POPULAR INSTRUCTION,

ETC.

LESSON I.

ON CERTAINTY AND PROBABILITY.

WHEN different circumstances give rise to an event, they are called the *changes* of the event.

Example.—The drawing of one number in a lottery offers ninety chances; hence ninety different numbers may lead to the expected event.

In the throwing of some one point, an ordinary die presents six chances, as it must fall on one of its faces.

When the nature of the event which we hope for be declared, two sorts of chances arise, some favourable, others contrary or unfavourable to the event.

Example.—In a game with thirty-two cards, as in piquet, the chances of drawing a figured or court card are twelve, that is to say, as many as there are court cards, and twenty chances against.

Observation.—When all the chances are favourable to an expected event, their combination constitutes certainty.

Example.—Suppose an urn to contain three white balls, and you desire to draw one white ball; all the chances being favourable, we call the attainment of the proposed object a certainty.

Observation.—When there are only a certain number of favourable chances to the accomplishment of an event, the event is said to be *probable.*

Example.—If an urn contain three balls, one white and two black, the drawing of the white ball is probable; in three chances there is only one favourable.

The chances of drawing a king in a game with thirty-two cards, is again *probable;* out of thirty-two chances there are but four favourable, one or the other of the four kings may be drawn.

All events are not equally probable, and their different degrees of probability will be measured by the greater or less number of favourable chances.

Example.—If an urn contain twenty white balls and five black; the drawing of a white ball offers more favourable chances than that of a black, and we say that it is more probable than the other.

The same in a game of thirty-two cards, the drawing of a figured or court card is less probable than that of any other.

The calculation which teaches you to discover the degrees of probability of an event, is called the conjectural calculation, or calculation of probabilities.

In *some* cases the number of chances of an event are *limited*, and calculable, as in most of the games of Hazard; the probability of an event may generally be estimated with considerable facility, as we shall see in what follows.

In *other* cases the number of chances are *unlimited;* as in most of the sciences of observation.

We ought then to *estimate* the probability of the event, by means of a certain number of the chances which we obtain by experiment.

The consideration of this species of probability will form the second part of this essay: to give an example however of its existence, suppose an urn to contain an indefinite number of balls, the colours of which are unknown; and you find, that in a great number of trials, white balls only are drawn, and you are asked what the probabilities are that the urn contains only balls of this one colour.

You answer, *that it is probable* the urn contains only white balls; this assertion cannot however be considered as a certainty; for it may be, that one or several black balls, not yet drawn, remain in the urn. Thus, although the sun has been seen to rise with regularity for millions of years, yet it can only be *considered as probable* that this planet will again rise to-morrow: for some law may exist in nature not yet manifested, and which may prevent the sun from rising to-morrow.

We have not perhaps been in a condition to examine all the possible chances. We conceive, nevertheless, that there are probabilities so strong as to admit of their being considered as nearly certainties.

Example.—To the case where the probability of seeing the sun rise to-morrow, or of drawing a white ball from an urn, which, after a considerable number of trials has only yielded white balls, may be added also the probability of a man in health and strength living five minutes. There is little difficulty in receiving the event as certain, notwithstanding the fact that men, who had given promise of living many days, had been cut off by sudden death.

We always regard events as dependent on the causes which produce them; and *chance* can only be considered as the effect of our ignorance of those causes.

We say, that a grain of dust, that a simple molicule of air or vapour floats at *hazard*, notwithstanding that the curve described is, as observed by the illustrious De la Place, regulated with as great certainty as the orbits of the planets; there is no difference between them but what is found in our ignorance.

QUESTIONS TO LESSON I.

What do you term the chances of an event?

What do you term the favourable and unfavourable chances of an event?

What is certainty?

What is probability?

Give examples of certainty and of probability; are there different degrees of probability?

What is the conjectural calculation of probabilities?

Are the number of chances of an event always limited and calculable?

When the number of chances of an event is unlimited, can you estimate still the probability of that event?

Are probabilities ever so great as to permit of their being considered as certainties?

How ought we to view that which we call *hazard?*

LESSON II.

ON MATHEMATICAL PROBABILITY.

In cases where all the chances of an event are equally possible, "the Mathematical Probability is estimated by dividing the number of favourable chances by the total number of chances."

If an urn contain 3 white balls and 2 black, there are three favourable chances out of five for drawing a white ball: and we say that the mathematical probability of the expected event is $\frac{3}{5}$. In like manner, the mathematical probability of drawing one figure, or court-card, in a game with 32 cards, is $\frac{12}{32}$; since among 32 cards there are 12 figured ones.

The probability, *unfavourable* to an expected event, is estimated in a similar manner; viz. by dividing the number of unfavourable chances by the total number of chances.

In the preceding examples the probabilities offered to the two expected events are $\frac{2}{5}$ and $\frac{20}{32}$.

Generally, each unexpected event gives rise to two opposite probabilities, viz. that the event shall occur, and that it shall not; and the sum of these two probabilities is equal to unity.

The probability of getting a figure card in a game with 32 cards, is $\frac{12}{32}$; the probability against it is $\frac{20}{32}$, and the sum of these probabilities is $\frac{12}{32}$ plus $\frac{20}{32}$ or 1.

It is then only necessary to subtract from 1 the mathematical probability favourable to an event, to obtain the probability against it.

The mathematical probability of an event, ought, after what has been said, to be expressed by a proper fraction, since the number of favourable chances cannot surpass the total number of chances.

It is obvious, that the greater the number of favourable chances in proportion to the total number of chances possible, the stronger will be the probability of the event occurring.

Example.—The probability of $\frac{12}{32}$ is greater than that of $\frac{4}{32}$; the former being the mathematical probability of obtaining a figure card at a game of 32 cards; and the latter that of one of the four aces.

When all the chances become favourable the result is *certainty*, and the numerator of the fraction becomes equal to the denominator, so that *unity is the symbol of certainty.*

When it is required to compare two mathematical probabilities, they must be reduced to a common denominator.

Example.—If it were required to know which were the more probable, the throwing of an ace with a die having six faces, or the drawing of a figure card in hearts, in a game with 32 cards. There will be for the first probability, $\frac{1}{6}$, and for the second, $\frac{3}{32}$; these fractions being reduced to a common denominator give $\frac{32}{192}$ and $\frac{18}{192}$. The first event is therefore more probable than the second.

The great defect (from habit) of estimating the probabilities of uncertain events, causes gross mistakes in estimating their value; it is always requisite to obtain a term of comparison, which may serve to rectify our judgments: the most simple means seems to be, to conceive the favourable and unfavourable chances to be represented numerically by white and black balls, which may be contained in an urn: the accomplishment of an expected event may be assimilated in this manner to the drawing of a white ball.

Example.—What is the probability of throwing an ace with a die having six faces? As there is but one chance out of six, the probability will be $\frac{1}{6}$, the same as that of drawing a white ball from an urn which contains 6 balls, viz. one white and 5 black; or again, What will be the probability of getting a king in a game with 32 cards? As there are four chances favourable out of 32, the probability will be $\frac{4}{32}$, the same as that of drawing one white ball from an urn which contains 32 balls, viz. 4 white and 28 black.

Observe, that the increase and decrease of the number of favourable chances, and the total number of possible chances, in the same proportion, does not alter the probability, it remains the same; thus, in place of the probability $\frac{4}{32}$, we may say the probability $\frac{1}{8}$, which will be found an equivalent.

The probability of drawing one white ball from an urn which contains 32 balls: thus, 4 white and 28 black, is precisely the same as drawing one white ball from an urn which contains 8 balls, viz. one white and 7 black: this means of simplification, depending on the properties of fractions, will be found often useful.

The mode suggested of estimating the value of probabilities offers, however, an inconvenience, inasmuch as it would be difficult to appreciate the amount of value which may be attached to a probability, to permit of its being classed among those which we have been habituated to consider as certainties.

The best term of comparison seems to be the probability of living still through a certain space of time. This measure will be more appreciated in consequence of the value which we attach generally to life, than any other more precise mode that we are in the habit of using.

If we cast our eyes on a table of mortality, on that of the southern provinces of the Netherlands for instance, we shall see, that out of 51,956 persons who had attained to the age of 20, the tenth part ceased to exist at the termination of 7 years.

Thus: at this age the probability of dying in the space of 7 years is $\frac{1}{10}$, that is to say, something less than the probability of getting a king at the first cut with 32 cards. By making similar calculations for the periods at which the $\frac{1}{100}$ the $\frac{1}{1000}$ of these 51,956 young persons cease to exist, the following Table is formed, to which we shall have occasion to refer.

TABLE.

Probability	of dying before
$\frac{1}{10}$	7 years.
$\frac{1}{100}$	8 months.
$\frac{1}{1,000}$	25 days.
$\frac{1}{10,000}$	60 hours.
$\frac{1}{100,000}$	6 hours.
$\frac{1}{1,000,000}$	36 minutes.
$\frac{1}{10,000,000}$	4 minutes.
$\frac{1}{100,000,000}$	22 seconds.
$\frac{1}{200,000,000}$	1 second.

It must be observed that these results can be only received as of general application, not to individuals who may be in health, but rather as the probability that the new born infant, if he attain the age of 20 years, will die before a certain time designated in the Table.

Example.—Supposing it were required to know what the probability would be of the letters composing the word " Constantinople," after being thrown in the air should arrange themselves so as to recompose the same word. We know by calculations, which cannot be given here, that these 14 letters may be arranged more than 87,000,000,000 different ways, and yet only produce the same word 24 times: we have then for the probability of the throw $\frac{24}{87,000,000,000}$, in other words, a probability less than that of dying in the space of a second at the age of 20 years. It may therefore be regarded as cer-