

## CONTENTS OF VOL II

XII. ARISTARCHUS OF SAMOS . . . . .	PAGES 1-15
XIII. ARCHIMEDES . . . . .	16-109
Traditions	
(a) Astronomy . . . . .	17-18
(β) Mechanics . . . . .	18
Summary of main achievements	19-20
Character of treatises . . . . .	20-22
List of works still extant . . . . .	22-23
Traces of lost works . . . . .	23-25
The text of Archimedes . . . . .	25-27
Contents of <i>The Method</i> . . . . .	27-34
<i>On the Sphere and Cylinder</i> , I, II . . . . .	34-50
Cubic equation arising out of II. 4 . . . . .	43-46
(i) Archimedes's own solution . . . . .	45-46
(ii) Dionysodorus's solution . . . . .	46
(iii) Diocles's solution of original problem . . . . .	47-49
<i>Measurement of a Circle</i> . . . . .	50-56
<i>On Conoids and Spheroids</i> . . . . .	56-64
<i>On Spirals</i> . . . . .	64-75
<i>On Plane Equilibriums</i> , I, II . . . . .	75-81
<i>The Sand-reckoner (Psammites or Arenarius)</i> . . . . .	81-85
<i>The Quadrature of the Parabola</i> . . . . .	85-91
<i>On Floating Bodies</i> , I, II . . . . .	91-97
The problem of the crown . . . . .	92-94
Other works	
(a) The Cattle-Problem . . . . .	97-98
(β) On semi regular polyhedra . . . . .	98-101
(γ) <i>The Liber Assumptorum</i> . . . . .	101-103
(δ) Formula for area of triangle . . . . .	103
Eratosthenes . . . . .	104-109
Measurement of the Earth . . . . .	106-108
XIV. CONIC SECTIONS. APOLLONIUS OF PERGA . . . . .	110-196
A. HISTORY OF CONICS UP TO APOLLONIUS . . . . .	110-126
Discovery of the conic sections by Menaechmus . . . . .	110-111
Menaechmus's probable procedure . . . . .	111-116
Works by Aristaeus and Euclid . . . . .	116-117
'Solid loci' and 'solid problems' . . . . .	117-118
Aristaeus's <i>Solid Loci</i> . . . . .	118-119
Focus-directrix property known to Euclid . . . . .	119
Proof from Pappus . . . . .	120-121
Propositions included in Euclid's <i>Conics</i> . . . . .	121-122
Conic sections in Archimedes . . . . .	122-126

## XIV. CONTINUED.

B APOLLONIUS OF PERGA . . . . .	PAGES	126-196
The text of the <i>Conics</i> . . . . .		126-128
Apollonius's own account of the <i>Conics</i> . . . . .		128-133
Extent of claim to originality . . . . .		132-133
Great generality of treatment . . . . .		133
Analysis of the <i>Conics</i> . . . . .		133-175
Book I . . . . .		133-148
Conics obtained in the most general way from oblique cone . . . . .		134-138
New names, 'parabola', 'ellipse', 'hyperbola' . . . . .		138-139
Fundamental properties equivalent to Cartesian equations . . . . .		139-141
Transition to new diameter and tangent at its extremity . . . . .		141-147
First appearance of principal axes . . . . .		147-148
Book II . . . . .		148-150
Book III . . . . .		150-157
Book IV . . . . .		157-158
Book V . . . . .		158-167
Normals as maxima and minima . . . . .		159-163
Number of normals from a point . . . . .		163-164
Propositions leading immediately to determination of <i>evolute</i> of conic . . . . .		164-166
Construction of normals . . . . .		166-167
Book VI . . . . .		167-168
Book VII . . . . .		168-174
Other works by Apollonius . . . . .		175-194
(a) <i>On the Cutting off of a Ratio</i> (λόγου ἀποτομή), two Books . . . . .		175-179
(β) <i>On the Cutting-off of an Area</i> (χωρίου ἀποτομή), two Books . . . . .		179-180
(γ) <i>On Determinate Section</i> (διωρισμένη τομή), two Books . . . . .		180-181
(δ) <i>On Contacts or Tangencies</i> (ἐπαφαί), two Books . . . . .		181-185
(ε) <i>Plane Loci</i> , two Books . . . . .		185-189
(ζ) <i>Νεύσεις</i> ( <i>Vergings or Inclinations</i> ), two Books . . . . .		189-192
(η) <i>Comparison of dodecahedron with icosahedron</i> . . . . .		192
(θ) <i>General Treatise</i> . . . . .		192-193
(ι) <i>On the Cochlias</i> . . . . .		193
(κ) <i>On Unordered Irrationals</i> . . . . .		193
(λ) <i>On the Burning-mirror</i> . . . . .		194
(μ) Ὀκυτόκιον . . . . .		194
Astronomy . . . . .		195-196

## XV. THE SUCCESSORS OF THE GREAT GEOMETERS . 197-234

Nicomedes . . . . .	199
Diocles . . . . .	200-203
Perseus . . . . .	203-206
Isoperimetric figures. Zenodorus . . . . .	206-213
Hypsicles . . . . .	213-218
Dionysodorus . . . . .	218-219
Posidonius . . . . .	219-222

## CONTENTS

vii

Geminus . . . . .	PAGES 222–234
Attempt to prove the Parallel-Postulate . . . . .	227–230
On <i>Meteorologica</i> of Posidonius . . . . .	231–232
<i>Introduction to the Phaenomena</i> attributed to Geminus . . . . .	232–234
 XVI. SOME HANDBOOKS . . . . .	235–244
Cleomedes, <i>De motu circulari</i> . . . . .	235–238
Nicomachus . . . . .	238
Theon of Smyrna, <i>Expositio rerum mathematicarum ad legendum Platonem utilium</i> . . . . .	238–244
 XVII. TRIGONOMETRY: HIPPARCHUS, MENELAUS, PTO- LEMY . . . . .	245–297
Theodosius . . . . .	245–246
Works by Theodosius . . . . .	246
Contents of the <i>Sphaerica</i> . . . . .	246–252
No actual trigonometry in Theodosius . . . . .	250–252
The beginnings of trigonometry . . . . .	252–253
Hipparchus . . . . .	253–260
The work of Hipparchus . . . . .	254–256
First systematic use of trigonometry . . . . .	257–259
Table of chords . . . . .	259–260
Menelaus . . . . .	260–273
The <i>Sphaerica</i> of Menelaus . . . . .	261–273
(a) ‘Menelaus’s theorem’ for the sphere . . . . .	266–268
(β) Deductions from Menelaus’s theorem . . . . .	268–269
(γ) Anharmonic property of four great circles through one point . . . . .	269–270
(δ) Propositions analogous to Eucl. VI. 3 . . . . .	270
Claudius Ptolemy . . . . .	273–297
The <i>Μαθηματικὴ σύνταξις</i> (Arab. <i>Almagest</i> ) . . . . .	273–286
Commentaries . . . . .	274
Translations and editions . . . . .	274–275
Summary of contents . . . . .	275–276
Trigonometry in Ptolemy . . . . .	276–286
(a) Lemma for finding $\sin 18^\circ$ and $\sin 36^\circ$ . . . . .	277–278
(β) Equivalent of $\sin^2 \theta + \cos^2 \theta = 1$ . . . . .	278
(γ) ‘Ptolemy’s theorem’, giving the equivalent of $\sin(\theta - \phi) = \sin \theta \cos \phi - \cos \theta \sin \phi$ . . . . .	278–280
(δ) Equivalent of $\sin^2 \frac{1}{2} \theta = \frac{1}{2}(1 - \cos \theta)$ . . . . .	280–281
(ε) Equivalent of $\cos(\theta + \phi) = \cos \theta \cos \phi - \sin \theta \sin \phi$ . . . . .	281
(ζ) Method of interpolation based on formula $\sin a / \sin \beta < a / \beta$ ( $\frac{1}{2}\pi > a > \beta$ ) . . . . .	281–282
(η) Table of chords . . . . .	283
(θ) Further use of proportional increase . . . . .	283–284
(ι) Plane trigonometry in effect used . . . . .	284
Spherical trigonometry: formulae in solution of spherical triangles . . . . .	284–286
The <i>Analemma</i> . . . . .	286–292
The <i>Planisphaerium</i> . . . . .	292–293
The <i>Optics</i> . . . . .	293–295
A mechanical work, <i>Περὶ ῥοπῶν</i> . . . . .	295
Attempt to prove the Parallel-Postulate . . . . .	295–297

## XVIII. MENSURATION: HERON OF ALEXANDRIA. PAGES 298-354

Controversies as to Heron's date . . . . .	298-306
Character of works . . . . .	307-308
List of treatises . . . . .	308-310
Geometry	
(a) Commentary on Euclid's <i>Elements</i> . . . . .	310-314
(β) The <i>Definitions</i> . . . . .	314-316
Mensuration . . . . .	316-344
The <i>Metrica</i> , <i>Geometrica</i> , <i>Stereometrica</i> , <i>Geodaesia</i> , <i>Mensurae</i> . . . . .	316-320
Contents of the <i>Metrica</i> . . . . .	320-344
Book I. Measurement of areas . . . . .	320-331
(a) Area of scalene triangle . . . . .	320-321
Proof of formula $\Delta = \sqrt{\{s(s-a)(s-b)(s-c)\}}$ . . . . .	321-323
(β) Method of approximating to the square root of a non-square number . . . . .	323-326
(γ) Quadrilaterals . . . . .	326
(δ) Regular polygons with 3, 4, 5, 6, 7, 8, 9, 10, 11, or 12 sides . . . . .	326-329
(ε) The circle . . . . .	329
(ζ) Segment of a circle . . . . .	330-331
(η) Ellipse, parabolic segment, surface of cylinder, right cone, sphere and segment of sphere . . . . .	331
Book II. Measurement of volumes . . . . .	331-335
(a) Cone, cylinder, parallelepiped (prism), pyramid and frustum . . . . .	332
(β) Wedge-shaped solid ( <i>βαμίσκος</i> or <i>σφηνίσκος</i> ) . . . . .	332-334
(γ) Frustum of cone, sphere, and segment thereof . . . . .	334
(δ) Anchor-ring or tore . . . . .	334-335
(ε) The two special solids of Archimedes's 'Method' . . . . .	335
(ζ) The five regular solids . . . . .	335
Book III. Divisions of figures . . . . .	336-343
Approximation to the cube root of a non-cube number . . . . .	341-342
Quadratic equations solved in Heron . . . . .	344
Indeterminate problems in the <i>Geometrica</i> . . . . .	344
The <i>Dioptra</i> . . . . .	345-346
The <i>Mechanics</i> . . . . .	346-352
Aristotle's Wheel . . . . .	347-348
The parallelogram of velocities . . . . .	348-349
Motion on an inclined plane . . . . .	349-350
On the centre of gravity . . . . .	350-351
The five mechanical powers . . . . .	351
Mechanics in daily life: queries and answers . . . . .	351-352
Problems on the centre of gravity, &c. . . . .	352
The <i>Catoptrica</i> . . . . .	352-354
Heron's proof of equality of angles of incidence and reflection . . . . .	353-354

## XIX. PAPPUS OF ALEXANDRIA. . . . . 355-439

Date of Pappus . . . . .	356
Works (commentaries) other than the <i>Collection</i> . . . . .	356-357

## CONTENTS

ix

	PAGES
The <i>Synagoge</i> or <i>Collection</i> . . . . .	357–439
(a) Character of the work; wide range . . . . .	357–358
(β) List of authors mentioned . . . . .	358–360
(γ) Translations and editions . . . . .	360–361
(δ) Summary of contents . . . . .	361–439
Book III. Section (1). On the problem of the two mean proportionals . . . . .	361–362
Section (2). The theory of means . . . . .	363–365
Section (3). The ‘Paradoxes’ of Erycinus . . . . .	365–368
Section (4). The inscribing of the five regular solids in a sphere . . . . .	368–369
Book IV. Section (1). Extension of theorem of Pythagoras . . . . .	369–371
Section (2). On circles inscribed in the <i>ἀρβηλος</i> (‘shoemaker’s knife’) . . . . .	371–377
Sections (3), (4). Methods of squaring the circle and trisecting any angle . . . . .	377–386
(a) The Archimedean spiral . . . . .	377–379
(β) The conchoid of Nicomedes . . . . .	379
(γ) The <i>Quadratrix</i> . . . . .	379–382
(δ) Digression: a spiral on a sphere . . . . .	382–385
Trisection (or division in any ratio) of any angle Section (5). Solution of the <i>νέυσις</i> of Archimedes, <i>On Spirals</i> , Prop. 8, by means of conics . . . . .	385–386 386–388
Book V. Preface on the sagacity of Bees . . . . .	389–390
Section (1). Isoperimetry after Zenodorus . . . . .	390–393
Section (2). Comparison of volumes of solids having their surfaces equal. Case of sphere . . . . .	393–394
Section (3). Digression on semi-regular solids of Archimedes . . . . .	394
Section (4). Propositions on the lines of Archimedes, <i>On the Sphere and Cylinder</i> . . . . .	394–395
Section (5). Of regular solids with surfaces equal, that is greater which has more faces . . . . .	395–396
Book VI. . . . .	396–399
Problem arising out of Euclid’s <i>Optics</i> . . . . .	397–399
Book VII. On the ‘Treasury of Analysis’ . . . . .	399–427
Definition of Analysis and Synthesis . . . . .	400–401
List of works in the ‘Treasury of Analysis’ . . . . .	401
Description of the treatises . . . . .	401–404
Anticipation of Guldin’s Theorem . . . . .	403
Lemmas to the different treatises . . . . .	404–426
(a) Lemmas to the <i>Sectio rationis</i> and <i>Sectio</i> <i>spatii</i> of Apollonius . . . . .	404–405
(β) Lemmas to the <i>Determinate Section</i> of Apollonius . . . . .	405–412
(γ) Lemmas on the <i>Νέυσις</i> of Apollonius . . . . .	412–416
(δ) Lemmas on the <i>On Contacts</i> of Apollonius . . . . .	416–417
(ε) Lemmas to the <i>Plane Loci</i> of Apollonius . . . . .	417–419
(ζ) Lemmas to the <i>Porisms</i> of Euclid . . . . .	419–424
(η) Lemmas to the <i>Conics</i> of Apollonius . . . . .	424–425
(θ) Lemmas to the <i>Surface Loci</i> of Euclid . . . . .	425–426
(i) An unallocated lemma . . . . .	426–427
Book VIII. Historical preface . . . . .	427–429
The object of the Book . . . . .	429–430
On the centre of gravity . . . . .	430–433

## XIX. CONTINUED.

Book VIII (*continued*)

The inclined plane . . . . .	PAGES 433-434
Construction of a conic through five points . . . . .	434-437
Given two conjugate diameters of an ellipse, to find the axes . . . . .	437-438
Problem of seven hexagons in a circle . . . . .	438-439
Construction of toothed wheels and indented screws	439

## XX. ALGEBRA: DIOPHANTUS OF ALEXANDRIA . . . . . 440-517

Beginnings learnt from Egypt . . . . .	440
'Hau'-calculations . . . . .	440-441
Arithmetical epigrams in the Greek Anthology . . . . .	441-443
Indeterminate equations of first degree . . . . .	443
Indeterminate equations of second degree before Dio- phantus . . . . .	443-444
Indeterminate equations in Heronian collections . . . . .	444-447
Numerical solution of quadratic equations . . . . .	448
Works of Diophantus . . . . .	448-450
The <i>Arithmetica</i> . . . . .	449-514
The seven lost Books and their place . . . . .	449-450
Relation of 'Porisms' to <i>Arithmetica</i> . . . . .	451-452
Commentators from Hypatia downwards . . . . .	453
Translations and editions . . . . .	453-455
Notation and definitions . . . . .	455-461
Sign for unknown ( $= x$ ) and its origin . . . . .	456-457
Signs for powers of unknown &c. . . . .	458-459
The sign ( $\Lambda$ ) for <i>minus</i> and its meaning . . . . .	459-460
The methods of Diophantus . . . . .	462-479
I. Diophantus's treatment of equations . . . . .	462-476
(A) Determinate equations . . . . .	
(1) Pure determinate equations . . . . .	462-463
(2) Mixed quadratic equations . . . . .	463-465
(3) Simultaneous equations involving quadratics . . . . .	465
(4) Cubic equation . . . . .	465
(B) Indeterminate equations . . . . .	
(a) Indeterminate equations of the second degree . . . . .	466-473
(1) Single equation . . . . .	466-468
(2) Double equation . . . . .	468-473
1. Double equations of first degree . . . . .	469-472
2. Double equations of second degree . . . . .	472-473
(b) Indeterminate equations of degree higher than second . . . . .	473-476
(1) Single equations . . . . .	473-475
(2) Double equations . . . . .	475-476
II. Method of limits . . . . .	476-477
III. Method of approximation to limits . . . . .	477-479
Porisms and propositions in the Theory of Numbers . . . . .	479-484
(a) Theorems on the composition of numbers as the sum of two squares . . . . .	481-483
(b) On numbers which are the sum of three squares . . . . .	483
(c) Composition of numbers as the sum of four squares . . . . .	483-484
Conspectus of <i>Arithmetica</i> , with typical solutions . . . . .	484-514
The treatise on Polygonal Numbers . . . . .	514-517

## CONTENTS

xi

XXI. COMMENTATORS AND BYZANTINES . . . . .	PAGES 518-555
Serenus . . . . .	519-526
(a) <i>On the Section of a Cylinder</i> . . . . .	519-522
(b) <i>On the Section of a Cone</i> . . . . .	522-526
Theon of Alexandria . . . . .	526-528
Commentary on the <i>Syntaxis</i> . . . . .	526-527
Edition of Euclid's <i>Elements</i> . . . . .	527-528
Edition of the <i>Optics</i> of Euclid . . . . .	528
Hypatia . . . . .	528-529
Porphyry. Iamblichus . . . . .	529
Proclus . . . . .	529-537
Commentary on Euclid, Book I . . . . .	530-535
(a) Sources of the Commentary . . . . .	530-532
(b) Character of the Commentary . . . . .	532-535
<i>Hypotyposis of Astronomical Hypotheses</i> . . . . .	535-536
Commentary on the <i>Republic</i> . . . . .	536-537
Marinus of Neapolis . . . . .	537-538
Domninus of Larissa . . . . .	538
Simplicius . . . . .	538-540
Extracts from Eudemos . . . . .	539
Eutocius . . . . .	540-541
Anthemius of Tralles . . . . .	541-543
<i>On burning-mirrors</i> . . . . .	541-543
The Papyrus of Akhmim . . . . .	543-545
<i>Geodesia</i> of 'Heron the Younger' . . . . .	545
Michael Psellus . . . . .	545-546
Georgius Pachymeres . . . . .	546
Maximus Planudes . . . . .	546-549
Extraction of the square root . . . . .	547-549
Two problems . . . . .	549
Manuel Moschopoulos . . . . .	549-550
Nicolas Rhabdas . . . . .	550-554
Rule for approximating to square root of a non-square number . . . . .	553-554
Ioannes Pediasimus . . . . .	554
Barlaam . . . . .	554-555
Isaac Argyrus . . . . .	555
APPENDIX. On Archimedes's proof of the subtangent-property of a spiral . . . . .	556-561
INDEX OF GREEK WORDS . . . . .	563-569
ENGLISH INDEX . . . . .	570-586