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A History of the Mathematical Theories of Attraction and the Figure of the Earth

Newton's *Principia* paints a picture of the earth as a spinning, gravitating ball. However, the earth is not completely rigid and the interplay of forces will modify the earth's shape in subtle ways. Newton predicted a flattening at the poles, yet others disagreed. Plenty of books have described the expeditions which sought to measure the shape of the earth, but very little has appeared on the mathematics of a problem, which remains of enduring interest even in an age of satellites. Published in 1874, this two-volume work by Isaac Todhunter (1820–84), perhaps the greatest of Victorian historians of mathematics, takes the mathematical story from Newton, through the expeditions which settled the matter in Newton's favour, to the investigations of Laplace which opened a new era in mathematical physics. Volume 1 traces developments from Newton up to 1780, including coverage of the work of Maupertuis, Clairaut and d'Alembert.

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A History of the Mathematical Theories of Attraction and the Figure of the Earth

From the Time of Newton to that of Laplace

VOLUME 1

ISAAC TODHUNTER

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HISTORY OF THE THEORIES OF ATTRACTION AND THE FIGURE OF THE EARTH.

VOLUME I.

Cet admirable Ouvrage [Newton's *Principia*] contient les germes de toutes les grandes découvertes qui ont été faites depuis sur le système du monde: l'histoire de leur développement par les successeurs de ce grand géomètre serait à la fois le plus utile commentaire de son Ouvrage, et le meilleur guide pour arriver à de nouvelles découvertes.

LAPLACE. *Connaissance des Tems pour l'an* 1823.

A HISTORY

OF THE

MATHEMATICAL THEORIES OF ATTRACTION

AND

THE FIGURE OF THE EARTH,

FROM THE TIME OF NEWTON TO THAT OF LAPLACE.

BY

I. TODHUNTER, M.A., F.R.S.

IN TWO VOLUMES.

VOLUME I.

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PREFACE.

IN the volumes now offered to students I have written the history of an important branch of science in the manner in which I formerly treated the Calculus of Variations and the mathematical theory of Probability; and in the present work, as in those, I undertake a task hitherto unattempted. For although much has been published on the History of Astronomy, yet the progress of the mathematical development of the principle of Attraction has been left almost untouched. The last of the six volumes which constitute the great work of Delambre is devoted to the Astronomy of the eighteenth century; but the Astronomy discussed is almost entirely that of observation, and the investigations of the eminent mathematicians who contributed to fill up the outline traced by Newton are scarcely noticed. There are indeed interesting and valuable works in which the results obtained by theory are stated in popular language for the benefit of general readers; such is the well-known history by Bailly in French, with its continuation by Voiron; and in English we have various excellent productions of the same kind, especially Narrien's *Historical Account of the Origin and Progress of Astronomy,* and Grant's *History of Physical Astronomy.* But the object of these works is quite distinct from that which I have kept in view in my contributions to scientific history. I desire not merely to record the results which may have been obtained but to trace the analysis which led to those results, to estimate its value, and to discriminate between its failure and its success, its error and its truth. So far as I know the only example of a mathematical treatise bearing on the history of Physical Astronomy is Gautier's *Essai Historique sur le problême des trois corps:* but as this treats of the Lunar and Planetary Theories, omitting the Figure of the Bodies, it has nothing in common with the present work.

In the fifth volume of the *Mécanique Céleste* Laplace arranges the whole subject of Physical Astronomy in six divisions, and gives brief sketches of the progress of the theory of all: in every case sound knowledge practically begins with Newton. Laplace's first division is devoted to the Figure and Rotation of the Earth; and this has suggested to me the subject of the present work. I undertake accordingly to trace the history of the Theories of Attraction and of the Figure of the Earth from Newton to Laplace. The two subjects are necessarily associated in origin, and have been historically always united; they are discussed together by Laplace in the second volume of his great work. I have confined myself to a single division of the wide subject of Physical Astronomy, for the extent and difficulty of the whole might deter even a professional cultivator of the science; and the numerous unfinished fragments of works intended to bear on the *Mécanique Céleste* furnish an impressive warning against the rashness of any extravagant design.

I will now give an outline of the plan of my work. The first Chapter is necessarily occupied with Newton, the founder of Physical Astronomy. The power revealed in all his efforts is nowhere more conspicuous than in his treatment of our two subjects.

In the theory of attraction, among other important results, he shewed that the attraction of a spherical shell on an external particle is the same as if the shell were collected at its centre, and that the attraction on an internal particle is zero. These two propositions constitute a complete theory of the attraction of a sphere in which the density varies as the distance from the centre. Moreover the result with respect to an internal particle was extended by Newton to the case in which the bounding surfaces of the shell are similar, similarly situated, and concentric ellipsoids of revolution.

Newton originated the idea of investigating the Figure of the Earth on the supposition that it might be treated as a homogeneous fluid rotating with uniform angular velocity. He assumed as a postulate that there could be relative equilibrium in such a case if the form were that of an oblate ellipsoid of revolution; and he determined the ratio of the axes and the law of variation of gravity at the surface. The investigation, though not free from imperfection, is a rare example of success in the first discussion of a most difficult problem, and constitutes an enduring monument to the surpassing ability of its author.

The second Chapter is devoted to Huygens. To him we owe the important condition of fluid equilibrium, that the resultant force at any point of the free surface must be normal to the surface at that point; and this has indirectly promoted the knowledge of our subject. But Huygens never accepted the great principle of the mutual attraction of particles of matter; and thus he contributed explicitly only the solution of a theoretical problem, namely the investigation of the form of the surface of rotating fluid under the action of a force always directed to a fixed point.

The third Chapter treats of various miscellaneous investigations connected with the subject in the course of one generation after the publication of the *Principia*. No real addition was made to Newton's theoretical results, while the measurements of arcs of the meridian in France led the Cassinis to adopt the hypothesis that the form of the Earth was not oblate but oblong.

The fourth Chapter relates to Maupertuis. He wrote various memoirs, among which were two in the form of commentaries on Newton's theories of Attraction and the Figure of the Earth. These theories were rendered more accessible by the translation from their original geometrical expression into the familiar analytical language of the epoch. By adhering to Newton's conclusions Maupertuis must have contributed much to maintain the truth among his countrymen, in opposition to the errors recommended by the authority of Des Cartes and the Cassinis.

The important postulate assumed by Newton was first considered by Stirling, a mathematician of great power: the fifth Chapter shews that he obtained, at least implicitly, an approximate demonstration of the required result.

In the sixth Chapter an account is given of various memoirs by Clairaut which preceded the publication of his important work on the Figure of the Earth. Clairaut explicitly demonstrated the truth of Newton's postulate approximately. He also gave the theorem, called Clairaut's theorem, which establishes a connection between the ellipticity of the earth and the coefficient of the term expressing the increase of gravity in passing from the equator to the pole.

The seventh Chapter narrates briefly the circumstances of the measurement of an arc of the meridian in Lapland. I have undertaken to develop the progress of the Mathematical Theories of Attraction and of the Figure of the Earth; but I do not profess to include the practical operations conducive to our knowledge of the exact dimensions of the Earth. These consist mainly of observations of pendulums, and measurements of arcs; and an account of them drawn from the original sources would form an interesting and instructive work. But the more difficult matters to which I have devoted the present volumes have furnished ample employment without any serious divergence into the department of practical application. I have therefore limited myself to short notices of the earlier pendulum experiments, and of the two great measurements in Lapland and Peru; these measurements deserve some attention on account of their historical interest and their decisive testimony to the oblate form of the Earth.

The eighth Chapter treats of various miscellaneous investigations between 1721 and 1740. Desaguliers maintained, with a zeal not uniformly discreet, the oblate form against the Cassinian hypothesis; on the other hand, the measurements in France were still held to be in favour of that hypothesis. Towards the end of the period the Academy of Paris proposed the Tides as the subject of a Prize Essay; and this led to the important researches of Maclaurin.

The ninth Chapter is devoted to Maclaurin. He completely solved the problem of the attraction of an ellipsoid of revolution on an internal or superficial particle; and his method and results admitted of obvious extension to the case of an ellipsoid not of revolution. The extent to which he proceeded for the case of an external particle requires to be stated with accuracy, in order to correct errors of opposite kinds which are current. The most general result yet attained may be stated thus: the potentials of two confocal ellipsoids at a given point external to both are as their masses. This theorem was first established by Laplace, but Maclaurin demonstrated it for the particular case in which the external point is on the prolongation of an axis of the ellipsoids. In the theory of the Figure of the Earth, Maclaurin's main achievement was an exact demonstration of Newton's postulate, of which hitherto only approximate investigations had been given.

In the tenth Chapter the contributions of Thomas Simpson are noticed. This eminent mathematician explicitly shewed that if the angular velocity of rotation exceeds a certain value, the oblatum is not a possible form of relative equilibrium for a fluid mass; and it followed implicitly from his results that for any value of the angular velocity less than the limit, more than one figure for relative equilibrium would exist. Simpson also gave a remarkable investigation of the attraction at the surface of a very extensive class of nearly spherical bodies.

The eleventh Chapter consists of an analysis of the celebrated work by Clairaut. The first part of the work treats on the principles of fluid equilibrium; here Clairaut far surpassed his predecessors in extent and accuracy, and left the theory in the form which it still retains, with the single exception of the improvement effected by Euler, who introduced the notion of the pressure at any point of the fluid, together with the appropriate symbol by which it is denoted. The second part of the work treats on the Figure of the Earth. For the case of a homogeneous fluid Clairaut closely followed Maclaurin. The case of a heterogeneous fluid had been hitherto practically untouched, and Clairaut invented for it a beautiful process which has remained substantially unchanged to the present time; the chief result is a certain equation connecting the ellipticity of the strata with their density, which appears in two forms: these I have called respectively Clairaut's primary equation, and Clairaut's derived equation.

The twelfth Chapter narrates briefly the circumstances of the measurement of an arc of the meridian in Peru. I have carefully examined the extensive literature, much of which is controversial, arising from this memorable expedition; and by means of exact references I have afforded assistance to any student who wishes to render himself familiar with all the circumstances.

The thirteenth Chapter is devoted to the earlier half of the writings of D'Alembert which bear on our subjects. They are extensive in amount, and may have served indirectly to diffuse the interest in such investigations which the writer must have felt himself; but on account of errors in principle and inaccuracy of detail their direct value is small. In various attempts which D'Alembert made to criticise the work of Clairaut he was I believe almost uniformly wrong, so far as regards the Figure of the Earth, and barely right on some unimportant points of Hydrostatics. It is stated in the life of D'Alembert published in the *Biographical Dictionary of the Society for the Diffusion of Useful Knowledge* that "He and Clairaut were rivals, and no work of either appeared without finding a severe critic in the other; but D'Alembert, the more cautious and profound of the two, was generally on the right side of the question:..." The judgment is pronounced by a most eminent authority to which I usually bow with reverence; but so far as the subjects of the present work extend, I should venture to reverse it.

The fourteenth Chapter is devoted principally to Boscovich, whose writings furnish elementary accounts of the most important results which had been obtained up to their date. I have also given a brief notice of the poem by Stay, for which Boscovich supplied notes and supplementary dissertations.

The fifteenth Chapter treats of various miscellaneous investigations between the years 1741 and 1760. It includes a brief notice of a Prize Essay on the Figure of the Earth, published by Clairaut, some years after his treatise.

The sixteenth Chapter is occupied with the later half of the writings of D'Alembert. The general character is the same as of the earlier half; the investigations themselves are disfigured by serious errors, but they serve to suggest interesting and important matter.

The works of Frisi are noticed in the seventeenth Chapter: they resemble those of Boscovich in the fact that they served to teach the subject rather than to promote its progress.

The eighteenth Chapter treats of various miscellaneous investigations between the years 1761 and 1780. The first three of Laplace's memoirs belong to this period, but for convenience the consideration of them is postponed. The Chapter includes an account of a memoir by Lagrange in which he proceeded by analysis to the point Maclaurin had reached by geometry. The operations carried on at Schehallien for ascertaining the density of the Earth are noticed, and references are supplied to the subsequent labours on the same subject. Here the first volume ends, which contains the history of our subjects during the century which followed the publication of Newton's *Principia*.

The nineteenth Chapter takes the first three memoirs of Laplace. The principal object of these memoirs may be said to be the solution of a problem which is an extension of Newton's postulate. Newton assumed that an oblatum was a possible form of relative equilibrium for rotating fluid; the present problem is to shew that an oblatum is the *only* possible form, at least under certain restrictions. I call the problem Legendre's, because he was the first who solved it with tolerable success. D'Alembert attempted the investigation, but failed. Laplace did not solve the problem completely; but he shewed that for a very large class of nearly spherical figures, the relative equilibrium was impossible. He also obtained the expression for the law of gravity which would hold universally.

The twentieth Chapter is devoted to a memoir which is conspicuous in the history of the Theory of Attraction, namely the earliest of Legendre's. The limit reached by Maclaurin is now for the first time left behind; Legendre shews that the theorem with respect to confocal ellipsoids is true for *any position* of the external point when the ellipsoids are solids of revolution. Legendre introduces here the memorable expressions, hitherto unknown, which are now usually called *Laplace's coefficients;* and also, at the suggestion of Laplace, the function now called the Potential function takes its place in the subject.

The twenty-first Chapter brings before us a scarce treatise by Laplace, and gives an analysis of that half of it which relates to Attraction and the Figure of the Earth. Here was published for the first time, the demonstration of the theorem relating to the action of confocal ellipsoids at an external point which I call by Laplace's name. The subjects of the Attraction of Ellipsoids and of the homogeneous Figure of the Earth appear in this treatise in nearly the same form as in the *Mécanique Céleste.*

The twenty-second Chapter relates to Legendre's second memoir. Here Legendre solves the problem which I call by his name. He assumes that the fluid is in the form of a figure of revolution, and that it does not deviate widely from the spherical form.

The twenty-third Chapter notices Laplace's fourth, fifth, and sixth memoirs. The fourth and fifth memoirs contain the theory of the attraction of spheroids, and the theory of Laplace's functions, in the form they assume in the *Mécanique Céleste.* The sixth memoir relates to Saturn's ring.

The twenty-fourth Chapter is devoted to Legendre's third memoir. The object of this memoir is to demonstrate Laplace's theorem respecting confocal ellipsoids by a more direct process than Laplace himself had employed. Legendre does demonstrate the theorem, without expanding his expressions in series, but the process is excessively long and complicated.

The twenty-fifth Chapter analyses Legendre's fourth memoir. Here we have a great development of Clairaut's process for the case of heterogeneous fluid. A general equation is obtained analogous to Clairaut's primary equation; and from this it is shewn that the strata must be ellipsoidal.

The twenty-sixth Chapter is devoted to Laplace's seventh memoir. This contains some numerical discussion of the lengths of degrees, and of the lengths of the seconds pendulum; there is also a theory of the heterogeneous figure of the Earth, which substantially agrees with that in Legendre's fourth memoir.

The twenty-seventh Chapter treats of miscellaneous investigations between the years 1781 and 1800. Among other matters we have here to notice Cousin's Introduction to the study of Physical Astronomy, a memoir by Lagrange, and a memoir by Trembley; the last is of the same unsatisfactory character as various memoirs by the same writer which I have examined in my *History of the Mathematical Theory of Probability.*

The twenty-eighth Chapter gives an account of the first two volumes of the *Mécanique Céleste,* so far as they relate to our subjects. Laplace in effect reproduced with small change the last four of his seven memoirs; and the result is a treatise not yet superseded.

The twenty-ninth Chapter traces the history of investigation with respect to Laplace's Theorem. Ivory, Legendre, Gauss and Rodrigues all gave complete discussions of the attraction of ellipsoids; while Biot and Plana also commented on parts of the theory. The method of Ivory is the simplest of all, and has obtained a permanent position in our elementary works; insomuch that it is usual to speak of *Ivory's theorem,* although the more correct phrase would be *Ivory's demonstration of Laplace's theorem.*

The thirtieth Chapter treats on an equation which Laplace seems to have regarded with peculiar favour, and which occurs often in his writings. The equation however did not satisfy Ivory, and he criticised it with severity. The result of the discussion may be said to have established the accuracy of Laplace's equation when used, as he himself used it, with due caution. But at the same time the objects which Laplace sought by the aid of his equation are now generally obtained without it; so that practically the equation is at present rarely employed.

The thirty-first Chapter elucidates the partial differential equation for the symbol which denotes the potential function. Laplace had originally assumed that a certain equation held both for an external particle, and for a component particle of the body considered; but Poisson shewed that the two cases required different forms of the equation.

The thirty-second Chapter discusses a method which Laplace gave for solving Legendre's problem, with the objection brought against it by Liouville, and the treatment which Poisson substituted in place of Laplace's.

The thirty-third Chapter passes in review various memoirs which Laplace published during the first quarter of the present century.

The thirty-fourth Chapter is devoted to that part of the fifth volume of the *Mécanique Céleste* which relates to our subjects; it consists chiefly of a republication of the memoirs noticed in the thirty-third Chapter.

Strictly speaking the period of history which I proposed to describe closes here; but it seemed convenient to include within my range all the writings of three mathematicians who had already been prominent in my work, and who may be naturally associated with their predecessors, especially with Laplace. These writers are Poisson, Ivory and Plana.

The thirty-fifth Chapter contains an account of all Poisson's contributions which had not been previously examined. The most important of these are an elaborate memoir on the Attraction of Spheroids, and a memoir giving a new investigation of Laplace's theorem respecting confocal ellipsoids.

The thirty-sixth Chapter gives a brief sketch of the numerous articles and memoirs which Ivory produced, mainly in support of opinions of his own which were both peculiar and erroneous. The great promise which his early success held out was not followed by any corresponding merit in the essays of his later years.

The thirty-seventh Chapter is devoted to Plana, who wrote several papers chiefly in the form of comments on Lagrange, Legendre and Laplace.

The last Chapter treats of various miscellaneous investigations during the first quarter of the present century. It is by accident the history finishes with a paragraph relating to Bowditch; but on account of his moral and intellectual eminence, and of his unselfish devotion to science, the name of one of the most distinguished mathematicians beyond the Atlantic may justly close a roll which commences with that of Newton.

The period of time which I have traversed will be found to correspond with some accuracy to a distinct boundary line in the subject. The labours of more recent date present to us many indications of what may be more appropriately called new methods rather than mere developments of those already discussed. Among them we may mention the investigations respecting the Potential by Green and Gauss, and the numerous researches on the attraction of Ellipsoids by Chasles; all these writers will occupy conspicuous places in any future record of the subjects. Sir John Herschel spoke of my *History of Probability* as embracing the series of the *Pleiocene analysts* in distinction from the Post-Pleiocene; and the illustration might be similarly applied in the present case.

Such then is the outline of the history which the present volumes contain. The principles on which I have executed my task are the same as those adopted in my former works; and I may refer especially to the preface to my *History of Probability* for an account of them. I will only state here that I have not thought it necessary to preserve the exact notation of the original authors; that notation frequently varies much in various places, and it is really advantageous for the sake of brevity and clearness to use the same symbols throughout. For example the ratio of the centrifugal force at the equator to the gravity there is denoted in some English books by the letter m; Clairaut uses ϕ; D'Alembert in the sixth volume of his *Opuscules Mathématiques* uses ω; Laplace in the *Mécanique Céleste*, Vol. v. page 7, uses ϕ, and in Vol. v. page 23 he uses $\alpha\phi$. For the ratio of the centrifugal force at the equator to the attraction there, which is very approximately the same thing as the preceding ratio, the letter j is used throughout the present work.

I have been very sparing in the introduction of new terms, for this practice seems carried to an embarrassing extent in some modern mathematical works. I have however found it necessary to have short designations for two things which occur perpetually in these investigations. The body formed by the revolution of an ellipse round its minor axis I call an *oblatum*, and the body formed by the revolution of an ellipse round its major axis I call an *oblongum*. In English books the former has usually been called an *oblate spheroid*; and the latter a *prolate spheroid*. Something is gained in conciseness by using one word instead of two for a name which is frequently required; but the chief reason of the change arises from the fact that the word *spheroid* has been much used in a different sense, namely to denote a body which differs but little from a sphere. It would be very convenient if this sense of the word *spheroid* could be so established as to render superfluous the formal enunciation of the condition of resemblance to a sphere. Perhaps the use of a word to express a form only approximately determined is felt to be somewhat unlike the ordinary precision of mathematical language; and this may account for the frequent repetition of the condition even after it has been explicitly adopted. Moreover the great French writers have often employed the word *spheroid* in a sense so wide as to render it practically equivalent to *body;* an example will be found in the title of a memoir by Poisson on page 388 of the second volume.

I have found it convenient to give a name to a certain ratio which is of importance in our subject, namely the ratio of the difference of gravity at the equator and at the pole to gravity at the equator. This ratio is one of the elements connected by Clairaut's theorem, and I have accordingly called it *Clairaut's fraction.*

There is one term, perhaps the most objectionable of all that have become permanent in mixed mathematics, which is used throughout the work, namely *centrifugal force.* It is with great reluctance that I have felt myself constrained to yield to universal authority and to employ language which experience shews to be most perplexing and misleading. The well-trained student will however have learned that the so-called centrifugal force is a fiction; the simple fact is that a dynamical problem relating to a body which is rotating uniformly, can be reduced to a statical problem by supposing the rotation to cease and a certain force to be introduced.

This History assumes on the part of the reader some elementary acquaintance with the subjects on which it treats. For the Theory of Attractions the Chapter in my work on *Statics*, to which I have occasionally referred, will be sufficient. For the Figure of the Earth the student may consult three well-known English treatises, namely one in Airy's *Mathematical Tracts*, one in O'Brien's *Mathematical Tracts*, and Pratt's Chapter on the subject in his *Mechanical Philosophy*, afterwards enlarged and published separately in a Treatise on *Attractions, Laplace's Functions and the Figure of the Earth:* Pratt's Treatise is the most comprehensive of these English treatises, and the easiest to procure. An interesting work was published at Paris in 1865, entitled *Traité Elémentaire de Mécanique Céleste. Par H. Resal.* About a third of this volume is devoted to our subjects; and it gives a very instructive account of them: but the extreme inaccuracy of the printing is a serious diminution of the value of the work.

The mathematical expressions which are called *Laplace's coefficients* and *Laplace's functions* play a very important part in the higher investigations of our subjects. The treatises of O'Brien, Pratt, and Resal, which have just been cited contain a sufficient account of these expressions for elementary purposes. The student who wishes to become intimately acquainted with them will have recourse to the work by Heine which is named on page 24 of the second volume; this is an admirable volume enriched with numerous references to the original authorities.

It may be naturally expected that a person who has devoted much time to the study of the history of science will feel disposed to attribute considerable value to the pursuit. The interest which attaches to the struggle of the human mind with serious difficulties, to its gradual progress and final triumph, may be at least as great as that which is excited by an account of the vicissitudes of civil history. An acquaintance with the origin and the course of any science will often give great assistance in the comprehension of its present state, and may even point out the most promising direction for future efforts. Moreover a familiarity with what has been already accomplished or attempted in any subject is conducive to a wise economy of labour; for it may often prevent a writer from investigating afresh what has been already settled, or it may warn him by the failure of his predecessors, that he should not too lightly undertake a labour of well-recognised difficulty. The opinions of Laplace and Arago, which are quoted in my title-pages, are justly entitled to great weight on these points.

That the subjects here treated historically are of no common importance and influence may be easily seen. A knowledge of the figure and dimensions of the Earth is the basis of all the numerical results of Astronomy, and therefore of the greatest practical value. Moreover the researches into the theories of Attraction and of the Figure of the Earth have been fertile in yielding new resources for mathematicians; it will be sufficient to point to the Transformation of Multiple Integrals, the theory of the Potential, and the elaborate doctrine of Laplace's functions, which have all sprung up in the cultivation of this field of Physical Astronomy. Humboldt has drawn attention to this circumstance in his *Cosmos;* the following passage occurs on pages 156 and 157 of the fifth edition of Sabine's translation of the first Volume: "Except the investigations concerning the parallax of the fixed stars, which led to the discovery of aberration and nutation, the history of science presents no problem in which the object obtained,—the knowledge of the mean compression of the Earth, and the certainty that its figure is not a regular one,—is so far surpassed in importance by the incidental gain which, in the course of its long and arduous pursuit, has accrued in the general cultivation and advancement of mathematical and astronomical knowledge."

It may appear that some apology is due for the extent to which the work has grown; this must be found in the extent and intricacy of the materials which had to be analysed. Indeed Ivory, who devoted much attention to the subject of the Figure of the Earth, asserts that it has been attended with greater difficulty and has occasioned a greater number of memoirs than any other branch of the system of the world. I have had some trouble in keeping within the limits of two volumes, and have been compelled to omit many developments which I should gladly have printed. I have also published separately various papers which have grown out of my historical studies; to these I refer in the appropriate places, but it may be convenient to give a list of them here. They are the following:

On Jacobi's Theorem respecting the relative equilibrium of a revolving ellipsoid of fluid, and on Ivory's discussion of the Theorem. *Proceedings of the Royal Society*, Vol. XIX.

Note relating to the Attraction of Spheroids. *Proceedings of the Royal Society*, Vol. XX.

Note on an erroneous extension of Jacobi's Theorem. *Proceedings of the Royal Society*, Vol. XXI.

On the Arc of the Meridian measured in Lapland. *Transactions of the Cambridge Philosophical Society*, Vol. XII.

On the equation which determines the form of the strata in Legendre's and Laplace's theory of the Figure of the Earth. *Transactions of the Cambridge Philosophical Society*, Vol. XII.

On the Proposition 38 of the Third Book of Newton's *Principia*. *Monthly Notices of the Royal Astronomical Society*, Vol. XXXII.

On the Arc of the Meridian measured in South Africa. *Monthly Notices of the Royal Astronomical Society*, Vol. XXXIII.

The account which is given of the memoirs and treatises will be found ample enough in most cases to supply all that a student will ever want to read of them; but this does not apply to the *Mécanique Celeste*, which I desire to illustrate not to supersede. In other words all that I say relative to that great work is intended as a commentary for the use of those who are consulting the original. I have usually cited it by *sections*, but in some cases, which occur almost exclusively in the fifth volume, I have for greater distinctness cited it by *pages*. The pages meant are those of Laplace's own edition; but the student who uses the national edition will be able to adjust the references by observing that in the fifth volume the 85 pages with which we are concerned correspond to 103 pages in the national edition.

It is well known that Laplace does not give any specific references to the labours of his predecessors and contemporaries; in his great treatises on Physical Astronomy and Probability he embodied with his own results much that he derived from others, and as these treatises have become the standards of authority for the subjects to which they relate, it has followed that with uncritical readers Laplace has not unfrequently obtained credit for what was not distinctively his own production. A student of the course of science will often discover that important investigations which first came under his notice in the works of Laplace, are really due to other mathematicians; and by a natural reaction the conjecture will arise that further research will lead to the restitution of much more to the rightful owners; and thus there may be a recoil from an undue admiration to a suspicious depreciation. But a complete evolution of the history will restore the reputation of Laplace to its just eminence. The advance of mathematical science is on the whole remarkably gradual, for with the single exception of Newton there is very little exhibition of great and sudden developments; but the possessions of one generation are received, augmented, and transmitted by the next. It may be confidently maintained that no single person has contributed more to the general stock than Laplace.

In the life of Laplace in the *English Cyclopædia*, which we may safely attribute to the late Professor De Morgan, there are some valuable remarks suggested by the want of specific information in the writings of Laplace as to what was done by himself and what was done by others; and it is stated that no one has yet supplied the deficiency. With respect to Laplace it is said: "Had he consulted his own glory, he would have taken care always to note exactly that part of his own work in which he had a forerunner; and it is not until this shall have been well and precisely done, that his labours will receive their proper appreciation." In the present history and in that of Probability I have gone over a third part of the collected mathematical works of Laplace; and to that extent the evidence of his great power and achievements is I hope fully and fairly manifested.

I have not hesitated to criticise all that has come before me; and there is scarcely any memoir or treatise of importance left without the suggestion of corrections or additions. I cannot venture to hope that I have uniformly escaped without any obscurity or error. My readers will I trust excuse such blemishes, arising partly from the nature of the task and partly from the circumstance that only such leisure could be found for it as remained amidst continuous occupation in elementary teaching and writing. The work has thus furnished ample employment for seven years of labour, with the exception of a necessary digression in order to explain and illustrate some peculiarities in the Calculus of Variations. It was perhaps rash for a mere volunteer to undertake so extensive a task; but in spite of the imperfections with which it may have been accomplished, I am willing to hope that the result will be a permanent addition to the literature of Physical Astronomy.

It is not from any desire to challenge comparisons with illustrious men, but merely to justify my estimate of the labour involved, that I venture to quote the following opinion expressed by the late Professor James Forbes in his *Review of the Progress of Mathematical and Physical Science*, and to extend its application from pure to mixed mathematics: "Specimens of what a history of pure mathematics would be, and must be, are to be found in the able 'Reports' of Dr Peacock and Mr Leslie Ellis, in the Transactions of the British Association for 1833, and 1846. A glance at these profound and very technical essays will shew the impossibility of a popular mode of treatment, while the difficulty and labour of producing such summaries may be argued from their exceeding rarity in this or any other language."

I have to record my great obligations to the Rev. J. Sephton, Head Master of the Liverpool Institute, formerly Fellow of St John's College, for his most valuable assistance in conducting the work through the Press. To the Syndics of the University Press I am indebted for their liberality in defraying the expenses of the printing.

I. TODHUNTER.

ST JOHN'S COLLEGE, CAMBRIDGE,
July, 1873.

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SIMPSON	1710	1761

THE following Table gives references to the principal numerical discussions of the Figure and Dimensions of the Earth in chronological order.

1755 BOSCOVICH. *De Litteraria Expeditione.*
1760 BOSCOVICH. Stay's *Philosophiæ Recentioris.*
1768 FRISI. *De Gravitate.*
1770 French translation of Boscovich's work.
1783 LAPLACE'S fifth Memoir.
1789 LAPLACE'S seventh Memoir.
1799 LAPLACE. *Mécanique Céleste*, Vol. II.
1826 AIRY. *Philosophical Transactions.*
1828 IVORY. *Philosophical Magazine.*
1830 AIRY. *Encyclopædia Metropolitana.*
1832 BOWDITCH. Translation of the second volume of the *Mécanique Céleste*, pages 450 to 455.
1837 BESSEL. *Astronomische Nachrichten*, Vol. XIV.
1842 BESSEL. *Astronomische Nachrichten*, Vol. XIX.
1844 BIOT. *Traité Elémentaire d'Astronomie Physique*, Vol. III.
1859 SCHUBERT. *Petersburg Mémoires*, seventh series, Vol. I.
1861 CLARKE. *Memoirs of the Royal Astronomical Society*, Vol. XXIX.
1866 HERSCHEL. *Familiar Lectures on Scientific Subjects.*