CONVERSATIONS ON ARITHMETIC.

Chapter I.

NOTATION AND NUMERATION.

Mrs. D. I think you are now old enough, my little boy, to be taught arithmetic; in learning which you see Alfred and Emily take so much pleasure.

Edmund. That means, I suppose, to answer questions about nuts and apples on my slate, instead of counting on my fingers?—I shall be very glad of that.

Mrs. D. As the first step, you must learn to write the figures, which are the signs of numbers. In the mean time these little fingers must again serve our purpose.

The figure which denotes, or is the sign for,

One, is written thus	1
One and one more, or two, are written .	2
Two and one more, or three, are written .	3
Three and one more, or four, are written .	4
Four and one more, or five, are written .	5
Five and one more, or six, are written .	6
Six and one more, or seven, are written .	7
Seven and one more, or eight, are written .	8
Eight and one more, or nine, are written .	9
Nought, or nothing, is expressed by a cipher, which is written	0

Now, when you have copied these on your slate, and can count each number on your fingers, you shall have another lesson.

Edmund. Mother, I can write all these figures now; and I want you to show me the signs for ten, eleven, and so on.

Mrs. D. Not so fast, my little man. Before you learn these, we must have a little conversation, according to our old method. Remember, my dear child, when I tell you anything, and you do not see clearly what I mean, you must ask me again and again, till you *quite* understand it. You must be attentive and patient, and not say you know when you do not know. Now, let me see you write your figures. Tolerably done.—These nine figures are called digits.

[*Alfred.* From the Latin word *digitus*, a finger.

Mrs. D. True; which is a corroborative proof that in the infancy of arithmetic the fingers were the principal reckoners. When men first began to cast up their accounts, there is little doubt that this method was adopted, as being the readiest and simplest.]

To count, and to recollect the number we have counted, with the help of our fingers, is certainly a very convenient plan, as long as our numbers do not exceed the number of our fingers; but when they are beyond these, we are obliged to have recourse to some other contrivance. You know, Edmund, you have often found that you have not nearly fingers enough to reckon the various things which you wish to count, and we have felt the want of some better way of finding out answers to the many questions you have asked. Thus, to express any number beyond ten, we must count your fingers a second time, and recollect that we have already used them once; unless, indeed, Emily will lend us one of her fingers, to show that we have counted yours once over. Each of your fingers will then represent one, or an unit, and each of hers ten ones, or ten units; so you can express ten units with yours, and she ten tens with

hers. But let me see in what situation our accounts stand at present. You have none of your fingers held up, and Emily has one of hers, to show that one ten has been counted.

Emily. While I am holding up my poor finger all this time, recollect, Edmund, that every one of my fingers are worth ten of yours.

Mrs. D. In order to express this in writing, we put down the sign for nothing, or nought, to show that there are no units, and the sign for one is put in what is called the place of tens, at the left-hand side of the place of units—the tens going before the units, as being of greater value: thus, one ten and no units are expressed in this manner, 10.

Edmund. Well, now we have Emily's little finger put down on the slate, let us go on counting. Here is another of my fingers held up.

Mrs. D. This will show that we have counted

	Tens.	Units.
Ten and one, or eleven, we therefore write	1	1.
In like manner,		
Ten and two, or twelve, are written .	1	2
Ten and three, or thirteen, are written .	1	3
Ten and four, or fourteen, are written .	1	4
Ten and five, or fifteen, are written .	1	5
Ten and six, or sixteen, are written .	1	6
Ten and seven, or seventeen, are written	1	7
Ten and eight, or eighteen, are written .	1	8
Ten and nine, or nineteen, are written .	1	9

Now you have got to hold up your last finger, and then Emily must let us have another of hers, to show that you have counted two tens, or that you have reckoned your fingers twice over. Then there will be two tens and no units or twenty, written . . . 20

In like manner,

Two tens and one unit, or twenty-one, are written . 21
Two tens and two units, or twenty-two, are written 22

And so on, till we have counted your fingers over three times, and then your sister's hand will show us three tens, or thirty, written 30

In like manner,

Three tens and one unit, or thirty-one, are written 31
and so on.
Four tens, or forty, are written 40
Four tens and one unit, or forty-one, are written . 41
and so on.
Five tens, or fifty, are written 50
Five tens and one unit, or fifty-one, are written . 51
and so on.
Six tens, or sixty, are written 60
Six tens and one unit, or sixty-one, are written . 61
and so on.
Seven tens, or seventy, are written . . . 70
Seven tens and one unit, or seventy-one, are written 71
and so on.
Eight tens, or eighty, are written 80
Eight tens and one unit, or eighty-one, are written 81
and so on.
Nine tens, or ninety, are written 90
Nine tens and one unit, or ninety-one, are written 91
and so on.

Edmund. But when we have used all Emily's fingers, what shall we do then?—we must certainly take Alfred's, and after he has no more to lend us, we must have yours, mother.

Mrs. D. Alfred's fingers may then be made each to represent ten tens, or one hundred, and each of mine may represent ten hundreds, or one thousand; but I suspect your brother and sister would not always be

quite ready to form part of your calculating machine, and as I think, likewise, that I can employ my own hands more advantageously, we must henceforth find some substitute for our digits. And now, Edmund, you have learnt enough for this morning; in the afternoon you shall write down some tens and units, and show me whether you remember what I have said to you.

[*Emily.* Why do we not say ten one and ten two, instead of eleven and twelve, they would be so much better understood by a beginner? Ten three, ten four, &c., would be better than thirteen, fourteen, and all the rest of the teens: these, however, are so like three and ten, four and ten, that there is no difficulty in recollecting that they mean the same thing; but eleven and twelve are barbarous words, and completely spoil the uniformity of numbers.

Mrs. D. Only of their names, Emily; and for this we must thank our Saxon ancestors, from whose language we have derived our numeral words. Eleven signifies one left, and twelve two left (after ten is taken away being understood). But though you may think these names of numbers capable of amendment, you cannot, I am sure, fail to give your unqualified admiration to the present system of notation; we are so accustomed to it from our earliest childhood, that we scarcely know how to appreciate its perfect beauty and simplicity—by which we are enabled, with only ten characters, to express any number whatever, and to work out the most difficult questions, merely by knowing how to marshal our men in their proper places, and to assign to each its due rank.

Alfred. There certainly seems little reason to doubt that this manner of reckoning by tens arose from the practice of counting with the fingers. Accordingly, if we had happened to have been made with a smaller or a

greater number of fingers, eight or twelve, for example, the system of notation would have been entirely different.

Mrs. D. The signs, but not the system, would have been different. In the one case we should only have had eight characters, and these would have recurred at intervals of eight numbers instead of ten; while in the other case we should have had twelve characters, recurring at every twelfth interval.

Alfred. How could these changes be managed, mother, and how would they affect the value of the present numbers?

Mrs. D. In no other way than by altering the signs or symbols of those numbers—the aggregate of units would have been exactly the same. If we count those sheep we now see grazing in the park, and find there are twenty, their number would still be the same, whether this were represented by 24, 20, or 18.

Alfred. It would be curious to follow this up, and find what these signs would be if the number of digits were different from what they are at present. Could I do this? Do I know enough to make this transformation?

Mrs. D. Yes, quite enough; but the inquiry is more curious than useful. It cannot by possibility lead to any practical good, and I advise you to dismiss it from your mind, as tending to confuse and bewilder you to no useful purpose. When you are "unco wise," in these matters, and like to indulge in fanciful speculations, from the mere love of the pursuit, then you may in this respect follow the example of many eminent mathematicians who have gone before you. Recollect, however, it is but a whim, and I own I should be better pleased to see my boy exercise his powers in a higher and more useful path of mathematics. It would be just as sensible to transpose the characters of the alphabet at your

arbitrary pleasure, and then to form words and sentences on the supposition. But enough, and perhaps too much, of this. It is well Edmund is gone to his play, or the discussion might have frightened him out of his love of arithmetic.]

Mrs. D. Here is a box full of white counters, and another box containing counters of all the colours of the rainbow. Now with these we shall be able to express more numbers than we shall have patience to reckon. The white counters we will call units, or unit-counters; the red, tens, or ten-counters; the blue, hundreds, or hundred-counters; the green, thousands, or thousand-counters, and so on. These seventeen white counters will therefore express the same number as this one red and seven white counters; these two red counters will mean the same as these twenty white; and these ten red, or tens, will be the same as this one blue, or hundred-counter.

Edmund. How am I to show on my slate one hundred?

Mrs. D. The hundreds, as being of a higher value, take precedence or stand before the tens, in the same manner as the tens stand before the units: thus, one hundred is written 100, the two noughts showing that there are no tens and no units. One hundred and one unit are written 101; one hundred and one ten, 110.

Emily. So, though we profess to consider these noughts as Mr. Nobodies, yet they act an important part in keeping the places of the different figures; without them, there would be a sad confusion. See, Edmund, you have written one unit and one ten, instead of one hundred and one unit: put the cipher between them, to show the place of tens.

Mrs. D. Now express two hundred and one ten—two hundred and one unit—one hundred and two tens, or twenty—one hundred and two units.

Edmund (*writes*). 210, 201, 120, 102.

Mrs. D. You perceive that, by using the same three figures, we can express four different numbers, and you can understand of how much importance it is that you should put each figure in its proper place.

Express the following numbers:—Three hundred and seventy-five; nine hundred and eighty-three; six hundred and forty-seven. Read 863; 740; 509.

Mrs. D. Tens of hundreds, or thousands, are placed before the hundreds, and occupy the fourth place on the left: thus, one thousand is expressed 1000, the noughts showing that there are no units, no tens, and no hundreds.

So two thousand three hundred and forty-one are expressed thus 2341

Three thousand two hundred and forty-one are expressed 3241

Two thousand four hundred and thirty-one . 2431

Read 2134; 1234: 1243.

Mrs. D. I think you now clearly understand that every number increases its value ten times by having a nought placed after it: thus, 20 are ten times more than 2; 200 ten times more than 20, and so on, whatever higher number we may use; and always every figure expresses ten times higher value than the same figure placed immediately on the right of it: this is called Notation. Numeration is the method of reading Notation. You now have been shown the first step in the knowledge of arithmetic; and the proper understanding of Notation is of great importance to all that

comes after it. The subject of your next lesson will be Addition.

[*Alfred.* But, mother, you have taught Edmund to enumerate only as far as thousands; you have forgotten to introduce to his notice all the *illions*.

Mrs. D. I think, for the present, we had better leave our little pupil to con over his thousands, and allow him to become quite familiar with them before we bewilder his tender mind with higher numbers. It is enough that he knows the principle of their increase; he can easily be made acquainted by slow degrees with the mere names, without requiring any further explanation. I have not, however, been so deficient in my enumeration lesson as you imagine, having prepared a table which will complete his education in this part of arithmetic, but which should not and must not be shown to him just at present. The value of figures, according to their places, and the names of these, will be readily seen from the following table.

9	3	4	5	7	8	9	1	0	5	6	2	8
Billions	Hundred thousands of millions	Ten thousands of millions	Thousands of millions	Hundreds of millions	Tens of millions	Millions	Hundreds of thousands	Tens of thousands	Thousands	Hundreds	Tens	Units

It is usual, when a number is composed of many places of figures, to break it into periods and half periods of six and of three figures. The first period em-

braces from units to hundreds of thousands; the second is the million period; the third the billion; the fourth the trillion; the fifth the quadrillion, &c.:—the above number would accordingly be written thus, 9,345,789,105,628.

Alfred. Will not your lesson on Notation and Numeration be incomplete, unless you take some notice of the Roman numerals?

Mrs. D. Very true. Be it your care, then, to teach these to Edmund as soon as he is a little more familiar with our own numbers.

Emily. Surely, mother, the Romans used our figures as well as their own, for they could not possibly multiply and divide with their strange numerals.

Mrs. D. They certainly could not accomplish these operations with as much simplicity as we perform them with our present Notation. But it is certain that both the Grecians and Romans were unacquainted with the characters now in so general use. The former made the letters of their alphabet to serve them also as signs for numbers; and the figures of the latter people are still generally known and used under the name of Roman numerals. In consequence of this imperfect mode of Notation, the operations in arithmetic were laborious and tedious, and could not well be performed to any extent without the aid of pebbles or counters: hence our word calculate, from the Latin word calculus, a small pebble. To the Roman accountant, as well as to the pupil in arithmetic, the *abacus* and *calculi*, or cipher-ing-board and counters, were as necessary aids as are the slates and pencils of the present day to our schoolboys.

Emily. When and how, then, did we obtain our numeral characters? It seems to me scarcely possible that any intricate calculations can be made without such a system.

Mrs. D. Antiquarians are disagreed as to the exact