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978-1-108-07049-2 - The Principles of Bridges: Containing the Mathematical Demonstrations of the Properties of the Arches, the Thickness of the Piers, the Force of the Water Against them, &c.

Charles Hutton

Excerpt

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T H E

P R I N C I P L E S

O F

S T O N E B R I D G E S .

S E C T I O N I .

*Of the Projects of Bridges, with the Design,
Estimate, &c.*

WHEN a bridge is deemed necessary to be built over a river, the first consideration is the place of it ; or what particular situation will contain a maximum of the advantages over the disadvantages.

In agitating this most important question, every circumstance, certain and probable, attending or likely to attend the bridge, should be separately, minutely, and impartially stated and examined ; and the advantage or disadvantage of it rated at a value proportioned to it : then the difference between the whole advantages and

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disadvantages, will be the neat value of that particular situation for which the calculation is made. And by doing the same for any other situations, all their neat values will be found, and of consequence the most preferable situation among them.—Or, in a competition between two places, if each one's advantage over the other be estimated or valued in every circumstance attending them, the sums of their advantages will shew whether of them is the better. And the same being done for this and a third, and so on, the best situation of all will be obtained.

In this estimation, a great number of particulars must be included; and nothing omitted that can be found to make a part of the consideration.

Among these, the situation of the town or place for the convenience of which the bridge is chiefly to be made, will naturally produce a particular of the first consequence; and a great many others ought to be sacrificed to it. If possible, the bridge should be placed where there can conveniently be opened and made passages or freets from the ends of it in every direction, and especially one as nearly in the direction of the bridge itself as possible, tending towards the body of the town, without narrows or crooked windings, and easily communicating with the chief streets, thoroughfares, &c.—And here
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every person, in judging of this, should divest himself of all partial regards or attachments whatever; think and determine for the good of the whole only, and for posterity as well as the present.

The banks or declivities towards the river are also of particular concern, as they affect the conveniency of the passage to and from the bridge, or determine the height of it, upon which in a great measure depends the expence.

The breadth of the river, the navigation upon it, and the quantity of water to be passed, or the velocity and depth of the stream, form also considerations of great moment; as they determine the bridge to be higher or lower, longer or shorter. However, in most cases, a wide part of the river ought rather to be chosen than a narrow one, especially if it is subject to great tides or floods; for, the increased velocity of the stream in the narrow part, being again augmented by the farther contraction of the breadth, by the piers of the bridge, will both incommode the navigation through the arches, and undermine the piers and endanger the whole bridge.

The nature of the bed of the river is also of great concern, it having a great influence on the expence; as upon it, and the depth and velocity

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of the stream, depend the manner of laying the foundations, and building the piers.

These are the chief and capital articles of consideration, and which will branch themselves out into other dependent ones, and so lead to the required estimate of the whole.

HAVING resolved on the place, the next considerations are the form, the estimate of the expence, and the manner of execution.

With respect to the form ; strength, utility, and beauty ought to be regarded and united ; the chief part of which lies in the arches. The form of the arches will depend on their height and span ; and the height on that of the water, the navigation, and the adjacent banks. They ought to be made so high, as that they may easily transmit the water at its greatest height either from tides or floods ; and their height and figure ought also to be such as will easily allow of a convenient passage of the craft through them. This and the disposition of it above, so as to render the passage over it also convenient, make up its utility.—Having fixed the heights of the arches, their spans are still necessary for determining their figure. Their spans will be known by dividing the whole breadth of the river into a convenient number of arches and piers, allowing at least the necessary thickness of
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the piers out of the whole. In fixing on the number of arches, take always an odd number, and rather take few and large ones than many and smaller, if convenient: For thus you will have not only fewer foundations and piers to make, with fewer arches and centers, which will produce great savings in the expence, but the arches themselves will also require much less materials and workmanship, and allow of more and better passage for the water and craft through them; and will appear at the same time more noble and beautiful, especially if constructed in elliptical, or in cycloidal forms: for the truth of which it may be sufficient to refer to that noble and elegant bridge lately built at Blackfriars, London, by Mr. Mylne. And here I can't help remarking that the Gentleman who, a few years since in a pamphlet on the Principles of Bridges, censured Mr. Mylne and Mr. Muller concerning elliptic arches, has very much exposed himself, and absurdly criticises them through his own want of mathematical knowledge, which he somewhere in the same pamphlet affects to despise. He brings to my mind an expression of (I think) Mr. Henry Fielding somewhere in his works, That a person does not speak the worse on a subject for knowing something about it. I do not however make this remark through any particular disrespect for this Gentleman, concerning whom I know nothing farther, any more than I do about the other two Gentlemen, but only to prevent

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prevent others from being prejudiced and misled by the authority of his *ipse dixit*.—If the top of the bridge be a straight horizontal line, let the arches be made all of a size; if it be a little lower at the ends than the middle, the arches must proportionally decrease from the middle towards the ends; but if higher at the ends than the middle, let them increase towards the ends. A choice of the most convenient arches is to be made from the 4th and 5th propositions, where their several properties, &c. are demonstrated and pointed out: Among them, the elliptic, cycloidal, and equilibrial arch in prop. 5, will generally claim the preference, both on account of their strength, beauty, and cheapness or saving in materials and labour: Other particulars also concerning them may be seen under the word ARCH in the Dictionary in the last section. And as the choice of the arch is of so great moment, let no person, either through ignorance or indolence, prefer a worse arch because it may seem to him easier to construct; for he would very ill deserve the name or employment of an Architect, who is incapable of rendering the exact construction of these curves easy and familiar to himself; but if, by chance, a Bridge-builder should be employed who is incapable of doing that, he ought at least to be endowed with such a share of honesty as to procure some person to go through the calculations which he cannot make for himself.

Next

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Next find what thickness at the keystone or top will be necessary for the arches. For which see the word KEYSTONE in the Dictionary in the last section.

Having thus obtained all the parts of the arches, with the height of the piers, the necessary thickness of the piers themselves are next to be computed by prop. 10.

This done, the chief and material requisites are found; the elevation and plans of the design can then be drawn, and the calculations of the expence from thence made, including the foundations, with such ornamental or accidental appendages as shall be thought fit; which I shall leave to the discretion of the Practical Architect, as being no part of the plan of my undertaking, together with the practical methods of carrying the design into execution. I shall however, in the Dictionary in the last section, not only describe the terms, parts, machines, &c. but also speak of their dimensions, properties, and any thing else material belonging to them; and to which therefore I from hence refer for more explicit information in each particular article, as well as to these immediately following propositions, in which the theory of the arches, piers, &c. are fully and strictly demonstrated.

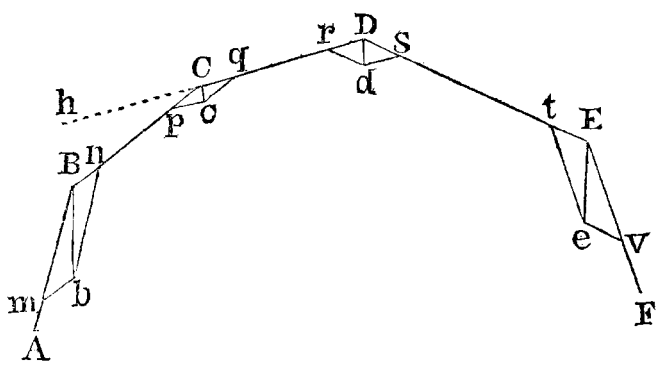
S E C.

S E C T I O N I I .

Of the Arches.

P R O P O S I T I O N I .

***L**ET there be any number of lines AB, BC, CD, DE, &c. all in the same vertical plane, connected together and moveable about the joints or angles A, B, C, D, E, F; the two extreme points A and F being fixed: It is required to find the proportions of the weights to be laid upon the angles B, C, D, &c. so that the whole may remain in equilibrium.*



Solution.

FROM the several angles having drawn the lines Bb, Cc, Dd, &c. perpendicular to the horizon; about them, as diagonals, constitute paral-

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parallelograms such, that those sides of each two that are upon the same one of the given lines, may be equal to each other; viz. having made one parallelogram mn , take $Cp = Bn$, and form the parallelogram pq ; then take $Dr = Cq$, and make the parallelogram rs ; and take $Et = Ds$, and form the parallelogram tv ; and so on: Then the said vertical diagonals Bb , Cc , Dd , Ee , &c. of those parallelograms, will be proportional to the weights, as required.

Demonstration.

By the resolution of forces, each of the weights or forces Bb , Cc , Dd , &c. in the diagonals of the parallelograms, is equal to, and may be resolved into two forces expressed by two adjacent sides of the parallelogram; viz. the force Bb will be resolved into the two forces Bm , Bn , and in those directions; the force Cc into the two forces Cp , Cq , and in those directions; the force Dd into the two forces Dr , Ds , and in those directions; and so on: Then, since two forces that are equal, and in opposite directions, do mutually balance each other; therefore the several pairs of forces Bn and Cp , Cq and Dr , Ds and Et , &c. being equal and opposite, by the construction, do mutually destroy or balance each other; and the extreme forces Bm , Ev , are balanced by the opposite resistances of the fixed points A , F . Wherefore there is no force

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to change the position of any one of the lines, and consequently they will all remain in equilibrium. *Q.E.D.*

Corollary.

HENCE, if one of the weights and the positions of all the lines be given, all the other weights may be found.

PROPOSITION II.

IF any number of lines, that are connected together and moveable about the points of connection, be kept in equilibrium by weights laid upon the angles, as in the last proposition: Then will the weight on any angle C be universally as $\frac{\text{fine of the } \angle BCD}{s. \angle BCc \times s. \angle cCD}$; that is, directly as the fine of that angle, and reciprocally as the fines of the two parts or angles into which that angle is divided by a line drawn through it perpendicular to the horizon.

Demonstration.

By the last proposition the weights are as Bb, Cc, Dd, &c. when Bn = pC, Cq = rD, Ds = tE, &c. But, since the angle A Bb is = the angle Bbn,