

Cambridge University Press

978-1-108-06307-4 - A History of Greek Mathematics: Volume 2

T. L. Heath

Frontmatter

[More information](#)

CAMBRIDGE LIBRARY COLLECTION

Books of enduring scholarly value

Mathematics

From its pre-historic roots in simple counting to the algorithms powering modern desktop computers, from the genius of Archimedes to the genius of Einstein, advances in mathematical understanding and numerical techniques have been directly responsible for creating the modern world as we know it. This series will provide a library of the most influential publications and writers on mathematics in its broadest sense. As such, it will show not only the deep roots from which modern science and technology have grown, but also the astonishing breadth of application of mathematical techniques in the humanities and social sciences, and in everyday life.

A History of Greek Mathematics

'If one would understand the Greek genius fully, it would be a good plan to begin with their geometry.' As early as the sixth century BCE, Thales of Miletus used geometrical principles to calculate distance and height. Within a few hundred years, Euclid had produced his seminal *Elements*, which was still used as a textbook when this two-volume work was first published in 1921. A distinguished civil servant as well as an expert on ancient Greek mathematics, Sir Thomas Little Heath (1861–1940) includes here sufficient detail for a modern mathematician to grasp ancient methodology, alongside explanatory sections aimed at classicists. This remains a rigorous and essential exposition of a vast topic. Volume 2 focuses on post-Euclidian mathematics, beginning with the work of Aristarchus of Samos and extending to that of Diophantus of Alexandria. Heath had previously published separate studies on these two thinkers (also reissued in this series).

Cambridge University Press

978-1-108-06307-4 - A History of Greek Mathematics: Volume 2

T. L. Heath

Frontmatter

[More information](#)

Cambridge University Press has long been a pioneer in the reissuing of out-of-print titles from its own backlist, producing digital reprints of books that are still sought after by scholars and students but could not be reprinted economically using traditional technology. The Cambridge Library Collection extends this activity to a wider range of books which are still of importance to researchers and professionals, either for the source material they contain, or as landmarks in the history of their academic discipline.

Drawing from the world-renowned collections in the Cambridge University Library and other partner libraries, and guided by the advice of experts in each subject area, Cambridge University Press is using state-of-the-art scanning machines in its own Printing House to capture the content of each book selected for inclusion. The files are processed to give a consistently clear, crisp image, and the books finished to the high quality standard for which the Press is recognised around the world. The latest print-on-demand technology ensures that the books will remain available indefinitely, and that orders for single or multiple copies can quickly be supplied.

The Cambridge Library Collection brings back to life books of enduring scholarly value (including out-of-copyright works originally issued by other publishers) across a wide range of disciplines in the humanities and social sciences and in science and technology.

Cambridge University Press

978-1-108-06307-4 - A History of Greek Mathematics: Volume 2

T. L. Heath

Frontmatter

[More information](#)

A History of Greek Mathematics

VOLUME 2

T.L. HEATH



CAMBRIDGE
UNIVERSITY PRESS

Cambridge University Press
978-1-108-06307-4 - A History of Greek Mathematics: Volume 2
T. L. Heath
Frontmatter
[More information](#)

CAMBRIDGE
UNIVERSITY PRESS

University Printing House, Cambridge, CB2 8BS, United Kingdom

Published in the United States of America by Cambridge University Press, New York

Cambridge University Press is part of the University of Cambridge.
It furthers the University's mission by disseminating knowledge in the pursuit of
education, learning and research at the highest international levels of excellence.

www.cambridge.org

Information on this title: www.cambridge.org/9781108063074

© in this compilation Cambridge University Press 2014

This edition first published 1921
This digitally printed version 2014

ISBN 978-1-108-06307-4 Paperback

This book reproduces the text of the original edition. The content and language reflect
the beliefs, practices and terminology of their time, and have not been updated.

Cambridge University Press wishes to make clear that the book, unless originally published
by Cambridge, is not being republished by, in association or collaboration with, or
with the endorsement or approval of, the original publisher or its successors in title.

Cambridge University Press

978-1-108-06307-4 - A History of Greek Mathematics: Volume 2

T. L. Heath

Frontmatter

[More information](#)

A HISTORY
OF
GREEK MATHEMATICS
VOLUME II

Cambridge University Press

978-1-108-06307-4 - A History of Greek Mathematics: Volume 2

T. L. Heath

Frontmatter

[More information](#)

Cambridge University Press

978-1-108-06307-4 - A History of Greek Mathematics: Volume 2

T. L. Heath

Frontmatter

[More information](#)

A HISTORY
OF
GREEK MATHEMATICS

BY

SIR THOMAS HEATH

K.C.B., K.C.V.O., F.R.S.

SC.D. CAMB. ; HON. D.SC. OXFORD

HONORARY FELLOW (FORMERLY FELLOW) OF TRINITY COLLEGE, CAMBRIDGE

. . . An independent world,
Created out of pure intelligence.'
WORDSWORTH.

VOLUME II

FROM ARISTARCHUS TO DIOPHANTUS

O X F O R D
AT THE CLARENDON PRESS

1921

Cambridge University Press

978-1-108-06307-4 - A History of Greek Mathematics: Volume 2

T. L. Heath

Frontmatter

[More information](#)

OXFORD UNIVERSITY PRESS

London Edinburgh Glasgow Copenhagen

New York Toronto Melbourne Cape Town

Bombay Calcutta Madras Shanghai

HUMPHREY MILFORD

Publisher to the University

CONTENTS OF VOL II

| | |
|---|------------|
| XII. ARISTARCHUS OF SAMOS | PAGES 1-15 |
| XIII. ARCHIMEDES | 16-109 |
| Traditions | |
| (a) Astronomy | 17-18 |
| (s) Mechanics | 18 |
| Summary of main achievements | 19-20 |
| Character of treatises | 20-22 |
| List of works still extant | 22-23 |
| Traces of lost works | 23-25 |
| The text of Archimedes | 25-27 |
| Contents of <i>The Method</i> | 27-34 |
| <i>On the Sphere and Cylinder</i> , I, II | 34-50 |
| Cubic equation arising out of II. 4 | 43-46 |
| (i) Archimedes's own solution | 45-46 |
| (ii) Dionysodorus's solution | 46 |
| (iii) Diocles's solution of original problem | 47-49 |
| <i>Measurement of a Circle</i> | 50-56 |
| <i>On Conoids and Spheroids</i> | 56-64 |
| <i>On Spirals</i> | 64-75 |
| <i>On Plane Equilibriums</i> , I, II | 75-81 |
| <i>The Sand-reckoner (Psammites or Arenarius)</i> | 81-85 |
| <i>The Quadrature of the Parabola</i> | 85-91 |
| <i>On Floating Bodies</i> , I, II | 91-97 |
| The problem of the crown | 92-94 |
| Other works | |
| (a) The Cattle-Problem | 97-98 |
| (β) On semi regular polyhedra | 98-101 |
| (γ) <i>The Liber Assumptorum</i> | 101-103 |
| (δ) Formula for area of triangle | 103 |
| Eratosthenes | 104-109 |
| Measurement of the Earth | 106-108 |
| XIV. CONIC SECTIONS. APOLLONIUS OF PERGA | 110-196 |
| A. HISTORY OF CONICS UP TO APOLLONIUS | 110-126 |
| Discovery of the conic sections by Menaechmus | 110-111 |
| Menaechmus's probable procedure | 111-116 |
| Works by Aristaeus and Euclid | 116-117 |
| 'Solid loci' and 'solid problems' | 117-118 |
| Aristaeus's <i>Solid Loci</i> | 118-119 |
| Focus-directrix property known to Euclid | 119 |
| Proof from Pappus | 120-121 |
| Propositions included in Euclid's <i>Conics</i> | 121-122 |
| Conic sections in Archimedes | 122-126 |

XIV. CONTINUED.

| B APOLLONIUS OF PERGA | PAGES | 126-196 |
|---|-------|---------|
| The text of the <i>Conics</i> | | 126-128 |
| Apollonius's own account of the <i>Conics</i> | | 128-133 |
| Extent of claim to originality | | 132-133 |
| Great generality of treatment | | 133 |
| Analysis of the <i>Conics</i> | | 133-175 |
| Book I | | 133-148 |
| Conics obtained in the most general way from oblique cone | | 134-138 |
| New names, 'parabola', 'ellipse', 'hyperbola' | | 138-139 |
| Fundamental properties equivalent to Cartesian equations | | 139-141 |
| Transition to new diameter and tangent at its extremity | | 141-147 |
| First appearance of principal axes | | 147-148 |
| Book II | | 148-150 |
| Book III | | 150-157 |
| Book IV | | 157-158 |
| Book V | | 158-167 |
| Normals as maxima and minima | | 159-163 |
| Number of normals from a point | | 163-164 |
| Propositions leading immediately to determination of <i>evolute</i> of conic | | 164-166 |
| Construction of normals | | 166-167 |
| Book VI | | 167-168 |
| Book VII | | 168-174 |
| Other works by Apollonius | | 175-194 |
| (a) <i>On the Cutting off of a Ratio</i> (λόγου ἀποτομή), two Books | | 175-179 |
| (β) <i>On the Cutting-off of an Area</i> (χωρίου ἀποτομή), two Books | | 179-180 |
| (γ) <i>On Determinate Section</i> (διωρισμένη τομή), two Books | | 180-181 |
| (δ) <i>On Contacts or Tangencies</i> (ἐπαφαί), two Books | | 181-185 |
| (ε) <i>Plane Loci</i> , two Books | | 185-189 |
| (ζ) <i>Νεύσεις</i> (<i>Vergings or Inclinations</i>), two Books | | 189-192 |
| (η) <i>Comparison of dodecahedron with icosahedron</i> | | 192 |
| (θ) <i>General Treatise</i> | | 192-193 |
| (ι) <i>On the Cochlias</i> | | 193 |
| (κ) <i>On Unordered Irrationals</i> | | 193 |
| (λ) <i>On the Burning-mirror</i> | | 194 |
| (μ) Ὀκυτόκιον | | 194 |
| Astronomy | | 195-196 |

XV. THE SUCCESSORS OF THE GREAT GEOMETERS . 197-234

| | |
|--|---------|
| Nicomedes | 199 |
| Diocles | 200-203 |
| Perseus | 203-206 |
| Isoperimetric figures. Zenodorus | 206-213 |
| Hypsicles | 213-218 |
| Dionysodorus | 218-219 |
| Posidonius | 219-222 |

CONTENTS

vii

| | |
|--|---------------|
| Geminus | PAGES 222–234 |
| Attempt to prove the Parallel-Postulate | 227–230 |
| On <i>Meteorologica</i> of Posidonius | 231–232 |
| <i>Introduction to the Phaenomena</i> attributed to Geminus | 232–234 |
| XVI. SOME HANDBOOKS | 235–244 |
| Cleomedes, <i>De motu circulari</i> | 235–238 |
| Nicomachus | 238 |
| Theon of Smyrna, <i>Expositio rerum mathematicarum ad legendum Platonem utilium</i> | 238–244 |
| XVII. TRIGONOMETRY: HIPPARCHUS, MENELAUS, PTO- LEMY | 245–297 |
| Theodosius | 245–246 |
| Works by Theodosius | 246 |
| Contents of the <i>Sphaerica</i> | 246–252 |
| No actual trigonometry in Theodosius | 250–252 |
| The beginnings of trigonometry | 252–253 |
| Hipparchus | 253–260 |
| The work of Hipparchus | 254–256 |
| First systematic use of trigonometry | 257–259 |
| Table of chords | 259–260 |
| Menelaus | 260–273 |
| The <i>Sphaerica</i> of Menelaus | 261–273 |
| (α) ‘Menelaus’s theorem’ for the sphere | 266–268 |
| (β) Deductions from Menelaus’s theorem | 268–269 |
| (γ) Anharmonic property of four great circles through one point | 269–270 |
| (δ) Propositions analogous to Eucl. VI. 3 | 270 |
| Claudius Ptolemy | 273–297 |
| The <i>Μαθηματικὴ σύνταξις</i> (Arab. <i>Almagest</i>) | 273–286 |
| Commentaries | 274 |
| Translations and editions | 274–275 |
| Summary of contents | 275–276 |
| Trigonometry in Ptolemy | 276–286 |
| (α) Lemma for finding $\sin 18^\circ$ and $\sin 36^\circ$ | 277–278 |
| (β) Equivalent of $\sin^2 \theta + \cos^2 \theta = 1$ | 278 |
| (γ) ‘Ptolemy’s theorem’, giving the equivalent of $\sin(\theta - \phi) = \sin \theta \cos \phi - \cos \theta \sin \phi$ | 278–280 |
| (δ) Equivalent of $\sin^2 \frac{1}{2} \theta = \frac{1}{2} (1 - \cos \theta)$ | 280–281 |
| (ε) Equivalent of $\cos(\theta + \phi) = \cos \theta \cos \phi - \sin \theta \sin \phi$ | 281 |
| (ζ) Method of interpolation based on formula $\sin a / \sin \beta < a / \beta$ ($\frac{1}{2} \pi > a > \beta$) | 281–282 |
| (η) Table of chords | 283 |
| (θ) Further use of proportional increase | 283–284 |
| (ι) Plane trigonometry in effect used | 284 |
| Spherical trigonometry: formulae in solution of spherical triangles | 284–286 |
| The <i>Analemma</i> | 286–292 |
| The <i>Planisphaerium</i> | 292–293 |
| The <i>Optics</i> | 293–295 |
| A mechanical work, <i>Περὶ ῥοπῶν</i> | 295 |
| Attempt to prove the Parallel-Postulate | 295–297 |

XVIII. MENSURATION: HERON OF ALEXANDRIA. PAGES 298-354

| | |
|---|---------|
| Controversies as to Heron's date | 298-306 |
| Character of works | 307-308 |
| List of treatises | 308-310 |
| Geometry | |
| (a) Commentary on Euclid's <i>Elements</i> | 310-314 |
| (β) The <i>Definitions</i> | 314-316 |
| Mensuration | 316-344 |
| The <i>Metrica</i> , <i>Geometrica</i> , <i>Stereometrica</i> , <i>Geodaesia</i> , <i>Mensurae</i> | 316-320 |
| Contents of the <i>Metrica</i> | 320-344 |
| Book I. Measurement of areas | 320-331 |
| (a) Area of scalene triangle | 320-321 |
| Proof of formula $\Delta = \sqrt{\{s(s-a)(s-b)(s-c)\}}$ | 321-323 |
| (β) Method of approximating to the square root of a non-square number | 323-326 |
| (γ) Quadrilaterals | 326 |
| (δ) Regular polygons with 3, 4, 5, 6, 7, 8, 9, 10, 11, or 12 sides | 326-329 |
| (ε) The circle | 329 |
| (ζ) Segment of a circle | 330-331 |
| (η) Ellipse, parabolic segment, surface of cylinder, right cone, sphere and segment of sphere | 331 |
| Book II. Measurement of volumes | 331-335 |
| (a) Cone, cylinder, parallelepiped (prism), pyramid and frustum | 332 |
| (β) Wedge-shaped solid (<i>βαμίσκος</i> or <i>σφηνίσκος</i>) | 332-334 |
| (γ) Frustum of cone, sphere, and segment thereof | 334 |
| (δ) Anchor-ring or tore | 334-335 |
| (ε) The two special solids of Archimedes's 'Method' | 335 |
| (ζ) The five regular solids | 335 |
| Book III. Divisions of figures | 336-343 |
| Approximation to the cube root of a non-cube number | 341-342 |
| Quadratic equations solved in Heron | 344 |
| Indeterminate problems in the <i>Geometrica</i> | 344 |
| The <i>Dioptra</i> | 345-346 |
| The <i>Mechanics</i> | 346-352 |
| Aristotle's Wheel | 347-348 |
| The parallelogram of velocities | 348-349 |
| Motion on an inclined plane | 349-350 |
| On the centre of gravity | 350-351 |
| The five mechanical powers | 351 |
| Mechanics in daily life: queries and answers | 351-352 |
| Problems on the centre of gravity, &c. | 352 |
| The <i>Catoptrica</i> | 352-354 |
| Heron's proof of equality of angles of incidence and reflection | 353-354 |

XIX. PAPPUS OF ALEXANDRIA. 355-439

| | |
|---|---------|
| Date of Pappus | 356 |
| Works (commentaries) other than the <i>Collection</i> | 356-357 |

CONTENTS

ix

| | PAGES |
|--|--------------------|
| The <i>Synagoge</i> or <i>Collection</i> | 357–439 |
| (a) Character of the work; wide range | 357–358 |
| (β) List of authors mentioned | 358–360 |
| (γ) Translations and editions | 360–361 |
| (δ) Summary of contents | 361–439 |
| Book III. Section (1). On the problem of the two mean proportionals | 361–362 |
| Section (2). The theory of means | 363–365 |
| Section (3). The ‘Paradoxes’ of Erycinus | 365–368 |
| Section (4). The inscribing of the five regular solids in a sphere | 368–369 |
| Book IV. Section (1). Extension of theorem of Pythagoras | 369–371 |
| Section (2). On circles inscribed in the <i>ἀρβηλος</i> (‘shoemaker’s knife’) | 371–377 |
| Sections (3), (4). Methods of squaring the circle and trisecting any angle | 377–386 |
| (a) The Archimedean spiral | 377–379 |
| (β) The conchoid of Nicomedes | 379 |
| (γ) The <i>Quadratrix</i> | 379–382 |
| (δ) Digression: a spiral on a sphere | 382–385 |
| Trisection (or division in any ratio) of any angle Section (5). Solution of the <i>νέυσις</i> of Archimedes, <i>On Spirals</i> , Prop. 8, by means of conics | 385–386 386–388 |
| Book V. Preface on the sagacity of Bees | 389–390 |
| Section (1). Isoperimetry after Zenodorus | 390–393 |
| Section (2). Comparison of volumes of solids having their surfaces equal. Case of sphere | 393–394 |
| Section (3). Digression on semi-regular solids of Archimedes | 394 |
| Section (4). Propositions on the lines of Archimedes, <i>On the Sphere and Cylinder</i> | 394–395 |
| Section (5). Of regular solids with surfaces equal, that is greater which has more faces | 395–396 |
| Book VI. | 396–399 |
| Problem arising out of Euclid’s <i>Optics</i> | 397–399 |
| Book VII. On the ‘Treasury of Analysis’ | 399–427 |
| Definition of Analysis and Synthesis | 400–401 |
| List of works in the ‘Treasury of Analysis’ | 401 |
| Description of the treatises | 401–404 |
| Anticipation of Guldin’s Theorem | 403 |
| Lemmas to the different treatises | 404–426 |
| (a) Lemmas to the <i>Sectio rationis</i> and <i>Sectio</i> <i>spatii</i> of Apollonius | 404–405 |
| (β) Lemmas to the <i>Determinate Section</i> of Apollonius | 405–412 |
| (γ) Lemmas on the <i>Νέυσις</i> of Apollonius | 412–416 |
| (δ) Lemmas on the <i>On Contacts</i> of Apollonius | 416–417 |
| (ε) Lemmas to the <i>Plane Loci</i> of Apollonius | 417–419 |
| (ζ) Lemmas to the <i>Porisms</i> of Euclid | 419–424 |
| (η) Lemmas to the <i>Conics</i> of Apollonius | 424–425 |
| (θ) Lemmas to the <i>Surface Loci</i> of Euclid | 425–426 |
| (i) An unallocated lemma | 426–427 |
| Book VIII. Historical preface | 427–429 |
| The object of the Book | 429–430 |
| On the centre of gravity | 430–433 |

XIX. CONTINUED.

Book VIII (*continued*)

| | | |
|--|-------|---------|
| The inclined plane | PAGES | 433-434 |
| Construction of a conic through five points | | 434-437 |
| Given two conjugate diameters of an ellipse, to find the axes | | 437-438 |
| Problem of seven hexagons in a circle | | 438-439 |
| Construction of toothed wheels and indented screws | | 439 |

XX. ALGEBRA: DIOPHANTUS OF ALEXANDRIA 440-517

| | |
|---|---------|
| Beginnings learnt from Egypt | 440 |
| 'Hau'-calculations | 440-441 |
| Arithmetical epigrams in the Greek Anthology | 441-443 |
| Indeterminate equations of first degree | 443 |
| Indeterminate equations of second degree before Dio- phantus | 443-444 |
| Indeterminate equations in Heronian collections | 444-447 |
| Numerical solution of quadratic equations | 448 |
| Works of Diophantus | 448-450 |
| The <i>Arithmetica</i> | 449-514 |
| The seven lost Books and their place | 449-450 |
| Relation of 'Porisms' to <i>Arithmetica</i> | 451-452 |
| Commentators from Hypatia downwards | 453 |
| Translations and editions | 453-455 |
| Notation and definitions | 455-461 |
| Sign for unknown ($= x$) and its origin | 456-457 |
| Signs for powers of unknown &c. | 458-459 |
| The sign (Λ) for <i>minus</i> and its meaning | 459-460 |
| The methods of Diophantus | 462-479 |
| I. Diophantus's treatment of equations | 462-476 |
| (A) Determinate equations | |
| (1) Pure determinate equations | 462-463 |
| (2) Mixed quadratic equations | 463-465 |
| (3) Simultaneous equations involving quadratics | 465 |
| (4) Cubic equation | 465 |
| (B) Indeterminate equations | |
| (a) Indeterminate equations of the second degree | 466-473 |
| (1) Single equation | 466-468 |
| (2) Double equation | 468-473 |
| 1. Double equations of first degree | 469-472 |
| 2. Double equations of second degree | 472-473 |
| (b) Indeterminate equations of degree higher than second | 473-476 |
| (1) Single equations | 473-475 |
| (2) Double equations | 475-476 |
| II. Method of limits | 476-477 |
| III. Method of approximation to limits | 477-479 |
| Porisms and propositions in the Theory of Numbers | 479-484 |
| (a) Theorems on the composition of numbers as the sum of two squares | 481-483 |
| (b) On numbers which are the sum of three squares | 483 |
| (c) Composition of numbers as the sum of four squares | 483-484 |
| Conspectus of <i>Arithmetica</i> , with typical solutions | 484-514 |
| The treatise on Polygonal Numbers | 514-517 |

CONTENTS

xi

| | |
|---|---------------|
| XXI. COMMENTATORS AND BYZANTINES | PAGES 518-555 |
| Serenus | 519-526 |
| (a) <i>On the Section of a Cylinder</i> | 519-522 |
| (b) <i>On the Section of a Cone</i> | 522-526 |
| Theon of Alexandria | 526-528 |
| Commentary on the <i>Syntaxis</i> | 526-527 |
| Edition of Euclid's <i>Elements</i> | 527-528 |
| Edition of the <i>Optics</i> of Euclid | 528 |
| Hypatia | 528-529 |
| Porphyry. Iamblichus | 529 |
| Proclus | 529-537 |
| Commentary on Euclid, Book I | 530-535 |
| (a) Sources of the Commentary | 530-532 |
| (b) Character of the Commentary | 532-535 |
| <i>Hypotyposis of Astronomical Hypotheses</i> | 535-536 |
| Commentary on the <i>Republic</i> | 536-537 |
| Marinus of Neapolis | 537-538 |
| Dominus of Larissa | 538 |
| Simplicius | 538-540 |
| Extracts from Eudemus | 539 |
| Eutocius | 540-541 |
| Anthemius of Tralles | 541-543 |
| <i>On burning-mirrors</i> | 541-543 |
| The Papyrus of Akhmim | 543-545 |
| <i>Geodesia</i> of 'Heron the Younger' | 545 |
| Michael Psellus | 545-546 |
| Georgius Pachymeres | 546 |
| Maximus Planudes | 546-549 |
| Extraction of the square root | 547-549 |
| Two problems | 549 |
| Manuel Moschopoulos | 549-550 |
| Nicolas Rhabdas | 550-554 |
| Rule for approximating to square root of a non-square number | 553-554 |
| Ioannes Pediasimus | 554 |
| Barlaam | 554-555 |
| Isaac Argyrus | 555 |
| APPENDIX. On Archimedes's proof of the subtangent-property of a spiral | 556-561 |
| INDEX OF GREEK WORDS | 563-569 |
| ENGLISH INDEX | 570-586 |