INTRODUCTION.

ON VULGAR FRACTIONS.

ARTICLE 1. A *fraction* is a quantity which represents a part or parts of an integer or whole.

(2.) A *simple fraction* consists of two members, the *numerator* and the *denominator*; the denominator shews into how many equal parts the whole, or unity, is divided; and the numerator, the number of those parts taken. The numerator is usually placed over the denominator with a line between them. Thus, $\frac{2}{3}$, two thirds, signifies that unity is divided into three equal parts, and that two of those parts are taken.

It must be observed, that we suppose every integer to be divisible into any number of equal parts at pleasure.

(3.) A *proper fraction* is one whose numerator is less than it's denominator, as $\frac{7}{8}$.

(4.) An *improper fraction* is one whose numerator is equal to, or greater than it's denominator, as $\frac{6}{6}$; $\frac{7}{5}$

(5.) A *compound fraction* is a fraction of a fraction, as $\frac{3}{4}$ of $\frac{5}{6}$, where $\frac{5}{6}$ is the whole quantity of which $\frac{3}{4}$ is to be taken; also, $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{9}{11}$, is a compound fraction; &c.

(6.) A quantity consisting of a whole number and a fraction is called a *mixed number*, as $7\frac{3}{10}$, which signifies 7 integers together with $\frac{3}{10}$ of an integer.

(7.) COR. 1. Every integer may be considered as a fraction whose denominator is 1; thus 5, or five units, is $\frac{5}{1}$.

(8.) COR. 2. *To multiply a fraction by any number, multiply the numerator by that number and retain the same denominator.* Thus, $\frac{2}{15}$ multiplied by 7 is $\frac{14}{15}$. For, the unit, in each of the fractions $\frac{2}{15}$ and $\frac{14}{15}$, is divided into 15 equal parts, and 7 times as many of those parts are taken in the latter case as in the former.

(9.) COR. 3. *To divide a fraction by any number, multiply the denominator by that number and retain the same numerator.* Thus, $\frac{3}{5}$ divided by 4 is $\frac{3}{20}$. For, the unit being divided into four times as many equal parts in $\frac{3}{20}$ as it is in $\frac{3}{5}$, each of the parts in the latter case is four times as great as in the former, and the

the same number of parts is taken in both cases; therefore the former fraction is one fourth of the latter.

(10.) A simple fraction may be considered as representing the quotient arising from the division of the numerator by the denominator; thus the fraction $\frac{3}{4}$ represents the quotient of 3 divided by 4; for 3 is $\frac{3}{1}$ (Art. 7), and this divided by 4 is the fraction $\frac{3}{4}$ (Art. 9). If the integer be supposed a pound, or twenty shillings, $\frac{3}{4}$ of £.1, which is 15 shillings, is equal to $\frac{1}{4}$ of £.3, which is also 15 shillings.

(11.) If the numerator and denominator of a fraction be both multiplied by the same number, its value is not altered. For, if the numerator be multiplied by any number, the fraction is multiplied by that number (Art. 8); and if the denominator be multiplied by the same number, the fraction is divided by it (Art. 9); and if a quantity be both multiplied and divided by the same number, it's value is not altered. Thus, $\frac{5}{14} = \frac{15}{42} = \frac{150}{420}$, &c. Hence, if the numerator and denominator be both divided by the same number, it's value is not altered; since $\frac{150}{420} = \frac{15}{42} = \frac{5}{14}$ *.

* To avoid repetition, the Reader is referred to the first section of the Algebra, for the explanation of the signs +, −, ×, and =.

ON REDUCTION.

The operation by which a quantity is changed from one denomination to another, without altering it's value, is called *Reduction.*

(12.) *To reduce a whole number to a fraction with a given denominator.*

Multiply the proposed number by the given denominator, and the product will be the numerator of the fraction required.

Ex. Reduce 5 to a fraction whose denominator is 6.

This is $\frac{5 \times 6}{6}$ or $\frac{30}{6}$; because 5 may be considered as a fraction $\frac{5}{1}$ (Art. 7), the numerator and denominator of which are multiplied by 6, therefore it's value is not altered. (Art. 11).

(13.) *To reduce a mixed number to an improper fraction.*

Multiply the integral by the denominator of the fractional part, to this product add the numerator of the fractional part, and make it's denominator the denominator of the sum.

Ex. 1. Reduce $7\frac{4}{5}$ to an improper fraction.

The quantity $7\frac{4}{5}$ is equal to $\frac{35+4}{5} = \frac{39}{5}$; for 7 (by the last Art.) is equal to $\frac{35}{5}$, and if to this $\frac{4}{5}$ be added, the whole is $\frac{39}{5}$.

Ex. 2. Also, $23\frac{9}{11} = \frac{253+9}{11} = \frac{262}{11}$.

(14.) *To*

(14.) *To reduce an improper fraction to a mixed number.*

Divide the numerator by the denominator for the integral part, and make the remainder the numerator of the fractional part, and the divisor it's denominator.

Ex. Reduce $\frac{39}{5}$ to an improper fraction. The fraction $\frac{39}{5} = 7\frac{4}{5}$; because the unit being divided into 5 parts, 39 such parts are to be taken, that is, 7 units and 4 such parts.

(15.) *To reduce a compound fraction to a simple one.*

Multiply all the numerators together for a new numerator, and all the denominators for a new denominator.

Ex. 1. $\frac{2}{3}$ of $\frac{4}{5} = \frac{8}{15}$; for, one third of $\frac{4}{5}$ is $\frac{4}{15}$, (Art. 9); therefore two thirds, which must be twice as great, is $\frac{8}{15}$ (Art. 8).

Ex. 2. $\frac{3}{4}$ of $5 = \frac{3}{4}$ of $\frac{5}{1} = \frac{15}{4}$.

Mixed numbers must be reduced to improper fractions, before the rule can be applied.

Ex. 3. $\frac{5}{8}$ of $\frac{2}{9}$ of $3\frac{1}{12} = \frac{10}{72}$ of $3\frac{1}{12} = \frac{10}{72}$ of $\frac{37}{12}$ $= \frac{370}{864}$.

(16.) *To reduce a fraction to lower terms.*

Whenever the numerator and denominator of a fraction have a common measure (or number which divides each of them without remainder) greater than unity.

unity, the fraction may be reduced to lower terms, by dividing both the numerator and denominator by this common measure.

Ex. $\frac{105}{120}$ is reduced to $\frac{21}{24}$, by dividing both the numerator and denominator by 5; and $\frac{21}{24}$ is again reduced to $\frac{7}{8}$, by dividing it's numerator and denominator by 3. That the value of the fraction is not altered, appears from Art. 11.

In the same manner, $\frac{168}{210} = \frac{84}{105} = \frac{28}{35} = \frac{4}{5}$.

(17.) *The greatest common measure of two numbers is found by dividing the greater by the less, and the preceding divisor by the remainder, continually, till nothing is left. the last divisor is the greatest common measure required.*

To find the greatest common measure of 189 and 224.

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189)224(1
    189
    ---
     35)189(5
        175
        ---
         14)35(2
            28
            --
             7)14(2
               14
               --
                *
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By proceeding according to the rule, it appears that 7 is is the last divisor, or the greatest common measure sought. The proof of this rule will be given hereafter*.

(18.) *A fraction is reduced to it's* lowest *terms, by dividing it's numerator and denominator by their* greatest *common measure.*

Ex. To reduce $\frac{385}{396}$ to it's lowest terms.

By the last Art. the greatest common measure of the numerator and denominator is found to be 11, and therefore $\frac{35}{36}$ is the fraction in it's lowest terms.

Cor. If unity be the greatest common measure of the numerator and denominator, the fraction is in it's lowest terms.

(19.) *To reduce fractions to a common denominator.*

Having reduced, if necessary, compound fractions to simple ones, and mixed numbers to improper fractions, multiply each numerator by all the denominators except it's own, for the new numerator, and all the denominators together for a common denominator.

Ex. 1. Reduce $\frac{1}{2}$, $\frac{2}{3}$ and $\frac{3}{4}$ to a common denominator.

$\frac{1\times3\times4}{2\times3\times4}$, $\frac{2\times2\times4}{2\times3\times4}$, and $\frac{3\times2\times3}{2\times3\times4}$, or $\frac{12}{24}$, $\frac{16}{24}$ and $\frac{18}{24}$ are the fractions required. These fractions are respectively equal to the former, the numerator and denominator in each case, having been multiplied by the same numbers, namely, the denominators of the rest.

$\frac{1\times3\times4}{2\times3\times4}=\frac{1}{2}$; $\frac{2\times2\times4}{2\times3\times4}=\frac{2}{3}$; and $\frac{3\times2\times3}{2\times3\times4}=\frac{3}{4}$.

Ex. 2.

* Art. 90.

Ex. 2. Reduce $\frac{2}{5}$ of $\frac{3}{4}$ and $4\frac{1}{3}$ to a common denominator.

These are $\frac{6}{20}$ and $\frac{13}{3}$, or $\frac{3}{10}$ and $\frac{13}{3}$; therefore $\frac{9}{30}$ and $\frac{130}{30}$ are the fractions required.

(20.) *If the denominator of one of two fractions contain the denominator of the other a certain number of times exactly, multiply the numerator and denominator of the latter by that number, and it will be reduced to the same denominator with the former.*

Ex. Reduce $\frac{5}{12}$ and $\frac{2}{3}$ to a common denominator.

Since 12 contains 3 four times exactly, multiply both the numerator and denominator of $\frac{2}{3}$ by 4, and it becomes $\frac{8}{12}$, a fraction having the same denominator with $\frac{5}{12}$.

(21.) Cor. By reducing two fractions to a common denominator their values may be compared.

Thus, $\frac{4}{7}$ and $\frac{7}{12}$ when reduced to a common denominator are $\frac{48}{84}$ and $\frac{49}{84}$; that is, the fractions have the same relative values that 48 and 49 have.

(22.) *To find the value of a fraction of a proposed denomination in terms of a lower denomination.*

Multiply

Multiply the fraction by the number of integers of the lower denomination contained in one integer of the higher, and the product is the value required. The value of any fractional part of the lower denomination may be obtained in the same manner, till we come to the lowest.

Ex. 1. What is the value of $\frac{5}{7}$ of a pound?

First, $\frac{5}{7}$ of £. 1 is $\frac{5}{7}$ of 20 shillings, or $\frac{5}{7}$ of $\frac{20}{1}$ shillings $= \frac{100}{7} = 14\frac{2}{7}$ shillings;

Next, $\frac{2}{7}$ of a shilling $= \frac{2}{7}$ of $\frac{12}{1}$ pence $= \frac{24}{7}$ pence $= 3\frac{3}{7}$ pence;

Lastly, $\frac{3}{7}$ of a penny $= \frac{3}{7}$ of 4 farthings $= \frac{3}{7}$ of $\frac{4}{1}$, or $\frac{12}{7}$ farthings $= 1\frac{5}{7}$ farthings: hence, $\frac{5}{7}$ of a pound is $\overset{s.}{14} : \overset{d.}{3} : \overset{q.}{1}\frac{5}{7}$.

The operation is usually performed in the following manner:

£.5
20
7)100
14 - - 2*s*.
12
7)24
3 - - 3*d*.
4
7)12
1 - - 5*q*.

Ans. $\overset{s.}{14} : \overset{d.}{3} : \overset{q.}{1}\frac{5}{7}$.

Ex. 2. What is the value of $\frac{5}{9}$ of a crown?

5 *C*
5
9)25
2 - - 7*s*.
12
9)84
9 - - 3*d*.
4
9)12
1 - - 3*q*.

Ans. $\overset{s.}{2} : \overset{d.}{9} : \overset{q.}{1}\frac{3}{9}$.

(23.) *To*