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ELECTRIC ILLUMINATION.

I.

ELECTRICAL MEASUREMENT.

GENERAL PRINCIPLES.—The adoption of a system of units (which cannot yet be regarded as perfect) has developed the study of electricity from a mere observation of phenomena into a science of exact measurement. The electrical quantities of Resistance, Electromotive Force, Strength of Current, and Capacity can be accurately determined; and—thanks to the labours of successive Committees of the British Association and to the Congress of Electricians—expressed in practical units.

The electrician can now give the capacity of a submarine cable in Microfarads, or its insulation resistance in Megohms; and he can determine the electrical efficiency of a dynamo, or the mechanical energy expended in an arc, or in an incandescence, lamp with ease and precision. Such measurements involve a knowledge of certain general principles, which it will be useful to briefly consider before describing the more important instruments and practical methods commonly used in this branch of physical science.

The fundamental law of this science was first enunciated by Dr. G. S. Ohm, in a work on the mathematical theory of the electric current, published in 1827.

It establishes the relation connecting the Electromotive Force, the Resistance, and the Current, viz., $C = \frac{E}{R}$.

This law shows:

(*a*) That when a given E.M.F. is impressed at the ends of different conductors, the current through these varies inversely as their respective resistances; and

(*b*) That for a given current passing through the conductors, the difference of the potentials at their ends varies directly as their resistance.

This great electrical law cannot be deduced from purely theoretical considerations. It was experimentally, though roughly, verified by Ohm himself, and subsequently by several eminent investigators, including Pouillet, Becquerel, Fechner, and Kohlrausch. More recently it has occupied the attention of a Committee of the British Association, in consequence of its accuracy being suspected by Weber, Lorenz, and Schuster.

The experimental verification was entrusted to Mr. G. Chrystal, of the Cavendish Laboratory, now Professor of Mathematics in the University of Edinburgh; and most carefully conducted experiments have shown it to be not merely a close approximation, but a strict physical law.*

Kohlrausch and Nippoldt, in a classical series of experiments, have shown it to hold for electrolytes throughout a great range of electromotive force.

Besides direct verifications, we have evidence for the accuracy of the law in the many discoveries to which it has led, as well as in a great variety of measurements which have been made with a degree of precision rarely approached in other physical determinations.

1. *Resistance.*—The idea implied in the term "*electrical resistance*" is that of an actual force opposing the E.M.F. which maintains the current. It is analogous to friction in mechanics, for it tends to diminish the available energy of the current.

Ohm's law furnishes a definition of resistance, and consequently also of unit resistance. It defines the resistance of a conductor to be the ratio of the numerical value of the E.M.F. to that of the current which it produces; and hence a conductor of unit resistance is one in which unit current is produced by unit electromotive force.

The resistance of a conductor may also be defined as the work done in it by the passage of unit current for unit time; for, as will be seen further on,

$$W = C\,T\,E = C\,T.C\,R = C^2\,R\,T.$$

Hence, if C be unit current and T unit time, we have

$$W = R,$$

that is, the work done is numerically equal to the resistance.

The resistance of metallic conductors depends in no way upon the current which is passing, or the E.M.F. which is impressed. It is a physical

* B.A. Report for 1876.

quantity, which is perfectly constant so long as the molecular conditions of the conductor remain unaltered.

The resistance of a conductor is affected by heat, strain, tempering, and even by magnetisation.

Resistance, in the electromagnetic system, has the dimensions of, and is therefore expressible as, a velocity.* The standard resistance, constructed in 1863 by the Committee of the British Association, is called the *ohm*, and was intended to represent the velocity of a body which, in one second of time, would move over a quadrant of a terrestrial meridian; that is to say, a velocity of 10^9 centimetres per second.

Some doubt having arisen as to the accuracy of this standard, redeterminations of its value have been made by Kohlrausch, Lorenz, H. Weber, Rowland, Glazebrook, Carey Foster, and Lord Rayleigh. The numerical results obtained by different methods are not perfectly concordant. That given by Lord Rayleigh from his latest determination (1883) makes the B. A. unit $= .9868 \dfrac{\text{earth quadrant}}{\text{second}}$ or .9868 of the theoretic ohm.

A *megohm* is a million ohms; a *microhm* is the millionth part of an ohm.

Conductivity is plainly the reciprocal of resistance. No special unit has yet been introduced.

Let A B be a conductor of length l and sectional area s, then the flow of electricity (the current) along A B will vary

(*a*) directly as the difference of potentials $V_a - V_b$;

(*b*) directly as s;

(*c*) directly as a constant c depending upon the material of the conductor; and

(*d*) inversely as l, *i.e.*:

$$C = \frac{V_a - V_b}{l} s c$$

$$= \frac{V_a - V_b}{\frac{l}{s c}} = \frac{V_a - V_b}{R}$$

* A coil of wire, in revolving about a vertical axis, cuts the horizontal component H of the earth's magnetic force. If C be the induced current, r the mean radius of the coil, n the number of windings, R the resistance, and ω the angular velocity of the coil, we have $C = \frac{H \cdot r^2 \pi n \cdot \omega}{R}$. Also, from the deflection of the magnet suspended at the centre of the spinning coil, we have $C = \frac{r H}{2 \pi n} \tan \phi$. By equating these values, we get $R = 2 \pi^2 n^2 r \omega \cot \phi$; that is, R is expressed as a velocity.

where

$$R = \frac{l}{s\,c} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

From (1) we conclude that the resistance of a conductor varies directly as its length and inversely as its cross section. If l = unit length (1 cm.) and s = unit area (1 sq. cm.), then R, the resistance of 1 cm. cube, is called the *specific resistance* of the conductor; and therefore, also, $c = \frac{1}{R}$ will be the *specific conductivity.*

The resistance of a cylindrical conductor may be defined with reference to its weight. Thus:

The weight W of a body is numerically the product of its mass M by the acceleration due to gravity g, *i.e.*, $W = M\,g$.

M is the product of the volume V by the density ρ, *i.e.*, $M = V\,\rho$, whence

$$W = V\,\rho\,g = s\,l\,\rho\,g \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The specific gravity of a body is the ratio of its weight to that of an equal volume of some standard substance. For solid bodies the standard of reference is water at the temperature of maximum density, 4° C.

In the centimetre-gramme-second (the c.g.s.) system of units, the gramme is the unit of mass; and as 1 cub. cm. of water at 4° C. weighs 1 gramme, the specific gravity of any body is numerically the weight of one cub. cm. of that body.

Putting $l\,s = 1$, equation (2) becomes $W = \rho\,g$, *i.e.*, $\rho\,g$ is the specific gravity of the body. Denoting it by σ, we have

$$W = l\,s\,\sigma \;\therefore\; s = \frac{W}{l\,\sigma}$$

And since R varies as $\frac{l}{s}$, R will also vary as $\frac{l^2\,\sigma}{W}$.

Ex. Let it be required to compare the resistances R_1 and R_2 of two copper wires, one of 2 metres long and weighing $\frac{1}{4}$ gm., and the other 5 metres long and weighing $\frac{2}{3}$ of a gm. Assuming that the wires have the same specific gravity σ,

$$\begin{aligned} R_1 : R_2 &= \frac{(2)^2\,\sigma}{\frac{1}{4}} : \frac{(5)^2\,\sigma}{\frac{2}{3}} \\ &= 16\,\sigma : 37.5\,\sigma \\ &= 32 : 75 \end{aligned}$$

Ex. It is required to find the specific resistance of a wire 437 mm. long, which has a resistance of .1257 ohm, and which weighs .411 gm. in air and .365 gm. in water.

Here .411 − .365 = .046 = weight of an equal volume of water ∴ specific gravity $= \frac{.411}{.046} = \frac{411}{46}$

But $W = V\sigma$ ∴ $.411 = V \times \frac{411}{46}$ ∴ $V = .046$ cub. cm.; and $V = r^2 \pi l$ $= r^2 \pi \times 43.7$ ∴ $r^2 \pi = \frac{.046}{43.7} = .001053$

The resistance of 43.7 cm. of the wire of area .001053 = .1257 ohm ∴ the resistance of conductor 1 cm. long and 1 cm. sectional area,

$$= \frac{.1257}{43.7} \times .001053 \text{ ohm} = .000003029 \text{ ohm}$$
$$= 3.029 \text{ microhms}$$
$$= \text{specific resistance required.}$$

The *joint resistance* of any number of conductors arranged *in series* is plainly the sum of their separate resistances, that is, $R = r_1 + r_2 + r_3 \ldots$

When arranged in *multiple arc*, the joint resistance is found as follows:

Let V_a and V_b be the potentials at A and B respectively.

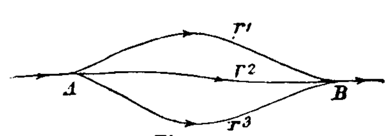

Fig. 1.

Then $V_a - V_b = E =$ the E.M.F. producing the current C.

But $C = \frac{E}{R}$ ∴ $C R = E$.

Let r_1, Fig. 1, be the resistance of one conductor and c_1 the current through it, with similar notation for the other branches. Then

$$V_a - V_b = c_1 r_1 = c_2 r_2 = c_3 r_3$$
$$= \frac{c_1}{\frac{1}{r_1}} = \frac{c_2}{\frac{1}{r_2}} = \frac{c_3}{\frac{1}{r_3}}$$
$$= \frac{c_1 + c_2 + c_3}{\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}}$$

But $c_1 + c_2 + c_3 = C$.

Whence $\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} = \frac{1}{R}$. This gives the resistance R of the multiple arc.

Ex. Find the joint resistance of three conductors of 10, 12, 18 ohms arranged in multiple arc. Here

$$\frac{1}{R} = \frac{1}{10} + \frac{1}{12} + \frac{1}{18} = \frac{43}{180}$$
$$\therefore R = \frac{180}{43} = 4.18 \text{ ohms.}$$

The resistance of metallic conductors is invariably increased by heat. Dr. Matthiessen* and Sir William Siemens† have given formulæ for finding the resistance of a conductor throughout an extensive range of temperature.

It is important to notice that the resistance of carbon falls with a rise of temperature. Thus, the resistance of a filament when cold was 71 ohms, and when heated to incandescence it was found to be only 47 ohms. The decrease when white hot is generally about $\frac{1}{3}$ the resistance when cold.

The electrical properties of selenium and tellurium, in the crystalline state, are modified by exposure to light, their resistance being considerably and instantaneously diminished. Professor W. G. Adams has shown that, under certain circumstances, light can even set up an E.M.F. and start a current in a bar of selenium.‡

The *internal resistance* of a voltaic cell practically depends upon the nature of the liquids, the size of the plates, and their distance apart. There appears to be some reason for thinking that the resistance is also a function of the current itself.§

The internal resistance is diminished

(*a*) by increasing the active area of the plates, and
(*b*) by bringing them closer together.

Heat affects the resistance of liquids, a rise of temperature diminishing the resistance of electrolytes generally.

Mr. W. H. Preece has recently shown that of the batteries in general use, the Daniell is most affected by variations of temperature. It becomes then necessary, where accurate measurement is required, to keep the temperature of the battery constant, or else to make frequent determinations of resistance and allow for the change.

2. *Electromotive Force.*—This is the name given to that force which tends to displace electricity from one point of a conductor to another.

Electromotive, must be distinguished from ordinary mechanical, force: the latter tends to set a certain *mass* in motion; the former to cause the transfer of *electricity* constituting the phenomenon of the electric current.

The nature of this force‖ and its causes are entirely unknown, whilst even its seat is yet a debatable subject among physicists.

* B. A. Report, 1864. † Bakerian Lecture, 1871. ‡ Phil. Trans. 1877.
§ Prof. G. Chrystal, in Encycl. Brit., art. Electricity, p. 50.
‖ According to Newton's definition of force, the term electromotive force is a misnomer.

Unit Electromotive Force exists between two points when it requires the expenditure of unit work to bring a unit of positive electricity from one of the points to the other.

This unit is exceedingly small, and accordingly a "practical" unit called the *volt* has been adopted, which is intended to represent 10^8 such absolute units. The E.M.F. of a Daniell's cell is a little more than a volt.

There is yet no absolutely constant standard of E.M.F.; a Daniell, a Clark, or a De la Rue cell is generally used when a definite E.M.F. is required.

Latimer Clark's standard cell consists of pure mercury, the surface of which is covered over with a mercurous paste; on this rests a plate of pure zinc. The positive pole is the platinum wire which is in contact with the mercury. The E.M.F. of this cell is given as 1.457 volts.

Dr. De la Rue's cell is shown in Fig. 2, where A is a rod of chemically pure zinc, B a cylinder of silver chloride into which is let a silver electrode C; B is usually enclosed in a small bag of parchment paper. The active liquid is a solution of pure sal-ammoniac in water. The E.M.F. is said to be 1.068 volts.

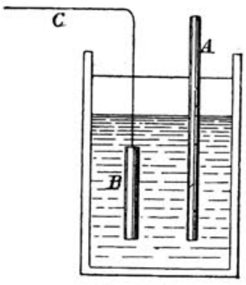

Fig. 2.

At the Southampton Meeting of the British Association, 1882, Sir William Thomson proposed a modified form of Daniell's cell to serve as a standard of E.M.F.

The zinc plate, Fig. 3, rests on the bottom of the vessel, and is immersed in a saturated solution of zinc sulphate. On the surface of this liquid is poured a quantity of copper sulphate, only half saturated, so that it may not rapidly flow down and intermix with the zinc sulphate. The copper plate is suspended horizontally in the upper stratum.

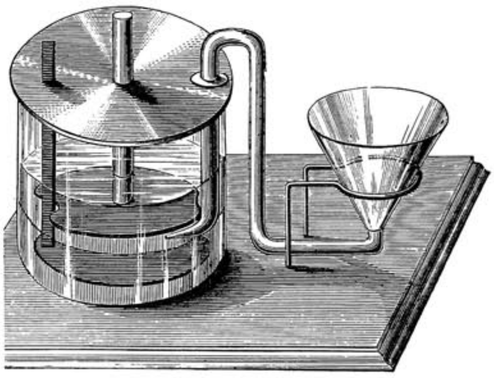

Fig. 3.

The vessel is furnished with a bent glass tube, drawn out at one extremity to a fine bore, and connected at the other with a piece of india-rubber tubing attached to a funnel.

By raising the funnel, the copper sulphate solution which it contains flows gently into the vessel and forms a clear horizontal surface of separation; by lowering it, the liquid is withdrawn.

This must be done as soon as a measurement is finished, and the cell no longer required.

The E.M.F. of this cell has been carefully determined, and found to be, at ordinary temperatures, 1.07 volts.

The E.M.F. of a battery, unlike its resistance, is but very little affected by heat. Latimer Clark found that the E.M.F. of his cell decreased slightly with rise of temperature. Mr. Preece has recently* shown that the E.M.F. of the batteries in common use—the Daniell, Bichromate, Leclanché—is practically unaffected by the ordinary variations of temperature.

The experiments of Beetz and others show, however, that the counter E.M.F. set up in secondary batteries during the process of charging, and consequently also the charge itself, is sensibly diminished by heat; it becomes, then, very important to keep such cells cool while charging.

The E.M.F. of a cell depends upon the nature of the materials used, but not upon the size of the plates. Thus, the E.M.F. of a Bunsen pint cell is the same as that of a quart cell. But when two similar cells are connected up in series, the resulting E.M.F. is twice that of either of them.

Let E be the E.M.F. of one cell, then that of n cells arranged in parallel circuit is simply E; if arranged in series, it is n E.

The E.M.F. of a battery also depends upon the resistance through which it is worked, being greatest when this is infinite, that is, when the poles are free. The E.M.F. of a battery is defined as the difference of potentials of its poles when these are free or insulated. When worked through a low resistance, polarisation sets up an appreciable inverse E.M.F., which diminishes that of the battery. Hence standard cells should never be used over a low resistance. They are especially useful for determining the value of electrometer readings, charging condensers, and for standards of comparison as in the "compensation methods."

3. *Current.*—The strength of the current is given by Ohm's law $C=\frac{E}{R}$, E being measured in volts and R in ohms, C is given in ampères.

Let there be n similar cells, each of E.M.F. E and resistance R, connected up in parallel circuit and maintaining a current through an external resistance r. The E.M.F. of the battery is E, and its internal resistance $\frac{R}{n}$, then $C=\frac{E}{\frac{R}{n}+r}$. If the poles of the battery be connected by a short thick

* Proc. Roy. Soc., Feb. 1883.