THE

MECHANICAL PRINCIPLES

OF

CIVIL ENGINEERING.

PART I.

STATICS.

1. FORCE is that which *tends* to cause or to destroy motion, or which actually causes or destroys it.

The *direction* of a force is that straight line in which it tends to cause motion in the point to which it is applied, or in which it tends to destroy the motion in it.

When more forces than one are applied to a body, and their respective tendencies to communicate motion to it counteract one another, so that the body remains at rest, these forces are said to be in EQUILIBRIUM, and are called PRESSURES.

It is found by experiment, that the effect of a pressure when applied to a solid body, is the same at whatever point in the line of its direction it is applied; so that the conditions of the equilibrium of that pressure, in respect to other pressures applied to the same body, are not altered, if, without altering the direction of the pressure, we remove its point of application, provided only the point to which we remove it be in the straight line in the direction of which it acts.

The science of STATICS is that which treats of the *equilibrium of pressures*. When two pressures *only* are applied to a body, and hold it at rest, it is found by experiment that these pressures act in opposite directions, and have their directions always in the same straight line. Two such pressures are said to be *equal.*

If instead of applying two pressures which are thus equal in *opposite* directions, we apply them both in the *same* direction, the single pressure which must be applied in a direction opposite to the *two* to sustain them, is said to be *double* of either of them. If we take a third pressure equal to either of the two first, and apply the three in the same direction, the single pressure, which must be applied in a direction opposite to the three to sustain them, is said to be *triple* of either of them; and so of any number of pressures. Thus fixing upon any one pressure, and ascertaining how many pressures equal to this are necessary, when applied in an opposite direction, to sustain any other greater pressure, we arrive at a true conception of the amount of that greater pressure in terms of the first.

That single pressure, in terms of which the amount of any other greater pressure is thus ascertained, is called an UNIT of pressure.

Pressures, the amount of which are determined in terms of some known unit of pressure, are said to be *measured.*

Different pressures, the amounts of which can be determined in terms of the *same* unit, are said to be *commensurable.*

The units of pressure which it is found most convenient to use, are the weights of certain portions of matter, or the pressures with which they tend towards the centre of the earth. The units of pressure are different in different countries. With us the unit of pressure from which all the rest are derived is the weight of 22·815 * cubic inches of distilled water. This weight is one pound troy; being divided into 5760 equal parts, the weight of each is a grain troy, and 7000 such grains constitute the pound avoirdupois.

If straight lines be taken in the directions of any number

* This standard was fixed by Act of Parliament in 1824. The temperature of the water is supposed to be 62° Farenheit, the weight to be taken in air, and the barometer to stand at 30 inches.

of pressures, and have their lengths proportional to the numbers of units in those pressures respectively, then these lines having to one another the same proportion in length that the pressures have in magnitude, and being moreover drawn in the directions in which those pressures respectively act, are said to *represent them in magnitude and direction.*

A system of pressures being in equilibrium, let any number of them be imagined to be taken away and replaced by a single pressure, and let this single pressure be such that the equilibrium which before existed may remain, then this single pressure, producing the same effect in respect to the equilibrium that the pressures which it replaces produced, is said to be their RESULTANT.

The pressures which it replaces are said to be the COMPONENTS of this single pressure; and the act of replacing them by such a single pressure, is called the COMPOSITION of pressures.

If, a single pressure being removed from a system in equilibrium, it be replaced by any number of other pressures, such, that whatever effect was produced by that which they replace singly, the same effect (in respect to the conditions of the equilibrium) may be produced by those pressures conjointly, then is that single pressure said to have been RESOLVED into these, and the act of making this substitution of two or more pressures for one, is called the RESOLUTION of pressures.

The Parallelogram of Pressures.

2. *The resultant of any two pressures applied to a point, is represented in direction by the diagonal of a parallelogram, whose adjacent sides represent those pressures in magnitude and direction.**

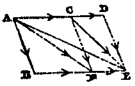
(Duchayla's Method.)

To the demonstration of this proposition, after the excellent method of Duchayla, it is necessary in the first place

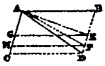
* This proposition constitutes the foundation of the entire science of Statics.

to show, that if there be any two pressures P_2 and P_3 whose directions are in the same straight line, and a third pressure P_1 in any other direction, and if the proposition be true in respect to P_1 and P_2, and also in respect to P_1 and P_3, then it will be true in respect to P_1 and P_2+P_3.

Let P_1, P_2, and P_3, form part of any system of pressures in



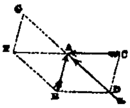
equilibrium, and let them be applied to the point A; take AB and AC to represent, in magnitude and direction, the pressures P_1 and P_2, and CD the pressure P_3, and complete the parallelograms CB and DF. Suppose the proposition to be true with regard to P_1 and P_2, the resultant of P_1 and P_2 will then be in the direction of the diagonal AF of the parallelogram BC, whose adjacent sides AC and AB represent P_1 and P_2 in magnitude and direction. Let P_1 and P_2 be replaced by this resultant. It matters not to the equilibrium where in the line AF it is applied; let it then be applied at F. But thus applied at F it may, without affecting the conditions of the equilibrium, be in its turn replaced by (or resolved into) two other pressures acting in CF and BF, and these will manifestly be equal to P_1 and P_2, of which P_1 may be transferred without altering the conditions to C, and P_2 to E. Let this be done, and let P_3 be transferred from A to C, we shall then have P_1 and P_3 acting in the directions CF and CD at C, and P_2, in the direction FE at E, and the conditions of the equilibrium will not have been affected by the transfer of them to these points. Now suppose that the proposition is also true in respect to P_1 and P_3 as well as P_1 and P_2. Then since CF and CD represent P_1 and P_3 in magnitude and direction, therefore their resultant is in the direction of the diagonal CE. Let them be replaced by this resultant, and let it be transferred to E, and let it then be resolved into two other pressures acting in the directions DE and FE; these will evidently be P_1 and P_3. We have now then transferred all the three pressures P_1, P_2, P_3, from A to E, and they act at E in directions parallel to the directions in which they acted at A, and this has been done without affecting the conditions of the equilibrium; or, in other words, it has been shown that the pressures P_1, P_2, P_3, produce the same effect as it respects the conditions of the equilibrium, whether they be applied at A or E. The *resultant* of P_1, P_2, P_3, must therefore produce the same effect, as it regards the conditions of the equilibrium, whether it be applied at A or E. But in order that this resultant may thus produce the same effect when acting at A or E, it must act in the straight line AE, because a pressure produces the same effect when applied at two different points, only when both those points are in the line of its direction. On the supposition made therefore, the resultant of P_1, P_2, and P_3, or of P_1 and P_2+P_3 acts in the direction of the diagonal AE of the parallelogram BD whose adjacent sides AD and AB represent P_2+P_3 and P_1 in magnitude and direction; and it has been shown, that if the proposition be true in respect to P_1 and P_2, and also in respect to P_1 and P_3, then it is true in respect to P_1 and P_2+P_3. Now this being the case for all values of P_1, P_2, P_3, it is the case when P_1, P_2, and P_3, are equal to one another. But if P_1 be equal to P_2 their resultant will manifestly have its direction as much towards one of these pressures as the other; that is, it will have its direction midway between them, and it will bisect the angle BAC: but the diagonal AF in this case also bisects the angle BAC, since P_1 being equal to P_2, AC is equal to AB; so that in this particular case the direction of the resultant *is* the direction of the diagonal, and the proposition is true, and similarly it is true of P_1 and P_3, since these pressures are equal. Since then it is true of P_1 and P_2 when they are equal, and also of P_1 and P_3, therefore it is true in this case of P_1 and P_2+P_3, that is of P_1 and $2P_1$. And since it is true of P_1 and P_2, and also of P_1 and $2P_1$, therefore it is true of P_1 and P_2+2P_1, that is of P_1 and $3P_1$; and so of P_1 and mP_1, if m be any whole number; and similarly since it is true of mP_1 and P_1, therefore it is true of mP_1 and $2P_1$ &c., and of mP_1 and nP_1 where n is any whole number. Therefore the proposition is true of any two pressures mP_1 and nP_1 which are *commensurable*.



It is moreover true when the pressures are *incommensurable.* For let AC and AB represent any two such pressures P_1 and P_2 in magnitude and direction, and complete the parallelogram ABDC, then will the direction of the resultant of P_1 and P_2 be in AD; for if not, let its direction be AE, and draw EG parallel to CD. Divide AB into equal parts, each less than GC, and set off on AC parts equal to those from A towards C. One of the divisions of these will manifestly fall in GC. Let it be H, and complete the parallelogram AHFB. Then the pressure P_2 being conceived to be divided into as many equal units of pressure as there are equal parts in the line AB, AH may be taken to represent a pressure P_3 containing as many of these units of pressure as there are equal parts in AH, and these pressures P_2 and P_3 will be *commensurable*, being measured in terms of the same unit. Their resultant is therefore in the direction AF, and this resultant of P_3 and P_2 has its direction nearer to AC than the resultant AE of P_1 and P_2 has; which is absurd, since P_1 is greater than P_3.

Therefore AE is not in the direction of the resultant of P_1 and P_2; and in the same manner it may be shown that no other than AD is in that direction. Therefore, &c.

3. *The resultant of two pressures applied in any directions to a point, is represented in magnitude as well as in direction by the diagonal of the parallelogram whose adjacent sides represent those pressures in magnitude and in direction.*

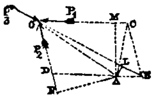


Let BA and CA represent, in magnitude and direction, any two pressures applied to the point A. Complete the parallelogram BC. Then by the last proposition AD will represent the resultant of these pressures in direction. It will also represent it in magnitude; for, produce DA to G, and conceive a pressure to be applied in GA equal to the resultant of BA and CA, and opposite to it, and let this pressure be represented in magnitude by the line GA. Then will the pressures represented by the lines BA, CA, and GA, manifestly be pressures in equilibrium. Complete the parallelogram BG, then is the resultant of GA and BA in the direction FA; also since GA and BA are in equilibrium with CA, therefore this resultant is in equilibrium with CA, but when *two* pressures are in equilibrium, their directions are in the same straight line; therefore FAC is a straight line. But AC is parallel to BD, therefore FA is parallel to BD, and FB is, by construction, parallel to GD, therefore AFBD is a parallelogram, and AD is equal to FB and therefore to AG. But AG represents the resultant of CA and BA in magnitude, AD therefore represents it in *magnitude*. Therefore, &c.

THE PRINCIPLE OF THE EQUALITY OF MOMENTS.

4. DEFINITION. If any number of pressures act in the same plane, and any point be taken in that plane, and perpendiculars be drawn from it upon the directions of all these pressures, produced if necessary, and if the number of units in each pressure be then multiplied by the number of units in the corresponding perpendicular, then this product is called the *moment* of that pressure *about* the point from which the perpendiculars are drawn, and these moments are said to be measured from that point.

5. *If three pressures be in equilibrium, and their moments be taken about any point in the plane in which they act, then the sum of the moments of those two pressures which tend to turn the plane in one direction about the point from which the moments are measured, is equal to the moment of that pressure which tends to turn it in the opposite direction.*



Let P_1, P_2, P_3, acting in the directions P_1O, P_2O, P_3O, be any three pressures in equilibrium. Take any point A in the plane in which they act, and measure their moments from A, then will the sum of the moments of P_2 and P_3, which tend to turn the plane in one direction about A, equal the moment of P_1, which tends to turn it in the opposite direction.

Through A draw DAB parallel to OP_1, and produce OP_2 to meet it in D. Take OD to represent P_2, and take DB of such a length that OD may have the same proportion to DB that P_2 has to P_1. Complete the parallelogram ODBC, then will OD and OC represent P_2 and P_1 in magnitude and direction. Therefore OB will represent P_3 in magnitude and direction.

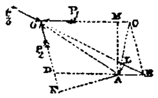
Draw AM, AN, AL, perpendiculars on OC, OD, OB, and join AO, AC. Now the triangle OBC is equal to the triangle OAC, since these triangles are upon the same base and between the same parallels.

Also, $\Delta\ ODA + \Delta\ OAB = \Delta\ ODB = \Delta\ OBC$,

$\therefore\ \Delta\ ODA + \Delta\ OAB = \Delta\ OAC$,

$\therefore\ \frac{1}{2}\,\overline{OD}\times\overline{AN} + \frac{1}{2}\,\overline{OB}\times\overline{AL} = \frac{1}{2}\,\overline{OC}\times\overline{AM}$,

$\therefore\ P_2\times\overline{AN} + P_3\times\overline{AL} = P_1\times\overline{AM}$.



Now $P_1\times\overline{AM}$, $P_2\times\overline{AN}$, $P_3\times\overline{AL}$, are the moments of P_1, P_2, P_3, about A (Art. 4.)

$$\therefore\ m^t\ P_2 + m^t\ P_3 = m^t\ P_1 \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ (1).$$

Therefore, &c. &c.

6. If R be the resultant of P_2 and P_3, then since R is equal to P_1 and acts in the same straight line, $m^t\ R = m^t P_1$,
$\therefore\ m^t\ P_2 + m^t\ P_3 = m^t\ R$. (8)

The sum of the moments therefore, about any point, of two pressures, P_2 and P_3, in the same plane, which tend to turn it in the same direction about that point, is equal to the moment of their resultant about that point.

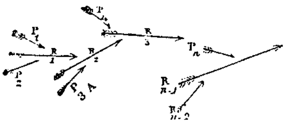
If they had tended to turn it in opposite directions, then the *difference* of their moments would have equalled the moment of their resultant. For let R be the resultant of P_1 and P_3, which tend to turn the plane in opposite directions about A, &c. Then is R equal to P_2, and in the same straight line with it, therefore moment R is equal to moment P_2.

But by equation (1) $m^t\ P_1 - m^t\ P_3 = m^t\ P_2$; $\therefore\ m^t\ P_1 - m^t\ P_3 = m^t\ R$.

Generally therefore, $m^t\ P_1 + m^t\ P_2 = m^t\ R$ . . . . . (2), *the moment therefore of the resultant of any two pressures in the same plane is equal to the sum or difference of the moments of its components, according as they act to turn the plane in the same direction about the point from which the moments are measured, or in opposite directions.*

7. *If any number of pressures in the same plane be in equilibrium, and any point be taken, in that plane, from which their moments are measured, then the sum of the moments of those pressures which tend to turn the plane in one direction about that point is equal to the sum of the moments of those which tend to turn it in the opposite direction.*

Let P_1, P_2, P_3 . . . . . P_n be any number of pressures in the same plane which are in equilibrium, and A any point in the plane from which their moments are measured, then will the sum of the moments of those pressures which tend to turn the plane in one direction about A equal the sum of the moments of those which tend to turn it in the opposite direction.



Let R_1 be the resultant of P_1 and P_2,

R_2 . . . . R_1 and P_3,

R_3 . . . . R_2 and P_4,

&c. . . . . &c.

R_{n-1} . . . . . . R_{n-2} and P_n.

Therefore by the last proposition, it being understood that the moments of those of the pressures P_1, P_2, which tend to turn the plane to the left of A, are to be taken negatively, we have

$$\begin{aligned} m^t\ R_1 &= m^t\ P_1 + m^t\ P_2, \\ m^t\ R_2 &= m^t\ R_1 + m^t\ P_3, \\ m^t\ R_3 &= m^t\ R_2 + m^t\ P_4, \\ \&c. &= \&c. \quad \&c. \\ m^t\ R_{n-1} &= m^t R_{n-2} + m^t\ P_n. \end{aligned}$$

Adding these equations together, and striking out the terms common to both sides, we have

$$m^t R_{n-1} = m^t P_1 + m^t P_2 + m^t P_3 + \ldots\ldots + m^t P_n \ldots (3),$$

where R_{n-1} is the resultant of all the pressures P_1, P_2, ... P_n.

But these pressures are in equilibrium; they have, therefore, no resultant.

$$\therefore R_{n-1} = 0 \therefore m^t R_{n-1} = 0,$$

$$\therefore m^t P_1 + m^t P_2 + m^t P_3 + \ldots\ldots m^t P_n = 0 \ldots (4).$$

Now in this equation the moments of those pressures which tend to turn the system to the left hand are to be taken negatively. Moreover, the sum of the negative terms must equal the sum of the positive terms, otherwise the whole sum could not equal zero. It follows, therefore, that the sum of the moments of those pressures which tend to turn the system to the right must equal the sum of the moments of those which tend to turn it to the left. Therefore, &c. &c.

8. *If any number of pressures acting in the same plane be in equilibrium, and they be imagined to be moved parallel to their existing directions, and all applied to the same point, so as all to act upon that point in directions parallel to those in which they before acted upon different points, then will they be in equilibrium about that point.*

For (see the preceding figure) the pressure R_1 at whatever point in its direction it be conceived to be applied, may be resolved at that point into two pressures parallel and equal to P_1 and P_2: similarly, R_2 may be resolved, at any point in its direction, into two pressures parallel and equal to R_1 and P_3, of which R_1 may be resolved into two, parallel and equal to P_1 and P_2, so that R_2 may be resolved at any point of its direction into three pressures parallel and equal to P_1, P_2, P_3: and in like manner, R_3 may be resolved into two pressures parallel and equal to R_2 and P_4, and therefore into four pressures parallel and equal to P_1, P_2, P_3, P_4, and