A

Treatise on Mills.

PART I.

OF THE LAWS OF CIRCULAR MOTION, THE RATIOS OF PROJECTILE AND CENTRIFUGAL FORCES, THE PERIODIC TIMES, &c.

FIG. I.

In the larger circle,

D = AD the diameter in feet.

V = AC the projectile force, or orbit velocity in feet.

A = BC = AE the space through which the body would move towards the centre, while it i describing AC, or the central force compared with the projectile.

B

F = BC the centrifugal force compared with gravity, or with the weight of the revolving body, when on the surface of the earth.

T = the periodic time, or time of a revolution, in seconds.

p = the time describing AE or AC.

S = 16 feet, the space through which a body falls in 1 second.

q = 3.1416, the circumference of a circle, when the diameter is 1.

In the lesser circle,

$d = ad$ the diameter.

$v = ac$ the velocity.

$a = ae$ the central force, &c.

$f = ae = bc$.

t = the time of a revolution.

Every body in motion endeavours to move in a straight line; the force which causes it to leave that line is called the *centripetal*, and the resistance which it affords the *centrifugal* force.

1. If a body at A, moving towards B, is drawn to C, BC represents the centrifugal *force*, and AE the centripetal *force*, which are equal; and as AE is to AC, so is the centrifugal to the projectile force, or circular velocity.

2. From the property of the circle,

As AD : AC :: AC : AE $=\frac{AC^2}{AD}=$ A.

Or, As D : V :: V : $\frac{V^2}{D}=$ A.

3. And in equable motion it will be,

As the time of a revolution is to the circumference of the orbit, so is any other time to the space passed over in that time; or so is 1 second to the velocity per second.

Viz. As T : Dq :: p : $\frac{Dqp}{T}=$ V.

And by substituting $\frac{Dqp}{T}$ for its equal AC or V we have, As D : $\frac{Dqp}{T}$ :: $\frac{Dqp}{T}$: $\frac{Dq^2p^2}{T^2}=$ AE $=$ A.

Or, As D : V :: V : $\frac{V^2}{D}=$ AE $=$ A.

4. Also, As $1^{2''}$: s :: p^2 : sp^2, the space through which a body near the surface of the earth would fall in the time p.

5. Then, to compare the centrifugal force with the gravity, or weight of the revolving body, which put $=1$.

It will be, As sp^2 : AE :: 1 : $\frac{AE}{sp^2}=$ F.

Or, As sp^2 : $\frac{Dq^2p^2}{T^2}$ :: 1 : $\frac{Dq^2}{sT^2}=$ F.

Or, As sp^2 : $\frac{V^2}{D}$ :: 1 : $\frac{V^2}{sDp^2}=$ F.

When the central force is equal to the weight of the body, $\frac{Dq^2}{ST^2} = 1$, and $sp^2 = 16$, if $p = 1$ second; also $A = S$.

From these proportions we obtain the following Theorems:

Theo. 1. $A = \frac{V^2}{D} = \frac{Vq}{T} = \frac{Dq^2}{T^2} = SF.$

Theo. 2. $V = \sqrt{AD} = \frac{Dq}{T} = \sqrt{FDS}.$

Theo. 3. $T = \frac{Vq}{A} = \frac{Dq}{V} = q\sqrt{\frac{D}{A}} = q\sqrt{\frac{D}{SF}} = \frac{Vq}{SF}.$

Theo. 4. $D = \frac{V^2}{A} = \frac{V^2}{SF} = \frac{T^2A}{q^2} = \frac{SFT^2}{q^2}.$

Theo. 5. $F = \frac{Dq^2}{ST^2} = \frac{A^2D}{SV^2} = \frac{V^2}{SD} = \frac{A}{S}.$

6. If two bodies, in different circles, revolve in the same time, the velocities will be directly as the diameters of the circles. ($T = t$.)

For, As $Dq : V$ (AC) :: $dq : v$ $(ac) = \frac{Vd}{D}$;

and, As $D : d :: V : v = \frac{Vd}{D.}$ Q. E. D.

And, As $AC : AE :: ac : ae$.

Or, As $V : A :: v : a = \frac{Av}{V.}$

Or, As $V : F :: v : \frac{vF}{} = f.$

Hence, As $V : v :: F : f :: D : d.$

From which it appears that, if the times are equal, the central force is directly as the diameter.

7. If $V = v$, then $Dq : dq :: T : \frac{Td}{D} = t$.

Therefore, As $D : d :: T : t$.

And, As $\frac{D}{T} : \frac{d}{t} : V : v = \frac{VdT}{Dt}$.

$\frac{D}{T} : \frac{d}{t} :: F : f = \frac{FdT}{Dt}$. and because F is as $\frac{V^2}{D}$ we have, As $F : f :: \frac{V^2}{D} : \frac{v^2}{d}$. Or,

As $DF : df :: V^2 : v^2$, and $V : v :: \sqrt{DF} : \sqrt{df}$.

And $\frac{D}{T}$ being as V, we have $\frac{D}{T}\sqrt{df} = \frac{d}{t}\sqrt{DF}$.

Or, from the fifth Theorem, if $T = t$.

As $\frac{Dq^2}{ST^2} : \frac{dq^2}{ST^2} :: F : f = \frac{Fd}{D}$.

Theo. 6. $v = \frac{Vd}{D}$.

Theo. 7. $d = \frac{vD}{V}$.

Theo. 8. $a = \frac{V^2 d}{D}$.

Theo. 9. $f = \frac{V^2 d}{SD}$.

8. If the diameter remains the same; or if $D = d$,

Then, As $\frac{Dq^2}{ST^2} : \frac{Dq^2}{ST^2} :: F : \frac{FT^2}{t^2} = f$, and as $t^2 : T^2 :: F : f :: V^2 : v^2$, or the central forces, in the same circle, are reciprocally as the squares of the times; or directly as the squares of the velocities.

Theo. 10. $f = \frac{FT^2}{t^2}$.

Theo. 11. $t = \sqrt{\frac{FT^2}{f}}$.

9. When $F = f$; $\frac{Dq^2}{ST^2} = \frac{dq^2}{st^2}$, or $Dt^2 = dT^2$.

And $\frac{D}{T^2} = \frac{d}{t^2}$ (see Theo. 5.) $\frac{Dt^2}{T^2} = d$.

And, As $T^2 : t^2 :: D : d$, or $T : t :: \sqrt{D} : \sqrt{d}$.

Theo. 12. $d = \frac{Dt^2}{T^2}$ when the times are given.

Theo. 13. $t = T\sqrt{\frac{d}{D}}$ when the distances are given.

The foregoing Theory exemplified, in the solution of various Problems in circular motion.

PROBLEM I.

Given the diameter of the orbit, 10 feet, and the centrifugal force equal to the weight of the revolving body; required the time of a revolution, and velocity per second?

In this case, $A = S$; and $F = 1$ (see Art. 5.)

In Theo. 3. we have $T = q\sqrt{\frac{D}{SF}} = 3.1416\sqrt{\frac{10}{16}} = 2.4818''$; and per Theo. 2. $V = \frac{Dq}{T} = \frac{31.416}{2.4818} = \sqrt{FDS} = \sqrt{160} = 12.6491$.

PROBLEM II.

Given the time of a revolution, 3 seconds, the central force, 1; required the distance, and velocity?

$\left.\begin{matrix} T = 3. \\ F = 1. \end{matrix}\right\}$ Theo. 4. $D = \frac{T^2 S}{q^2} = \frac{144}{9.8696} = 14.59$ feet; half of which is the distance, $= 7.295$ feet.

And $V = \sqrt{FDS} = 15{\cdot}279$ feet per second.

PROBLEM III.

Given the diameter and periodic time, to find the central force and velocity?

$\left.\begin{matrix} D = 14.59. \\ T = 3''. \end{matrix}\right\}$ (Theo. 5.) $F = \frac{Dq^2}{ST^2} = \frac{144}{144} = 1.$

$V = \frac{Dq}{T} = \frac{45.837}{3} = 15.279$ feet.

PROBLEM IV.

Given the diameter, 14.59 feet, and the velocity, 15.279, to find the periodic time and central force?

(Per Theo. 3.) $T = \frac{Dq}{V} = \frac{45.837}{15.279} = 3''$.

(Per Theo. 5.) $F = \frac{V^2}{SD} = \frac{233.44}{16 \times 14.59} = 1$.

PROBLEM V.

Given the diameter, 14.59 feet, the central force equal to twice the weight of the body; what is the velocity, and time of a revolution?

$T = q\sqrt{\frac{D}{2FS}} = \sqrt{\frac{14.59}{32}} \times q = .6753 \times 9 = 2{\cdot}1214''$.

$V = \sqrt{2FDS} = 21.6074$ feet per second.

Or if we compare this problem with the last, we have (8)

As $f : F :: T^2 : t^2$, or $t = T\sqrt{\frac{F}{f}} = 3\sqrt{\frac{1}{2}} = 2.12$, &c.

And, As $F : f :. V^2 : v^2 = \frac{V^2 f}{F}$, or $v = V\sqrt{\frac{f}{F}} = 15.279 \times \sqrt{\frac{2}{1}} = 15.279 \times 1.414$, &c. $= 21.607$.

PROBLEM VI.

Let the diameter be 29.18 feet, (twice as much as in Prob. III.) the time of a revolution 3 seconds; required the velocity, and central force?

$$V = \frac{Dq}{T} = \frac{91.671}{3} = 30.5572.$$

$$F = \frac{Dq^2}{ST^2} = \frac{288}{144} = 2.$$

(Art. 6.) As $D : d :: V : v = \frac{Vd}{D} = 30.5572.$

Viz. As 14.59 : 29.18 :: 15.279 : 30.5572.

Again, As $D : d :: F : f = \frac{dF}{D}.$

As 14.59 : 29.18 :: 1 : 2, the centrifugal force, the same as above.

PROBLEM VII.

The stones on which they grind table knives at Sheffield, are about 44 inches diameter, and weigh about half a ton; the velocity of the surface is at the rate of 1250 yards in a minute, equal to 326 revolutions; required the centrifugal force, or the tendency which the stones have to burst?

$D = 2.59$ feet, the diameter of the circle of gyration.

$T = .184$ seconds, the time of one revolution.

$T^2 = .033856$.

$$F = \frac{Dq^2}{ST^2} = \frac{2.59 \times 9.8696}{16 \times .0338, \&c.} = \frac{25.5622}{.54169} = 47.18$$

times the weight of the stone, or $23\frac{1}{2}$ tons.

PROBLEM VIII.

If a fly, 12 feet diameter, and 3 tons weight, revolves in 8 seconds, and another of the same weight revolves in 3 seconds; what must be the diameter of the last, when they have the same centrifugal force?

$D = 12$.

$T = 8$.

$t = 3$.

$F = f$.

And per Theo. 12. $d = \frac{Dt^2}{T^2} = \frac{108}{64} = 1.6875$ feet, the diameter of the circle of percussion.

N. B. As the weight of each fly is the same, it does not enter into the solution; but if the diameters should be the same, as in the next Problem, the weight must be considered.

PROBLEM IX.

If a fly, 12 feet diameter, revolves in 8 seconds, and another of the same diameter in 3