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## ERRATA

- Vol. i, p. 120, line 7, *for* 'Laodamas' *read* 'Leodamas'.
- Vol. i, p. 161, line 5 from foot, *for* 'pentagon' *read* 'pentagram'.
- Vol. i, p. 290, line 9 from foot, *for* 'ideals' *read* 'ideas'.
- Vol. ii, p. 324, note 2, line 12, *for* '1853' *read* '1851'.
- Vol. ii, p. 360, line 8 from foot, *for* 'Breton le Champ' *read* 'Breton de Champ'.