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Mathematical Papers of the Late George Green

A miller's son, George Green (1793–1841) received little formal schooling yet managed to acquire significant knowledge of modern mathematics, especially French work. In 1828 he published his *Essay on the Application of Mathematical Analysis to the Theories of Electricity and Magnetism*, the work for which he is now celebrated. Admitted to Cambridge in 1833 as a mature student, Green went on to become a fellow of Gonville and Caius College. His early death, however, cut short a promising career as a mathematical physicist. While English contemporaries saw what he might have achieved, they did not understand what he had actually achieved. Only when William Thomson (later Lord Kelvin) rediscovered Green's first publication and shared it with the French mathematical elite was his greatness truly appreciated. Edited by the Cambridge mathematician Norman Macleod Ferrers (1829–1903) and published in 1871, this collection comprises Green's influential essay and nine further papers.

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Mathematical Papers of the Late George Green

EDITED BY N.M. FERRERS

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PREFACE.

HAVING been requested by the Master and Fellows of Gonville and Caius College to superintend an edition of the mathematical writings of the late George Green, I have fulfilled the task to the best of my ability. The publication may be opportune at present, as several of the subjects with which they are directly or indirectly concerned, have recently been introduced into the course of mathematical study at Cambridge. They have also an interest as being the work of an almost entirely self-taught mathematical genius.

George Green was born at Sneinton, near Nottingham, in 1793. He commenced residence at Gonville and Caius College, in October, 1833, and in January, 1837, took his degree of Bachelor of Arts as Fourth Wrangler. It is hardly necessary to say that this position, distinguished as it was, most inadequately represented his mathematical power. He laboured under the double disadvantage of advanced age, and of inability to submit entirely to the course of systematic training needed for the highest places in the Tripos. He was elected to a fellowship of his College in 1839, but did not long enjoy this position, as he died in 1841. The contents of the following pages will sufficiently shew the heavy loss which the scientific world sustained by his premature death.

A slight sketch of the papers comprised in this volume may not be uninteresting.

The first paper, which is also the longest and perhaps the most important, was published by subscription at Nottingham in 1828. It was in this paper that the term *potential* was first introduced to denote the result obtained by adding together the masses of all the particles of a system, each divided by its distance from a given point. In this essay, which is divided into three parts, the properties of this function are first considered, and they are then applied, in the second and third parts, to the theories of magnetism and electricity respectively. The full analysis of this essay which the author has given in his Preface, renders any detailed description in this place unnecessary. In connexion with this essay, the corresponding portions of Thomson and Tait's *Natural Philosophy* should be studied, especially Appendix A. to Chap. I., and Arts. 482—550, inclusive.

The next paper, "On the Laws of the Equilibrium of Fluids analogous to the Electric Fluid," was laid before the Cambridge Philosophical Society by Sir Edward Ffrench Bromhead, in 1832. The law of repulsion of the particles of the supposed fluid here considered is taken to be inversely proportional to the n^{th} power of the distance. This paper, though displaying great analytical power, is perhaps rather curious than practically interesting; and a similar remark applies to that which succeeds it, "On the determination of the attractions of Ellipsoids of variable Densities," which, like its predecessor, was communicated to the Cambridge Philosophical Society by Sir E. F. Bromhead. Space of n dimensions is here considered, and the surfaces of the attracting bodies are supposed to be represented by equations formed by equating to unity the sums of the squares of the n variables, each divided by an appropriate coefficient. It is of course possible to adapt the formula of this paper to the case of nature by supposing $n = 3$.

The next paper, "On the Motion of Waves in a variable canal of small depth and width," though short, is interesting. It was read before the Cambridge Philosophical Society, on May 15, 1837, and a Supplement to it on Feb. 18, 1839. On Dec. 11, 1837, were communicated two of his most valuable memoirs, "On the Reflexion and Refraction of Sound," and "On the Reflexion and Refraction of Light at the common surface of two non-crystallized media." These two papers should be studied together. The question discussed in the first is, in fact, that of the propagation of normal vibrations through a fluid. Particular attention should be paid to the mode in which, from the differential equations of motion, is deduced an explanation of a phenomenon analogous to that known in Optics as Total internal reflection when the angle of incidence exceeds the critical angle. By supposing that there are propagated, in the second medium, vibrations which rapidly diminish in intensity, and become evanescent at sensible distances, the change of phase which accompanies this phenomenon is clearly brought into view.

The immediate object of the next paper, "On the Reflexion and Refraction of Light at the common surface of two non-crystalline media," is to do for the theory of light what in the former paper has been done for that of sound. This is done in a manner which will present little difficulty to one who has mastered the former paper. But this paper has an interest extending far beyond this subject. For the purpose of explaining the propagation of transversal vibrations through the luminiferous ether, it becomes necessary to investigate the equations of motion of an elastic solid. It is here that Green for the first time enunciates the principle of the Conservation of work, which he bases on the assumption of the impossibility of a perpetual motion. This principle he enunciates in the following words: "In whatever manner the elements of any material system may act upon each other, if all the internal forces be multiplied by the elements of their respective directions, the total sum for any assigned portion of the mass will always be the exact differential of some function." This function, it will be seen, is what is now known under the name of Potential Energy, and the above principle is in fact equivalent to stating that the sum of the Kinetic and Potential Energies of the system is constant. This function, supposing the displacements so small that powers above the second may be neglected, is shewn for the most general constitution of the medium to involve twenty-one coefficients, which reduce to nine in the case of a medium symmetrical with respect to three rectangular planes, to five in the case of a medium symmetrical around an axis, and to two in the case of an isotropic or uncrystallized medium. The present paper is devoted to the consideration of the propagation of vibrations from one of two media of this nature. The two coefficients above mentioned, called respectively A and B, are shewn to be proportional to the squares of the velocities of propagation of normal and transversal vibrations respectively. It is to be regretted that the *statical* interpretation is not also given. It may however be shewn (see Thomson and Tait's *Natural Philosophy*, p. 711 (*m.*)) that $A - \frac{4}{3}B$ measures the resistance of the medium to compression or dilatation, or its *elasticity of volume*, while B measures its resistance to distortion, or its *rigidity*. The equilibrium of the medium, it may be shewn, cannot be stable, unless both of these quantities are positive*. A Supplement to this paper supplying certain omissions, immediately follows it.

In the next paper, "On the Propagation of Light in Crystalline Media," the principle of Conservation of Work is again assumed as a starting-point and applied to a medium of any description. It is first assumed that the medium is symmetrical with respect to three planes at right angles to one another, by which supposition the twenty-one coefficients previously mentioned are reduced to nine. Fresnel's supposition, that the vibrations affecting the eye are accurately in front of the wave, is then introduced, and a complete explanation of the phenomena of polarization is shewn to follow, on the hypothesis that the vibrations constituting a plane-polarized ray are *in* the plane of polarization. The hypothesis adopted in the former paper—that these vibrations are *perpendicular* to the plane of polarization—is then resumed, and an explanation arrived at, by the aid of a subsidiary assumption—unfortunately not of the same simple character as those previously introduced—that for the three principal waves the wave-velocity depends on the direction of the disturbance only, and is independent of the position of the wave's front. The paper concludes by taking the case of a perfectly general medium, and it is shewn that Fresnel's supposition of the vibrations being accurately in the wave-front, gives rise to fourteen re-

* In comparing Green's paper with the passage in Thomson and Tait's *Natural Philosophy* above referred to, it should be remarked that the A of the former is equal to the $m - \frac{1}{3}n$ of the latter, and that $B = n$.

lations among the twenty-one coefficients, which virtually reduce the medium to one symmetrical with respect to three planes at right angles to one another.

This paper, read May 20, 1839, was his last production. Another, "On the Vibrations of Pendulums in Fluid Media," read before the Royal Society of Edinburgh, on Dec. 16, 1833, will be found at the end of this collection. The problem here considered is that of the motion of an inelastic fluid agitated by the small vibrations of a solid ellipsoid, moving parallel to itself.

I have to express my thanks to the Council of the Cambridge Philosophical Society, and to that of the Royal Society of Edinburgh, for the permission to reproduce the papers published in their respective Transactions which they have kindly given.

N. M. FERRERS.

Gonville and Caius College,
Dec. 1870.

CONTENTS.

ERRATA.

Page 23, line 11, *for* there *read* these.
,, 23, ,, 25, *for* $dzdy$ *read* $dydz$.
,, 29, ,, 7, *for* $\frac{d\overline{V}}{dx}$ *read* $\frac{d\overline{V}}{dw}$.
,, 29, ,, 16, *for* $\frac{d\overline{V}}{dx'}$ *read* $\frac{d\overline{V}}{dw'}$.
,, 36, ,, 22, *after* co-ordinates, *insert* of.
,, 37, ,, 2 from bottom, *for* $\delta\overline{V}$, *read* $\delta'\overline{V}$.
,, 43, ,, 8, *for* axes, *read* axis.
,, 46, ,, 19, *after* radius, *insert* is.
,, 53, ,, 7, *for* ρ *read* (g).
,, 54, ,, 11, *for* $4\pi f^3$ *read* $2\pi f^3$.
,, 54, ,, 16, *for* $4\pi af^2$ *read* $4\pi af^3$.
,, 56, ,, 19, *for* $Q\left(\frac{1}{a}-\frac{1}{a}\right)=Q'\left(\frac{1}{a'}-\frac{1}{b}\right)$ *read* $Q\left(\frac{1}{a}-\frac{1}{b}\right)=Q'\left(\frac{1}{a'}-\frac{1}{b}\right)$.
,, 60, ,, 13, *for* $\int\frac{d\sigma}{f^3}V$ *read* $\int\frac{ds}{f^3}\overline{V}$.
,, 64, ,, 4 from bottom, *before* a potential *insert* of.
,, 71, throughout *for* dw and $d\omega$ read $d\varpi$.
,, 72, ,, 18, *for* $d\omega$ *read* $d\varpi$.
,, 74, ,, 24, *for* his *read* this.
,, 84, ,, 11, *for* $\frac{U^{(2)}}{r'^3}$ *read* $\frac{U^{(1)}}{r'^2}$.
,, 88, ,, 20, *for* r^2 *read* r^3.
,, 89, ,, 17, *for* $\sin\theta'$ *read* $\sin\theta$.
,, 89, ,, 18, *for* $U^{(0)}$ *read* $U^{(1)}$.
,, 90, ,, 2, *for* $\frac{d}{dx}$ *read* $\frac{d}{dx'}$.
,, 92 ,, 24, *for* $\frac{3}{4}\pi$ *read* $\frac{3}{4\pi}$.
,, 107 ,, 20, *for* $r^2\frac{d^2\phi}{dx^2}=$ *read* $r^2\frac{d^2\phi}{dx^2}+$
,, 123 ,, 19, *for* these *read* thus.
,, 129 ,, 24, *for* $\sin\varpi'_2$ *read* $\sin\varpi'$.