 1448. Demonstrates the theorem, 1449. Criticises a remark made by Biot, 1453. Erroneous statements, 1456. Ivory's assumed principle, 1459. Discusses erroneously Jacobi's theorem, 1460. Remarks on Poisson, 1461. General summary, 1464.

CHAPTER XXXVII. PLANA

413

List of his writings, 1465. Commentary on Lagrange, 1466. Problems in Attraction, 1468. Erroneous result, 1472. Solution of a problem, 1474. Neglect of the principle of dimensions, 1476. Saturn's ring, 1479. Solution of a problem, 1480. Two definite integrals, 1484. Law of

xxx	CONTENTS.	PAGE
	density and pressure, 1486. Formula given by Gauss, 1489. Law of density, 1491. Attraction of ellipsoid, 1492. Opinion of Legendre's process, 1495. Criticises Legendre and Pontécoulant, 1498. Remark on Rodrigues, 1502. Unsupported statement, 1503. Remark on Legendre, 1505. Refers to the example of Euler, 1507. Poisson and Legendre, 1508. The potential of an ellipsoid, 1509. Appendix to the memoir on the attraction of an ellipsoid, 1513. Notes on Newton, 1515. Opinion on Newton's method, 1519. Density of the superficial stratum, 1520. Difficulties, 1524. Density of a mountain, 1527. Force of gravity, 1528. Hypothesis of Huygens, 1529. Criticises Laplace, 1530. Attributes an error to Newton, 1534. Criticises Calandrini, 1535. Approximate solution, 1538. Oblongum is not an admissible figure, 1539. Jacobi's Theorem, 1540. Potential of an ellipsoid, 1543. Criticises Poisson, 1544. Hypothesis of uniformly increasing density, 1546. Formula of the Integral Calculus, 1550. Gravity at the surface of the sea, 1551. Laplace's equation, 1553. Remark on D'Alembert, 1556. Tides, 1558. Remark on Newton, 1559. General summary, 1560.	

CHAPTER XXXVIII. MISCELLANEOUS INVESTIGATIONS BETWEEN THE YEARS 1801 AND 1825	453
Clay, 1562. Benzenberg, 1563. Burckhardt's translation of Laplace, 1564. Other works of the same nature, 1565. Playfair, 1566. Length of arc of the meridian, 1568. The Survey of England, 1572. Svanberg, 1575. Von Zach, 1576. Base du Système Métrique, 1577. Dr Young, 1578. De Zach, 1579. Lagrange, 1580. Example of Fluid equilibrium, 1581. Error, 1582. Playfair, 1583. Silvabelle's problem, 1584. Error, 1587. General result, 1589. Knight, 1593. Attraction of a right-angled triangle, 1595. Extension of Silvabelle's problem, 1599. Modification of the problem, 1601. Formula of the Integral Calculus, 1603. Solution of an example, 1604. De Zach on the Attraction of mountains, 1605. Quotations, 1607. Tadino, 1608. Cauchy, 1609. Dissertation on pendulums, 1610. Luckcock, 1611. Adrain, 1612. Lambton's Indian arc, 1614. Nobili, 1615. Dr Young, 1616. The mean density of the Earth, 1617. Dr Young's Rule, 1618. Ellipticity calculated, 1619. Erroneous solution of a problem, 1621. General remarks, 1623. Wronski, 1624. Airy, 1625. Saturn's ring, 1627. Different numerical calculations, 1628. Form of Saturn, 1630. Beccaria's arc, 1631. Bowditch, 1632.	

CHRONOLOGICAL LIST OF AUTHORS.

The figures refer to the Articles of the Volumes.

	ART.		ART.
1637 NORWOOD	68, 71	1733 MAUPERTUIS	131
1669 PICARD	70, 102	1733 GODIN	213
RICHER	69	1733 LA CONDAMINE.....	214
VARIN	69	1733 J. CASSINI.....	215
DES HAYES	69	1733 CLAIRAUT	160
DU GLOS	69	1734 MAUPERTUIS	132
1686 HALLEY	72	1734 BRADLEY	217
1687 NEWTON	1	1734 MANFREDI.....	218
1690 HUYGENS	47	1734 BOUGUER	219
1691 EISENSCHMIDT	76	1734 J. CASSINI.....	220
1698 KEILL.....	73	1734 JOHN BERNOULLI	221
1699 KEILL	79	1735 MAUPERTUIS	140
1700 COUPLET	80	1735 STIRLING	151
1701 DES HAYES	81	1735 CLAIRAUT	161
1701 D. CASSINI	81	1735 J. CASSINI	222, 223
1702 J. CASSINI.....	82	1735 CASSINI DE THURY	224
1702 D. GREGORY	84	1735 MAIRAN	225
1703 DE LA HIRE	83	1735 GODIN	225
1708 KEILL.....	85	1735 BOUGUER	225
1708 FEUILLÉE	90	1735 LA CONDAMINE.....	225
1711 FREIND	91	1736 MAUPERTUIS	141
1713 NEWTON.....	1	1736 CLAIRAUT	162
1713 J. CASSINI.....	92	1736 CASSINI DE THURY	226
1716 HERMANN	93	1736 BOUGUER	227
1718 J. CASSINI.....	99	1737 MAUPERTUIS	142
1720 J. CASSINI.....	100	1737 CLAIRAUT	163
1720 MAIRAN	109	1737 DELISLE	228
1725 DESAGULIERS	200	1738 MAUPERTUIS	146, 150, 181
1726 NEWTON.....	1	1738 CLAIRAUT	167
1728 POLENI	209	1738 CELSIUS	181, 198
1729 DE LA CROYERE	210	1738 EULER	229
1732 MAUPERTUIS.....	117, 122, 128	1738 D. BERNOULLI	230
1732 J. CASSINI.....	211	1739 CLAIBAUT	177

Cambridge University Press

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Isaac Todhunter

Frontmatter

[More information](#)

xxxii

CHRONOLOGICAL LIST OF AUTHORS.

	ART.		ART.
1739 CASSINI DE THURY	231	1753 CLAIRAUT	535
1740 D. BERNOULLI	233	1753 LA LANDE	544
1740 EULER	234	1753 EULER	545
1740 MAUPERTUIS	235	1754 BOUGUER	352
1740 CASSINI DE THURY	236	1754 LA CONDAMINE.....	352
1740 WINSHEIM	240	1754 D'ALEMBERT	410
1740 MACLAURIN	275	1754 LA CAILLE	546
1741 MURDOCH	515	1754 HOLLMANNUS.....	547
1742 ZELLER	181	1754 BOUGUER and others	551
1742 LE SEUR	239	1755 BOSCOVICH.....	454
1742 JACQUIER	239	1755 EULER	548
1742 CALANDRINUS	239	1755 LA CAILLE	549
1742 MACLAURIN	241	1756 D'ALEMBERT.....	415, 564
1742 BREMOND	521	1756 LA CONDAMINE.....	550
1742 MAIRAN	526	1757 BOSCOVICH	489
1743 SIMPSON.....	277	1757 D'ALEMBERT.....	565
1743 CLAIRAUT	295	1758 WALMESLEY	552
1744 OUTHIER	182	1758 LA CAILLE	553
1744 BOUGUER	352	1758 D'ARCY	554
1744 D'ALEMBERT	365	1759 CLAIRAUT	555
1744 CASSINI DE THURY	527	1759 CHASTELLET	558
1745 LA CONDAMINE.....	352	1759 LAGRANGE.....	561
1745 BUFFON	528	1760 BOSCOVICH.....	490
1745 CLAIRAUT	528	1760 STAY	490
1746 BOUGUER	352	1760 LAGRANGE	562
1746 LA CONDAMINE	352	1761 D'ALEMBERT	566
1746 ZANOTTUS, E.	529	1761 FRISI	686
1747 D'ALEMBERT	370	1763 LA CAILLE	540
1747 LA CONDAMINE	530	1764 KRAFFT.....	687
1748 JUAN	352	1764 OSTERWALD	696
1748 ULLOA	352	1766 MICHELL	697
1748 CASSINI DE THURY	530	1767 CANTERZANUS	698
1749 BOUGUER	352	1767 E. ZANOTTUS	699
1749 D'ALEMBERT	384	1767 LAMBERT	701
1749 EULER	411	1768 D'ALEMBERT.....	570
1750 SIMPSON	293	1768 FRISI	660
1750 SILVABELLE	531	1768 J. A. EULER	700
1751 LA CONDAMINE.....	352	1768 LIESGANIG.....	702
1751 BOUGUER	538	1768 MASON	703
1751 LA CAILLE	539	1768 DIXON	703
1752 BOUGUER	352	1768 MASKELYNE	703
1752 LA CONDAMINE.....	352	1770 D'ALEMBERT.....	368
1752 D'ALEMBERT	392	1770 LIESGANIG.....	704
1752 LA CAILLE.....	539	1772 CAVENDISH	705
1752 LA LANDE	543	1772 LA CONDAMINE.....	706
1753 FRISI	532	1772 LAPLACE	751
1753 SHORT	532	1773 D'ALEMBERT.....	579

CHRONOLOGICAL LIST OF AUTHORS. xxxiii

	ART.		ART.		
1773	LAGRANGE.....	707	1791	PICTET	997
1773	LAPLACE	742	1791	WARING.....	998
1774	D'ALEMBERT.....	643	1791	DALBY	999
1774	BECCARIA	717	1791	CASSINI IV.	1000
1775	FRISI	668	1791	MÉCHAIN	1000
1775	LAGRANGE.....	720	1791	LEGENDRE	1000
1775	CASSINI DE THURY	723	1792	CAGNOLI	1001
1775	MASKELYNE	724	1792	TOPPING	1002
1775	LAPLACE	764	1792	LA LANDE	1003
1776	CASSINI DE THURY	718	1792	LAGRANGE	1004
1778	LA CONDAMINE.....	352	1796	RUMOVSKY	1012
1778	WHITEHURST	729	1797	PRONY	1014
1778	HUTTON	730	1798	CAVENDISH	1015
1778	COUSIN	734	1799	TREMBLEY	1019
1779	EULER	735	1799	FONTANA	1034
1780	D'ALEMBERT.....	644, 654	1799	VAN SWINDEN.....	1035
1780	HUTTON	736	1799	LAPLACE	1040
1782	LAPLACE	848	1802	CLAY	1562
1783	LAPLACE	859	1802	BURCKHARDT	1564
1784	EULER	970	1804	BENZENBERG	1563
1784	KRAFFT	971	1805	PLAYFAIR	1566
1784	ST PIERRE.....	972	1805	SVANBERG	1575
1784	LAPLACE	795	1806	BIOT.....	1130
1784	LEGENDRE	819	1806	A. VON ZACH	1576
1785	FRISI	680	1806	DELAMBRE	1577
1785	ROY	984	1807	LAPLACE	1275
1785	LA LANDE.....	985	1808	YOUNG.....	1578
1785	LEGENDRE	779	1809	IVORY	1140
1786	WILLIAMS	988	1809	LAGRANGE	1197
1787	COUSIN	973	1810	LEGENDRE	1149
1787	ROY	986	1810	DE ZACH	1579
1787	LA LANDE	989	1811	DE ZACH	717
1787	LEGENDRE	990	1811	PLAYFAIR	731
1787	MONGE	991	1811	PLANA	1147
1787	LAPLACE	864	1811	POISSON	1357
1788	LEGENDRE	874	1811	LAGRANGE	1580
1788	WILLIAMS	988	1812	BIOT	1148
1788	BORDA and others.....	992	1812	POISSON	1160
1788	CASSINI IV.	992	1812	IVORY	1203, 1219, 1417
1788	BRISSON.....	992	1812	PLANA	1466
1788	LEGENDRE	992	1812	PLAYFAIR	1583
1788	COULOMB	993	1812	KNIGHT	1593
1788	LAGRANGE.....	994	1813	GAUSS	1162
1789	LEGENDRE	891	1813	POISSON	1237
1789	LAPLACE	958	1814	DE ZACH	1605
1789	WARING.....	995	1814	TADINO.....	1608
1791	TRIESNECKER.....	996	1815	CAUCHY	1609

xxxiV CHRONOLOGICAL LIST OF AUTHORS.

		ART.			ART.
1815	TENGSTROM and BONSDORFF	1610	1830	IVORY.....	1452 to 1454
1816	RODRIGUES	1176, 1240	1831	OSTROGRADSKY	1247
1817	LAPLACE.....	1220, 1286	1831	POISSON	1385
1817	LUCKCOCK	1611	1831	IVORY	1455
1818	PLANA	867	1832	BOWDITCH	1232
1818	LAPLACE.....	1276, 1277, 1286	1833	POISSON	1390
1818	ADRAIN.....	1612, 1613	1835	POISSON	1391
1818	LAMBTON.....	1614	1837	MACCULLAGH	1233
1818	NOBILI.....	1615	1837	LIIOUVILLE	1255
1819	BAILY	1001	1837	POISSON	1265, 1400
1819	LAPLACE	1284	1838	REICH	733
1819	YOUNG.....	1616	1838	POISSON.....	1404, 1413
1820	LAPLACE	1287, 1288	1838	POINSOT	1404
1820	PLANA	1468	1838	LIIOUVILLE	1414
1820	YOUNG.....	1619	1838	IVORY	1460, 1461
1820	WRONSKI.....	1624	1839	WANTZEL.....	1272
1821	HUTTON.....	733	1839	IVORY.....	1460, 1462
1821	LAPLACE	1278, 1283	1840	MENABREA	733
1821	PLANA	1486	1840	GIULIO.....	733
1821	BIOT and ARAGO	1577	1840	GAUSS	1253
1822	IVORY.....	1224, 1419	1840	PLANA.....	1492, 1509
1822	LAPLACE	1285	1841	R. L. ELLIS.....	1422
1823	POISSON.....	1223, 1241	1843	BAILY	733
1823	LAPLACE	1289, 1291	1843	PLANA	1513
1824	CARLINI.....	733	1843	BIOT.....	1577
1824	IVORY.....	1421, 1435, 1436	1845	PLANA	1357
1824	POISSON	1422	1845	BENZENBERG	1563
1825	LAPLACE	1293	1847	HEARN	733
1825	IVORY.....	1437, 1438, 1439	1849	CAYLEY	890
1825	BOWDITCH	1632	1851	PLANA	1515
1826	IVORY.....	1439 to 1442	1852	REICH	733
1826	YOUNG.....	1621	1852	PLANA	1520, 1529
1827	AIRY.....	1227, 1625	1853	PLANA	1533, 1546
1827	POISSON	1384	1854	PLANA	1551
1827	BIOT.....	1577	1855	YOUNG	1622
1827	IVORY.....	1443 to 1445	1856	AIRY	733
1828	IVORY.....	1446 to 1448	1856	HAUGHTON	733
1829	PONTECOULANT	1231	1857	CAYLEY	1060, 1163
1829	POISSON.....	1246, 1358	1858	CAYLEY	1178
1829	IVORY.....	1449 to 1451	1861	HEINE	784
1830	SCHMIDT	733	1866	MACLEAR.....	542
1830	STURM	1251	1869	SCHELL.....	733

THE following Table gives the DATES OF BIRTH AND DEATH
of the principal writers on ATTRACTION and the FIGURE
OF THE EARTH :

BIOT	1777	1862
BOSCOVICH	1711	1787
BOUGUER	1698	1758
BOWDITCH	1773	1838
CASSINI, J. D.	1625	1712
CASSINI, J.	1677	1756
CASSINI DE THURY	1714	1784
CASSINI IV.	1748	1845
CAVENDISH	1731	1810
CLAIRAUT	1713	1765
COULOMB	1736	1806
COUSIN	1739	1800
D'ALEMBERT.....	1717	1783
DELAMBERE	1749	1822
EULER	1707	1783
FRISI.....	1728	1784
GAUSS	1777	1855
HUYGENS	1629	1695
IVORY.....	1765	1842
LA CAILLE	1713	1762
LA CONDAMINE	1701	1774
LAGRANGE.....	1736	1813
LA LANDE	1732	1807
LAPLACE	1749	1827
LEGENDRE	1752	1833
MACLAURIN	1698	1746
MAIRAN	1678	1771
MASKELYNE	1732	1811
MAUPERTUIS	1698	1759
NEWTON	1642	1727
PLANA	1781	1864
PLATFAIR	1748	1819
POISSON.....	1781	1840
SIMPSON	1710	1761

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Isaac Todhunter

Frontmatter

[More information](#)

THE following Table gives references to the principal numerical discussions of the Figure and Dimensions of the Earth in chronological order.

- 1755 BOSCOVICH. *De Litteraria Expeditione*.
- 1760 BOSCOVICH. *Stay's Philosophiæ Recentioris*.
- 1768 FRISI. *De Gravitate*.
- 1770 French translation of Boscovich's work.
- 1783 LAPLACE'S fifth Memoir.
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