ELEMENTS OF LOGIC.

CHAPTER I.

Firſt Notions.

THE firſt notion which a reader can form of Logic is by viewing it as the examination of that part of reaſoning which depends upon the manner in which inferences are formed, and the inveſtigation of general maxims and rules for conſtructing arguments, ſo that the concluſion may contain no inaccuracy which was not previouſly aſſerted in the premiſes. It has ſo far nothing to do with the truth of the facts, opinions, or preſumptions, from which an inference is derived; but ſimply takes care that the inference ſhall certainly be true, if the premiſes be true. Thus, when we ſay that all men will die, and that all men are rational beings, and thence infer that ſome rational beings will die, the *logical* truth of this ſentence is the ſame whether it be true or falſe that men are mortal and rational. This logical truth depends upon the *ſtructure of the ſentence*, and not upon the particular matters ſpoken of. Thus,

Inſtead of	Write,
All men will die.	Every Y is X.
All men are rational beings.	Every Y is Z.
Therefore ſome rational beings will die.	Therefore ſome Zs are Xs.

The ſecond of theſe is the ſame propoſition, logically conſidered, as the firſt; the conſequence in both is virtually contained in, and rightly inferred from, the premiſes. Whether the premiſes be true or falſe, is not a queſtion of logic, but of morals, philoſophy, hiſtory, or any other knowledge to which their ſubjectmatter belongs: the queſtion of logic is, does the concluſion certainly follow if the premiſes be true?

Every act of reaſoning muſt mainly conſiſt in comparing together different things, and either finding out, or recalling from previous knowledge, the points in which they reſemble or differ from each other. That particular part of reaſoning which is called *inference*, conſiſts in the compariſon of ſeveral and different things with one and the ſame other thing; and aſcertaining the reſemblances, or differences, of the ſeveral things, by means of the points in which they reſemble, or differ from, the thing with which all are compared.

There muſt then be ſome propoſitions already obtained before any inference can be drawn. All propoſitions are either aſſertions or denials, and are thus divided into *affirmative* and *negative*. Thus, X is Y, and X is not Y, are the two forms to which all propoſitions may be reduced. Theſe are, for our preſent purpoſe, the moſt ſimple forms; though it will frequently happen that much circumlocution is needed to reduce propoſitions to them. Thus, ſuppoſe the following aſſertion, 'If he ſhould come to-morrow, he will probably ſtay till Monday;' how is this to be reduced to the form X is Y? There is evidently ſomething ſpoken of, ſomething ſaid of it, and an affirmative connection between them. Something, if it happen, that is, the happening of ſomething, makes the happening of another ſomething probable; or *is* one of the things which render the happening of the ſecond thing probable.

X	is	Y
The happening of his arrival to-morrow	is	an event from which it may be inferred as probable that he will ſtay till Monday.

The forms of language will allow the manner of aſſerting to be varied in a great number of ways; but the reduction to the preceding form is always poſſible. Thus, 'ſo he ſaid' is an affirmation, reducible as follows:

What you have juſt ſaid (or whatever elſe 'ſo' refers to)	is	the thing which he ſaid.

By changing 'is' into 'is not,' we make a negative propoſition; but care muſt always be taken to aſcertain whether a propoſition which appears negative be really ſo. The principal danger is that of confounding a propoſition which is negative with another which is affirmative of ſomething requiring a negative to deſcribe it. Thus, 'he reſembles the man who was not in the room,' is affirmative, and muſt not be confounded with 'he does not reſemble the man who was in the room.' Again, 'if he ſhould come to-morrow, it is probable he will not ſtay till Monday,' does not mean the ſimple denial of the preceding propoſition, but the affirmation of a directly oppoſite propoſition. It is,

X	is	Y
The happening of his arrival to-morrow,	is	an event from which it may be inferred to be *im*probable that he will ſtay till Monday:

whereas the following,

The happening of his arrival to-morrow,	is *not*	an event from which it may be inferred as probable that he will ſtay till Monday,

would be expreſſed thus: 'If he ſhould come to-morrow, that is no reaſon why he ſhould ſtay till Monday.'

Moreover, the negative words not, no, &c., have two kinds of meaning which muſt be carefully diſtinguiſhed. Sometimes they deny, and nothing more: ſometimes they are uſed to affirm the direct contrary. In caſes which offer but two alternatives, one of which is neceſſary, theſe amount to the ſame thing, ſince the denial of one, and the affirmation of the other, are obviouſly equivalent propoſitions. In many idioms of converſation, the negative implies affirmation of the contrary in caſes which offer not only alternatives, but degrees of alternatives. Thus, to the queſtion, 'Is he tall?' the ſimple anſwer, 'No,' moſt frequently means that he is the contrary of tall, or conſiderably under the average. But it muſt be remembered, that, in all logical reaſoning, the negation is ſimply negation, and nothing more, never implying affirmation of the contrary.

The common propoſition that two negatives make an affirmative, is true only upon the ſuppoſition that there are but two poſſible things, one of which is denied. Grant that a man muſt be either able or unable to do a particular thing, and then *not unable* and able are the ſame things. But if we ſuppoſe various degrees of performance, and therefore degrees of ability, it is falſe, in the common ſenſe of the words, that two negatives make an affirmative. Thus, it would be erroneous to ſay, 'John is able to tranſlate Virgil, and Thomas is not unable; therefore, what John can do Thomas can do,' for it is evident that the premiſes mean that John is ſo near to the beſt ſort of tranſlation that an affirmation of his ability may be made, while Thomas is conſiderably lower than John, but not ſo near to abſolute deficiency that his ability may be altogether denied. It will generally be found that two negatives imply an affirmative of a weaker degree than the poſitive affirmation.

Each of the propoſitions, 'X is Y,' and 'X is not Y,' may be ſubdivided into two ſpecies: the *univerſal*, in which every poſſible caſe is included; and the *particular*, in which it is not meant to be aſſerted that the affirmation or negation is univerſal. The four ſpecies of propoſition are then as follows, each being marked with the letter by which writers on logic have always diſtinguiſhed it.

A	*Univerſal Affirmative*	Every X is Y
E	*Univerſal Negative*	No X is Y
I	*Particular Affirmative*	Some Xs are Ys
O	*Particular Negative*	Some Xs are not Ys

In common converſation the affirmation of a part is meant to imply the denial of the remainder. Thus, by 'ſome of the apples are ripe,' it is always intended to ſignify that ſome are not ripe. This is not the caſe in logical language, but every propoſition is intended to make its amount of affirmation or denial, and no more. When we ſay, 'Some X is Y,' or, more grammatically, 'Some Xs are Ys,' we do not mean to imply that ſome are not: this may or may not be. Again, the word ſome means, 'one or more, poſſibly all.' The following table will ſhew the bearing of each propoſition on the reſt.

Every X is Y affirms *Some Xs are Ys* and denies { *No X is Y* / *Some Xs are not Ys* }

No X is Y affirms *Some Xs are not Ys* and denies $\left\{\begin{array}{l}\textit{Every X is Y}\\ \textit{Some Xs are Ys}\end{array}\right.$

Some Xs are Ys does not contradict $\left\{\begin{array}{l}\textit{Every X is Y}\\ \textit{Some Xs are not Ys}\end{array}\right\}$ but denies *No X is Y*

Some Xs are not Ys does not contradict $\left\{\begin{array}{l}\textit{No X is Y}\\ \textit{Some Xs are Ys}\end{array}\right\}$ but denies *Every X is Y*

Contradictory propositions are those in which one denies *any thing* that the other affirms; *contrary* propositions are those in which one denies *every thing* which the other affirms, or affirms every thing which the other denies. The following pair are contraries,

Every X is Y and No X is Y

and the following are contradictories,

Every X is Y to Some Xs are not Ys
No X is Y to Some Xs are Ys

A contrary, therefore, is a complete and total contradictory; and a little consideration will make it appear that the decisive distinction between contraries and contradictories lies in this, that contraries may both be false, but of contradictories, one must be true and the other false. We may say, 'Either P is true, or *something* in contradiction of it is true;' but we cannot say, 'Either P is true, or *every thing* in contradiction of it is true.' It is a very common mistake to imagine that the *denial* of a proposition gives a right to *affirm* the contrary; whereas it should be, that the *affirmation* of a proposition gives a right to *deny* the contrary. Thus, if we deny that Every X is Y, we do not affirm that No X is Y, but only that Some Xs are not Ys; while, if we affirm that Every X is Y, we deny No X is Y, and also Some Xs are not Ys.

But, as to contradictories, affirmation of one is denial of the other, and denial of one is affirmation of the other. Thus, either Every X is Y, or Some Xs are not Ys: affirmation of either is denial of the other, and *vice versâ*.

Let the student now endeavour to satisfy himself of the following. Taking the four preceding propositions, A, E, I, O, let the simple letter signify the affirmation, the same letter in parentheses the denial, and the absence of the letter, that there is neither affirmation nor denial.

From A follow	(E), I, (O)	From (A) follow	O
From E	(A), (I), O	From (E)	I
From I	(E)	From (I)	(A), E, O
From O	(A)	From (O) . . .	A, (E), I

Theſe may be thus ſummed up: The affirmation of a univerſal propoſition, and the denial of a particular one, enable us to affirm or deny all the other three; but the denial of a univerſal propoſition, and the affirmation of a particular one, leave us unable to affirm or deny two of the others.

In ſuch propoſitions as 'Every X is Y,' 'Some Xs are not Ys,' &c., X is called the *ſubject*, and Y the *predicate*, while the verb 'is' or 'is not,' is called the *copula*. It is obvious that the words of the propoſition point out whether the ſubject is ſpoken of univerſally or partially, but not ſo of the predicate, which it is therefore important to examine. Logical writers generally give the name of *diſtributed* ſubjects or predicates to thoſe which are ſpoken of univerſally; but as this word is rather technical, I ſhall ſay that a ſubject or predicate enters wholly or partially, according as it is univerſally or particularly ſpoken of.

1. In A, or 'Every X is Y,' the ſubject enters wholly, but the predicate only partially. For it obviouſly ſays, 'Among the Ys are all the Xs,' 'Every X is part of the collection of Ys, ſo that all the Xs make a part of the Ys, the whole it *may* be.' Thus, 'Every horſe is an animal,' does not ſpeak of all animals, but ſtates that all the horſes make up a portion of the animals.

2. In E, or 'No X is Y,' both ſubject and predicate enter wholly. 'No X whatſoever is any one out of all the Ys;' 'ſearch the whole collection of Ys, and *every* Y ſhall be found to be ſomething which is not X.'

3. In I, or 'Some Xs are Ys,' both ſubject and predicate enter partially. 'Some of the Xs are found among the Ys, or make up a part (the whole poſſibly, but not known from the preceding) of the Ys.'

4. In O, or 'Some Xs are not Ys,' the ſubject enters partially, and the predicate wholly. 'Some Xs are none of them any whatſoever of the Ys; every Y will be found to be no one out of a certain portion of the Xs.'

It appears then that,

In affirmatives, the predicate enters partially.

Cambridge University Press
978-1-108-07078-2 - Formal Logic: Or, The Calculus of Inference, Necessary and Probable
Augustus De Morgan
Excerpt
More information

In negatives, the predicate enters wholly.

In contradictory propositions, both subject and predicate enter differently in the two.

The *converse* of a proposition is that which is made by interchanging the subject and predicate, as follows:

	The proposition.	Its converse.
A	Every X is Y	Every Y is X
E	No X is Y	No Y is X
I	Some Xs are Ys	Some Ys are Xs
O	Some Xs are not Ys	Some Ys are not Xs

Now, it is a fundamental and self-evident proposition, that no consequence must be allowed to assert more widely than its premises; so that, for instance, an assertion which is only of some Ys can never lead to a result which is true of all Ys. But if a proposition assert agreement or disagreement, any other proposition which asserts the same, to the same extent and no further, must be a legitimate consequence; or, if you please, must amount to the whole, or part, of the original assertion in another form. Thus, the converse of A is not true: for, in 'Every X is Y,' the predicate enters partially; while in 'Every Y is X,' the subject enters wholly. 'All the Xs make up a part of the Ys, then a part of the Ys are among the Xs, or some Ys are Xs.' Hence, the only *legitimate* converse of 'Every X is Y' is, 'Some Ys are Xs.' But in 'No X is Y,' both subject and predicate enter wholly, and 'No Y is X' is, in fact, the same proposition as 'No X is Y.' And 'Some Xs are Ys' is also the same as its converse 'Some Ys are Xs:' here both terms enter partially. But 'Some Xs are not Ys' admits of no converse whatever; it is perfectly consistent with all assertions upon Y and X in which Y is the subject. Thus neither of the four following lines is inconsistent with itself.

Some Xs are not Ys	and	Every Y is X
Some Xs are not Ys	and	No Y is X
Some Xs are not Ys	and	Some Ys are Xs
Some Xs are not Ys	and	Some Ys are not Xs.

Having thus discussed the principal points connected with the simple assertion, I pass to the manner of making two assertions give a third. Every inſtance of this is called a *ſyllogiſm*, the two aſſertions which form the baſis of the third are called *premiſes*, and the third itſelf the *concluſion.*

If two things both agree with a third in any particular, they agree with each other in the ſame; as, if X be of the ſame colour as Y, and Z of the ſame colour as Y, then X is of the ſame colour as Z. Again, if X differ from Y in any particular in which Z agrees with Y, then X and Z differ in that particular. If X be not of the ſame colour as Y, and Z be of the ſame colour as Y, then X is not of the colour of Z. But if X and Z both differ from Y in any particular, nothing can be inferred; they may either differ in the ſame way and to the ſame extent, or not. Thus, if X and Z be both of different colours from Y, it neither follows that they agree, nor differ, in their own colours.

The paragraph preceding contains the eſſential parts of all inference, which conſiſts in comparing two things with a third, and finding from their agreement or difference with that third, their agreement or difference with one another. Thus, Every X is Y, every Z is Y, allows us to infer that X and Z have all thoſe qualities in common which are neceſſary to Y. Again, from every X is Y, and 'No Z is Y,' we infer that X and Z differ from one another in all particulars which are eſſential to Y. The preceding forms, however, though they repreſent common reaſoning better than the ordinary ſyllogiſm, to which we are now coming, do not conſtitute the ultimate forms of inference. Simple *identity* or *non-identity* is the ultimate ſtate to which every aſſertion may be reduced; and we ſhall, therefore, firſt aſk, from what identities, &c., can other identities, &c., be produced? Again, ſince we name objects in ſpecies, each ſpecies conſiſting of a number of individuals, and ſince our aſſertion may include all or only part of a ſpecies, it is further neceſſary to aſk, in every inſtance, to what extent the concluſion drawn is true, whether of all, or only of part?

Let us take the ſimple aſſertion, 'Every living man reſpires;' or every living man is one of the things (however varied they may be) which reſpire. If we were to encloſe all living men in a large triangle, and all reſpiring objects in a large circle, the preceding aſſertion, if true, would require that the whole of the triangle ſhould be contained in the circle. And in the ſame way we may reduce any aſſertion to the expreſſion of a coincidence, total or partial, between two figures. Thus, a point in a circle may repreſent an individual of one ſpecies, and a point in a triangle an individual of another ſpecies: and we may expreſs that the whole of one ſpecies is aſſerted to be contained or not contained in the other by ſuch forms as, 'All the △ is in the ○'; 'None of the △ is in the ○'.

Any two aſſertions about X and Z, each expreſſing agreement or diſagreement, total or partial, with or from Y, and leading to a concluſion with reſpect to X or Z, is called a ſyllogiſm, of which Y is called the *middle term*. The plaineſt ſyllogiſm is the following:—

Every X is Y	All the △ is in the ○
Every Y is Z	All the ○ is in the □
Therefore Every X is Z	Therefore All the △ is in the □

In order to find all the poſſible forms of ſyllogiſm, we muſt make a table of all the elements of which they can conſiſt; namely—

X and Y		Z and Y
Every X is Y	A	Every Z is Y
No X is Y	E	No Z is Y
Some Xs are Ys	I	Some Zs are Ys
Some Xs are not Ys	O	Some Zs are not Ys
Every Y is X	A	Every Y is Z
Some Ys are not Xs	O	Some Ys are not Zs

Or their ſynonymes,

△ and ○		□ and ○
All the △ is in the ○	A	All the □ is in the ○
None of the △ is in the ○	E	None of the □ is in the ○
Some of the △ is in the ○	I	Some of the □ is in the ○
Some of the △ is not in the ○	O	Some of the □ is not in the ○
All the ○ is in the △	A	All the ○ is in the □
Some of the ○ is not in the △	O	Some of the ○ is not in the □

Now, taking any one of the ſix relations between X and Y, and combining it with either of thoſe between Z and Y, we have ſix pairs of premiſes, and the ſame number repeated for every different relation of X to Y. We have then thirty-ſix

Cambridge University Press
978-1-108-07078-2 - Formal Logic: Or, The Calculus of Inference, Necessary and Probable
Augustus De Morgan
Excerpt
More information

forms to conſider: but, thirty of theſe (namely, all but (A, A) (E, E), &c.,) are half of them repetitions of the other half. Thus, 'Every X is Y, no Z is Y,' and 'Every Z is Y, no X is Y,' are of the ſame form, and only differ by changing X into Z and Z into X. There are then only 15+6, or 21 diſtinct forms, ſome of which give a neceſſary concluſion, while others do not. We ſhall ſelect the former of theſe, claſſifying them by their concluſions; that is, according as the inference is of the form A, E, I, or O.

I. In what manner can a univerſal affirmative concluſion be drawn; namely, that one figure is entirely contained in the other? This we can only aſſert when we know that one figure is entirely contained in the circle, which itſelf is entirely contained in the other figure. Thus,

Every X is Y	All the △ is in the ○	A
Every Y is Z	All the ○ is in the □	A
Every X is Z	All the △ is in the □	A

is the only way in which a univerſal affirmative concluſion can be drawn.

II. In what manner can a univerſal negative concluſion be drawn; namely, that one figure is entirely exterior to the other? Only when we are able to aſſert that one figure is entirely within, and the other entirely without, the circle. Thus,

Every X is Y	All the △ is in the ○	A
No Z is Y	None of the □ is in the ○	E
No X is Z	None of the △ is in the □	E

is the only way in which a univerſal negative concluſion can be drawn.

III. In what manner can a particular affirmative concluſion be drawn; namely, that part or all of one figure is contained in the other? Only when we are able to aſſert that the whole circle is part of one of the figures, and that the whole, or part of the circle, is part of the other figure. We have then two forms.

Every Y is X	All the ○ is in the △	A
Every Y is Z	All the ○ is in the □	A
Some Xs are Zs	Some of the △ is in the □	I