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Abraham de Moivre  
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The Doctrine of Chances

A Huguenot exile in England, the French mathematician Abraham de Moivre (1667–1754) formed friendships with such luminaries as Edmond Halley and Isaac Newton. Making his living from private tuition, he became a fellow of the Royal Society in 1697 and published papers on a range of topics. Probability theory had been pioneered by Pascal, Fermat and Huygens, with further development by the Bernoullis. Originally published in 1718, *The Doctrine of Chances* was the first English textbook on the new science and so influential that for a time the whole subject was known by the title of the work. Reissued here is the revised and expanded 1738 second edition which contains the remarkable discovery that when a coin is tossed many times, the binomial distribution may be approximated by the normal distribution. This version of the central limit theorem stands as one of de Moivre’s most significant contributions to mathematics.

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# The Doctrine of Chances

*Or, a Method of Calculating  
the Probability of Events in Play*

ABRAHAM DE MOIVRE



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THE  
DOCTRINE  
OF  
CHANCES:  
OR,  
A METHOD of Calculating the Probabilities  
of Events in PLAY.

THE SECOND EDITION,  
*Fuller, Clearer, and more Correct than the First.*

BY A. DE MOIVRE,  
*Fellow of the ROYAL SOCIETY, and Member of the ROYAL ACADEMY  
OF SCIENCES of Berlin.*




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To the RIGHT HONOURABLE the  
  
Lord *CARPENTER*.

MY LORD,

 HERE are many People in the World who are prepossessed with an Opinion, that the Doctrine of Chances has a Tendency to promote Play, but they soon will be undeceived, if they think fit to look into the general Design of this Book : in the mean time, it will not be improper to inform them, that your Lordship is pleased to espouse the Patronage of this second Edition, which your strict Probity, and the distinguished Character you bear in the World, would not have permitted, were not their Apprehensions altogether groundless.

A 2 Your

## D E D I C A T I O N.

Your Lordship does easily perceive, that this Doctrine is so far from encouraging Play, that it is rather a Guard against it, by setting in a clear Light, the Advantages and Disadvantages of those Games wherein Chance is concerned.

Besides the Endowments of the Mind which you have in common with Those whose natural Talents have been cultivated by the best Education, you have this particular Happiness, that you understand, in an eminent Degree, the Principles of Political Arithmetic, the Nature of our Funds, the National Credit, and its Influence on public Affairs.

As one Branch of this useful Knowledge extends to the Valuation of Annuities founded on the Contingencies of Life, and that I have made it my particular Care to facilitate and improve the Rules I have formerly given on that Subject ; I flatter myself with a favourable Acceptance of what is now, with the greatest Deference, submitted to your Judgment, by,

MY LORD,

*Your Lordship's*

*Most Obedient and*

*Most Obligated,*

*Humble Servant,*

A. De Moivre.





# P R E F A C E.\*

**T**IS now about Seven Years, since I gave a Specimen in the Philosophical Transactions, of what I now more largely treat of in this Book. The occasion of my then undertaking this Subject was chiefly owing to the Desire and Encouragement of the Honourable Francis Robartes Esq; who, upon occasion of a French Traët, called, L'Analyse des Jeux de Hazard, which had lately been published, was pleased to propose to me some Problems of much greater difficulty than any he had found in that Book; which having solved to his Satisfaction, he engaged me to methodize those Problems, and to lay down the Rules which had led me to their Solution. After I had proceeded thus far, it was enjoined me by the Royal Society, to communicate to them what I had discovered on this Subject, and thereupon it was ordered to be published in the Transactions, not so much as a matter relating to Play, but as containing some general Speculations not unworthy to be considered by the Lovers of Truth.

I had not at that time read any thing concerning this Subject, but Mr. Huygens's Book de Ratiociniis in Ludo Aleæ, and a little English Piece (which was properly a Translation of it) done by a very ingenious Gentleman, who, tho' capable of carrying the matter a great deal farther, was contented to follow his Original; adding only to it the computation of the Advantage of the Setter in the Play called Hazard, and some few things more. As for the French Book, I had run it over but cursorily, by reason I had observed that the Author chiefly insisted on the

\* This Preface was written in 1717.

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*the Method of Huygens, which I was absolutely resolv'd to reject, as not seeming to me to be the genuine and natural way of coming at the Solution of Problems of this kind.*

*I had said in my Specimen, that Mr. Huygens was the first who had Published the Rules of this Calculation, intending thereby to do justice to a Man who had well deserved of the Public; but what I then said was misinterpreted, as if I had designed to wrong some Persons who had considered this matter before him, and a passage was cited against me out of Huygens's Preface, in which he saith, Sciendum vero quod jam pridem, inter Præstantissimos totâ Galliâ Geometras, Calculus hic fuerit agitatus; ne quis indebitam mihi primæ Inventionis gloriam hac in re tribuat. But what follows immediately after, had it been minded, might have cleared me from any Suspicion of injustice. The words are these, Cæterum illi difficillimis quibusque Quæstionibus se invicem exercere Soliti, methodum Suam quisque occultam retinere, adeo ut a primis elementis hanc materiam evolvere mihi necesse fuerit. By which it appears, that tho' Mr. Huygens was not the first who had applied himself to those sorts of Questions, he was nevertheless the first who had published Rules for their Solution; which is all that I affirmed.*

*Such a Treatise as this is may be useful to severals end; the first of which is, that there being in the World several inquisitive Persons, who are desirous to know what foundation they go upon, when they engage in Play, whether from a motive of Gain, or barely Diversion, they may, by the help of this or the like Treatise, gratify their curiosity, either by taking the pains to understand what is here Demonstrated, or else making use of the Conclusions, and taking it for granted that the Demonstrations are right.*

*Another use to be made of this Doctrine of Chances is, that it may serve in Conjunction with the other parts of the Mathematicks, as a fit Introduction to the Art of Reasoning; it being known by experience that nothing can contribute more to the attaining of that Art, than the consideration of a long Train of Consequences, rightly deduced from undoubted Principles, of which this Book affords many Examples. To this may be added, that some of the Problems about Chance having a great appearance of Simplicity, the Mind is easily drawn into a belief, that their Solution may be attained by the meer Strength of natural good Sense; which generally proving otherwise, and the Mistakes occasioned thereby being not unfrequent, 'tis presumed that a Book of this Kind, which teaches to distinguish Truth from what seems so nearly to resemble it, will be look'd upon as a help to good Reasoning.*

*Among*

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*Among the several Mistakes that are committed about Chance, one of the most common and least suspected, is that which relates to Lotteries. Thus, supposing a Lottery wherein the proportion of the Blanks to the Prizes is as five to one; 'tis very natural to conclude, that therefore five Tickets are requisite for the Chance of a Prize; and yet it may be proved, Demonstratively, that four Tickets are more than sufficient for that purpose, which will be confirmed by often repeated Experience. In the like manner, supposing a Lottery wherein the proportion of the Blanks to the Prizes is as Thirty-nine to One, (such as was the Lottery of 1710) it may be proved, that in twenty eight Tickets, a Prize is as likely to be taken as not; which tho' it may seem to contradict the common Notions, is nevertheless grounded upon infallible Demonstration.*

*When the Play of the Royal Oak was in use, some Persons who lost considerably by it, had their Losses chiefly occasioned by an Argument of which they could not perceive the Fallacy. The Odds against any particular Point of the Ball were One and Thirty to One, which intitled the Adventurers, in case they were winners, to have thirty two Stakes returned, including their own; instead of which they having but Eight and Twenty, it was very plain that on the single account of the disadvantage of the Play, they lost one eighth part of all the Money they play'd for. But the Master of the Ball maintained that they had no reason to complain; since he would undertake that any particular point of the Ball should come up in Two and Twenty Throws; of this he would offer to lay a Wager, and actually laid it when required. The seeming contradiction between the Odds of One and Thirty to One, and Twenty-two Throws for any Chance to come up, so perplexed the Adventurers, that they began to think the Advantage was on their side; for which reason they play'd on and continued to lose.*

*The Doctrine of Chances may likewise be a help to cure a Kind of Superstition, which has been of long standing in the World, viz. that there is in Play such a thing as Luck, good or bad. I own there are great many judicious people, who without any other Assistance than that of their own reason, are satisfied, that the Notion of Luck is meerly Chimerical; yet I conceive that the ground they have to look upon it as such, may still be farther inforced from some of the following Considerations.*

*If by saying that a Man has had good Luck, nothing more was meant than that he has been generally a Gainer at play, the Expression might be allowed as very proper in a short way of speaking: But if the Word Good Luck be understood to signifie a certain predominant quality, so inherent in a Man, that he must win whenever he Plays, or at least win oftner than lose, it may be denied that there is any such thing in nature.*

*The Asserters of Luck are very sure from their own Experience, that at some times they have been very Lucky, and that at other times they have had a prodigious Run of ill Luck against them, which whilst it continued obliged them to be very cautious in engaging with the Fortunate; but how Chance should produce those extraordinary Events, is what they cannot conceive: They would be glad, for Instance, to be Satisfied, how they could lose Fifteen Games together at Piquet, if ill Luck had not strangely prevailed against them. But if they will be pleased to consider the Rules deliver'd in this Book, they will see, that though the Odds against their losing so many times together be very great, viz. 32767 to 1, yet that the Possibility of it is not destroy'd by the greatness of the Odds, there being One Chance in 32768 that it may so happen; from whence it follows, that it was still possible to come to pass without the Intervention of what they call Ill Luck.*

*Besides, This Accident of losing Fifteen times together at Piquet, is no more to be imputed to ill Luck, than the Winning with one single Ticket the highest Prize, in a Lottery of 32768 Tickets, is to be imputed to good Luck, since the Chances in both Cases are perfectly equal. But if it be said that Luck has been concerned in this latter Case, the Answer will be easy; for let us suppose Luck not existing, or at least let us suppose its Influence to be suspended, yet the highest Prize must fall into some Hand or other, not by Luck, (for by the Hypothesis that has been laid aside) but from the meer necessity of its falling somewhere.*

*Those who contend for Luck, may, if they please, alledge other Cases at Play, much more unlikely to happen than the Winning or Losing fifteen Games together, yet still their Opinion will never receive any Addition of Strength from such Suppositions: For, by the Rules of Chance, a time may be computed, in which those Cases may as probably happen as not; nay, not only so, but a time may be computed in which there may be any proportion of Odds for their so happening.*

*But supposing that Gain and Loss were so fluctuating, as always to be distributed equally, whereby Luck would certainly be annihilated; would it be reasonable in this Case to attribute the Events of Play to Chance alone? I think, on the contrary, it would be quite otherwise, for then there would be more reason to suspect that some unaccountable Fatality did rule in it: Thus, if two Persons play at Cross and Pile, and Chance alone be suppos'd to be concern'd in regulating the fall of the Piece, is it probable that there should be an Equality of Heads and Crosses? It is Five to Three that in four times there will be an inequality; 'tis Eleven to Five in six, 93 to 35 in Eight, and about 12 to 1 in a hundred times: Wherefore Chance alone by its Nature constitutes the Inequalities of Play, and there is no need to have recourse to Luck to explain them.*

*Further,*

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*Further, The same Arguments which explode the Notion of Luck, may, on the other side, be useful in some Cases to establish a due comparison between Chance and Design: We may imagine Chance and Design to be, as it were, in Competition with each other, for the production of some sorts of Events, and may calculate what Probability there is, that those Events should be rather owing to one than to the other. To give a familiar Instance of this, Let us suppose that two Packs of Piquet-Cards being sent for, it should be perceived that there is, from Top to Bottom, the same Disposition of the Cards in both Packs; Let us likewise suppose that, some doubt arising about this Disposition of the Cards, it should be questioned whether it ought to be attributed to Chance, or to the Maker's Design: In this Case the Doctrine of Combinations decides the Question, since it may be proved by its Rules, that there are the Odds of above 26313083 Millions of Millions of Millions of Millions to One, that the Cards were designedly set in the Order in which they were found.*

*From this last Consideration we may learn, in many Cases, how to distinguish the Events which are the effect of Chance, from those which are produc'd by Design: The very Doctrine that finds Chance where it really is, being able to prove by a gradual Increase of Probability, till it arrive at Demonstration, that where Uniformity, Order and Constancy reside, there also reside Choice and Design.*

*Lastly, One of the principal Uses to which this Doctrine of Chances may be apply'd, is the discovering of some Truths, which cannot fail of pleasing the Mind, by their Generality and Simplicity; the admirable Connexion of its Consequences will increase the Pleasure of the Discovery; and the seeming Paradoxes wherewith it abounds, will afford very great matter of Surprise and Entertainment to the Inquisitive. A very remarkable Instance of this nature may be seen in the prodigious Advantage which the repetition of Odds will amount to; Thus, Supposing I play with an Adversary who allows me the Odds of 43 to 40, and agrees with me to play till 100 Stakes are won or lost on either side, on condition that I give him an Equivalent for the Gain I am intitled to by the Advantage of my Odds; the Question is, what I am to give him, on supposing we play a Guinea a Stake: The Answer is 99 Guineas and above 18 Skillings \*, which will seem almost incredible, considering the smallness of the Odds of 43 to 40. Now let the Odds be in any Proportion given, and let the Number of Stakes be played for be never so great, yet one general Conclusion will include all the possible Cases, and the application of it to Numbers may be wrought in less than a Minute's time.*

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\* Guineas were then at 21<sup>lb.</sup> 6<sup>d.</sup>



*I have explain'd, in my Introduction to the following Treatise, the chief Rules on which the whole Art of Chances depends; I have done it in the plainest manner that I could think of, to the end it might be (as much as possible) of general Use. I flatter my self that those who are acquainted with Arithmetical Operations, will, by the help of the Introduction alone, be able to solve a great Variety of Questions depending on Chance: I wish, for the sake of some Gentlemen who have been pleased to subscribe to the printing of my Book, that I could every where have been as plain as in the Introduction; but this was hardly practicable, the Invention of the greatest part of the Rules being intirely owing to Algebra; yet I have, as much as possible, endeavour'd to deduce from the Algebraical Calculation several practical Rules, the Truth of which may be depend upon, and which may be very useful to those who have contented themselves to learn only common Arithmetick.*

*On this occasion, I must take notice to such of my Readers as are well vers'd in Vulgar Arithmetick, that it would not be difficult for them to make themselves Masters, not only of all the practical Rules in this Book, but also of more useful Discoveries, if they would take the small Pains of being acquainted with the bare Notation of Algebra, which might be done in the hundredth part of the Time that is spent in learning to write Short-hand.*

*One of the principal Methods I have made use of in the following Treatise, has been the Doctrine of Combinations, taken in a Sense somewhat more extensive, than as it is commonly understood: The Notion of Combinations being so well fitted to the Calculation of Chance, that it naturally enters the Mind whenever any Attempt is made towards the Solution of any Problem of that kind. It was this that led me in course to the Consideration of the Degrees of Skill in the Adventurers at Play, and I have made use of it in most parts of this Book, as one of the Data that enter the Question; it being so far from perplexing the Calculation, that on the contrary it is rather a Help and an Ornament to it: It is true, that this Degree of Skill is not to be known any other way than from Observation; but if the same Observation constantly recur, 'tis strongly to be presumed that near Estimation of it may be made: However, to make the Calculation more precise, and to avoid causing any needless Scruples to those who love Geometrical Exactness, it will be easy, in the room of the word Skill, to substitute a Greater or Less Proportion of Chances among the Adventurers, so as each of them may be said to have a certain Number of Chances to win one single Game.*

*The general Theorem invented by Sir Isaac Newton, for raising a Binomial to any Power given, facilitates infinitely the Method of Combinations, representing*

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*representing in one View the Combination of all the Chances, that can happen in any given Number of Times. 'Tis by the help of that Theorem, joined with some other Methods, that I have been able to find practical Rules for the solving a great Variety of difficult Questions, and to reduce the Difficulty to a single Arithmetical Multiplication, whereof several Instances may be seen in the 21st Page of this Book.*

*Another Method I have made use of, is that of Infinite Series, which in many cases will solve the Problems of Chance more naturally than Combinations. To give the Reader a Notion of this, we may suppose two Men at Play throwing a Die, each in their Turns, and that he be to be reputed the Winner who shall first throw an Ace: It is plain, that the Solution of this Problem cannot so properly be reduced to Combinations, which serve chiefly to determine the proportion of Chances between the Gamesters, without any regard to the Priority of Play. 'Tis convenient therefore to have recourse to some other Method, such as the following: Let us suppose that the first Man, being willing to compound with his Adversary for the Advantage he is intitled to from his first Throw, should ask him what Consideration he would allow to yield it to him; it may naturally be supposed that the Answer would be one Sixth part of the Stake, there being but Five to One against him, and that this Allowance would be thought a just Equivalent for yielding his Throw. Let us likewise suppose the second Man to require in his Turn to have one sixth part of the remaining Stake for the Consideration of his Throw; which being granted, and the first Man's Right returning in course, he may claim again one sixth part of the Remainder, and so on alternately, till the whole Stake be exhausted: But this not being to be done till after an infinite number of Shares be thus taken on both Sides, it belongs to the Method of Infinite Series to assign to each Man what proportion of the Stake he ought to take at first, so as to answer exactly that fictitious Division of the Stake in infinitum; by means of which it will be found, that the Stake ought to be divided between the contending Parties into two parts, respectively proportional to the two Numbers 6 and 5. By the like Method it would be found that if there were Three or more Adventurers playing on the conditions above described, each Man, according to the Situation he is in with respect to Priority of Play, might take as his due such part of the Stake, as is expressible by the corresponding Term of the proportion of 6 to 5, continued to so many Terms as there are Gamesters; which in the case of Three Gamesters, for Instance, would be the Numbers 6, 5, and  $4\frac{1}{6}$ , or their Proportionals 36, 30, and 25.*

*Another Advantage of the Method of Infinite Series is, that every Term of the Series includes some particular Circumstance wherein the Gamesters*

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*Gamesters may be found, which the other Methods do not; and that a few of its Steps are sufficient to discover the Law of its Procefs. The only Difficulty which attends this Method, being that of summing up so many of its Terms as are requisite for the Solution of the Problem propos'd: But it will be found by experience, that in the Series resulting from the consideration of most Cases relating to Chance, the Terms of it will either constitute a Geometric Progression, which by the known Methods is easily summable; or else some other sort of Progression, whose nature consists in this, that every Term of it has to a determinate number of the preceding Terms, each being taken in order, some constant relation; in which case I have contriv'd some easy Theorems, not only for finding the Law of that Relation, but also for finding the Sums required; as may be seen in several places of this Book, but particularly from page 127 to page 128.*

*A Third Advantage of the Method of Infinite Series is, that the Solutions derived from it have a certain Generality and Elegancy, which scarce any other Method can attain to; those Methods being always perplexed with various unknown Quantities, and the Solutions obtained by them terminating commonly in particular Cases.*

*There are other Sorts of Series, which tho' not properly infinite, yet are called Series, from the Regularity of the Terms whereof they are compos'd; those Terms following one another with a certain uniformity, which is always to be defined. Of this nature is the Theorem given by Sir Isaac Newton, in the fifth Lemma of the third Book of his Principles, for drawing a Curve through any given number of Points; of which the Demonstration, as well as of other things belonging to the same Subject, may be deduced from the first Proposition of his Methodus Differentialis, printed with some other of his Tracts, by the care of my Intimate Friend, and very skilful Mathematician, Mr. W. Jones. The abovementioned Theorem being very useful in summing up any number of Terms whose last Differences are equal, (such as are the Numbers called Triangular, Pyramidal, &c. the Squares, the Cubes, or other Powers of Numbers in Arithmetic Progression) I have shewn in many places of this Book how it might be applicable to these Cases.*

*After having dwelt some time upon various Questions depending on the general Principle of Combinations, as laid down in my Introduction, and upon some others depending on the Method of Infinite Series, I proceed to treat of the Method of Combinations properly so called, which I shew to be easily deducible from that more general Principle which hath been before explained: Where it may be observed, that altho' the*

*Cases*

*N.B. The Numbers in the Margin, here and elsewhere, answer to the present Edition.*



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*Cases it is applied to are particular, yet the Way of Reasoning, and the Consequences derived from it, are general; that Method of Arguing about generals by particular Examples, being in my opinion very convenient for easing the Reader's Imagination.*

*Having explained the common Rules of Combinations, and given a Theorem which may be of use for the Solution of some Problems relating to that Subject, I lay down a new Theorem, which is properly a contraction of the former, whereby several Questions of Chance are resolved with wonderful ease, tho' the Solution might seem at first sight to be of insuperable difficulty.*

*It is by the Help of that Theorem so contracted, that I have been able to give a compleat Solution of the Problems of Pharaon and Bassete, which was never done before me: I own that some great Mathematicians have before me taken the pains of calculating the Advantage of the Banker, in any circumstance either of Cards remaining in his Hands, or of any number of times that the Card of the Ponte is contained in the Stock: But still the curiosity of the Inquisitive remained unsatisfied; The Chief Question, and by much the most difficult, concerning Pharaon or Bassete, being what it is that the Banker gets per Cent. of all the Money adventured at those Games, which now I can certainly answer is very near Three per Cent. at Pharaon, and Three fourths per Cent at Bassete, as may be seen in my xxiii Problem, where the precise Advantage is calculated. 32.*

*In the 24th and 25th Problems, I explain a new sort of Algebra, 34. 35. whereby some Questions relating to Combinations are solved by so easy a Process, that their solution is made in some measure an immediate consequence of the Method of Notation. I will not pretend to say that this new Algebra, is absolutely necessary to the Solving of those Questions which I make to depend on it, since it appears that Mr. Monmort, Author of the Analyse des Jeux de Hazard, and Mr. Nicholas Bernoulli have solved, by another Method, many of the cases therein proposed: But I hope I shall not be thought guilty of too much Confidence, if I assure the Reader, that the Method I have followed has a degree of Simplicity, not to say of Generality, which will hardly be attained by any other Steps than by those I have taken.*

*The 29th Problem, proposed to me, amongst some others, by the Honourable Mr. Francis Robartes, I had solved in my Tract De mensura Sortis; It relates, as well as the 24th and 25th, to the Method of Combinations, and is made to depend on the same Principle. When I began for the first time to attempt its Solution, I had nothing else to guide me but the common Rules of Combinations, such as they had been delivered by Dr. Wallis and others; which when I endeavoured to apply, I was surprized to find that my Calculation swelled by degrees to an intolerable Bulk: 38. 34. 35.*

a

For

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*For this reason I was forced to turn my Views another way, and to try whether the Solution I was seeking for might not be deduced from some easier considerations; whereupon I happily fell upon the Method I have been mentioning, which as it led me to a very great Simplicity in the Solution, so I look upon it to be an Improvement made to the Method of Combinations.*

39. *The 30th Problem is the reverse of the preceding; It contains a very remarkable Method of Solution, the Artifice of which consists in changing an Arithmetic Progression of Numbers into a Geometric one; this being always to be done when the Numbers are large, and their Intervals small. I freely acknowledge that I have been indebted long ago for this useful Idea, to my much respected Friend, That Excellent Mathematician Doctor Halley, Secretary to the Royal Society, whom I have seen practise the thing on another occasion: For this and other Instructive Notions readily imparted to me, during an uninterrupted Friendship of five and Twenty years, I return him my very hearty Thanks.*

44. *The 32 Problem, having in it a Mixture of the two Methods of Combinations and Infinite Series, may be proposed for a pattern of Solution, in some of the most difficult cases that may occur in the Subject of Chance, and on this occasion I must do that Justice to Mr. Nicholas Bernoulli, to own he had sent me the Solution of this Problem before mine was Published; which I had no sooner received, but I communicated it to the Royal Society, and represented it as a Performance highly to be commended: Whereupon the Society order'd that his Solution should be Printed; which was accordingly done some time after in the Philosophical Transactions, Numb. 341. where mine was also inserted.*

*The Problems which follow relate chiefly to the Duration of Play, or to the Method of determining what number of Games may probably be played out by two Adversaries, before a certain number of Stakes agreed on between them be won or lost on either side. This Subject affording a very great Variety of Curious Questions, of which every one has a degree of Difficulty peculiar to it self, I thought it necessary to divide it into several distinct Problems, and to illustrate their Solution with proper Examples.*

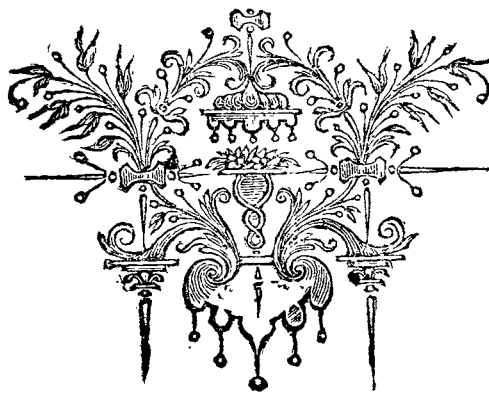
*Tho' these Questions may at first sight seem to have a very great degree of difficulty, yet I have some reason to believe, that the Steps I have taken to come at their Solution, will easily be followed by those who have a competent skill in Algebra, and that the chief Method of proceeding therein will be understood by those who are barely acquainted with the Elements of that Art.*

*When I first began to attempt the general Solution of the Problem concerning the Duration of Play, there was nothing extant that could give me any light into that Subject; for altho' Mr. de Monmort, in the first Edition of his Book, gives the Solution of this Problem, as limited to three*

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*Stakes to be won or lost, and farther limited by the Supposition of an Equality of Skill between the Adventurers; yet he having given no Demonstration of his Solution, and the Demonstration when discovered being of very little use towards obtaining the general Solution of the Problem, I was forced to try what my own Enquiry would lead me to, which having been attended with Success, the result of what I found was afterwards published in my Specimen before mentioned.*

*All the Problems which in my Specimen related to the Duration of Play, have been kept entire in the following Treatise; but the Method of Solution has received some Improvements by the new Discoveries I have made concerning the Nature of those Series which result from the Consideration of the Subject; however, the Principles of that Method having been laid down in my Specimen, I had nothing now to do, but to draw the Consequences that were naturally deducible from them.*





# ADVERTISEMENT

*Concerning this Second Edition.*

THE Advantages which this second Edition has over the first are many ; for 1°, The Arithmetical Principles laid down in the Introduction of the first have been very much enlarged, and made much more easy, than they were ; 2°, The Problems, of which the Solution requires more Skill than can be derived from the Rules of common Arithmetic, are treated in the clearest manner that the nature of the Subject will admit of ; 3°, The chief Questions which may relate to Lotteries are fully resolved, such as the following ; What is the Chance of getting a certain number of Prizes with a determinate number of Tickets ? What is the Advantage or Disadvantage of contracting about returning undrawn Tickets in case of Prizes, and how to state an Equality between the contending Parties ? What Consideration ought to be given in the beginning of a Lottery to have continually a new Chance supplied upon the determination of the old ? 4°, I have added the Solutions of some Problems relating to some Circumstances of the Game of *Quadrille*, whereby is shewn, the Probability of forcing all the Trumps, and of judging whether it be safe to undertake the *Vole* : 5°, The Values of Chances in *Raffling* are more easily calculated than in the first Edition, on which occasion this Question has been added ; Whether upon supposition of the Adventurers being equal Contributors to a common stock, it were better for the Caster of the first Chance that there were many or few Adventurers besides himself, and the true Limitation of the Number which would make that Chance as valuable as possible ? 6°, An easy Method of establishing at Hazard an Equality of Chance between the *Caster* and the *Setter* : 7°, This Question is resolved, What is at Hazard the Gain of the *Box*, or how many *Mains*, one with another, are requisite to make the Box-keeper be intitled to his Box-Money ? 8°, Some Problems concerning the Odds between two Players at *Piquet* according to the various Circumstances of a *Party* of three Games, together with other Questions relating to that Play. 9°, Some Questions relating to *Whisk*, as also the Resolution of a Question hitherto unanswered, *viz.* Whether one side being *Eight* of the Game, and

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and the other *Nine*, the Eight or the Nine have the best Chance? 10°, Whether at any Game it be more advantageous to have the *Odds* in Chance, upon an Equality of Skill, or to have the Odds laid to, upon an Equality of Chance? 11°, In this Book have been inserted the principal Rules of a former Book of mine concerning the Values of Annuities for Life, whether upon one, two or more Lives; the Values of Reversions, the Values of joint and successive Lives; Probabilities of Survivorship; Values of Expectations founded on the Contingency of the Duration of certain Lives for a limited Time: as also some Tables to facilitate the practice of the foregoing Rules. 12°, The Method of calculating the Value of a perpetual *Advowson*, according to the Age of the Incumbent. 13°, The Solution of a Question which cannot fail of interesting the Reader; it is, What reasonable Conjectures may be derived from Experiments, or what are the Odds that after a certain number of Experiments have been made concerning the happening or failing of Events, the Accidents of Contingency will not afterwards vary from those of Observation beyond certain Limits; which leads naturally to this Consequence, that altho' Chance has very great Influence on some Events, yet that it very little disturbs those which in their natural Institution were designed to happen according to fixt Laws, Chance vanishing as it were at long run in respect to the Constancy and Regularity of Order. 14°, Altho the Solution of the Question last mentioned was intended to conclude my Work, yet two or three curious Problems have been added to it, on some particular Occasions.

I suppose it is not expected from me that I should answer the Objections which a late anonymous Author has made against what I formerly writ concerning the Values of Lives; for he having declared more than once that he had never read my Book, but that what he had cited out of it, was from the Citation of others, I think he has thereby sufficiently answered himself, for which reason I'll content my self with telling him, that he neither understands what he would confute, nor knows what he would establish.

There is in the World a Gentleman of an older Date, who in the year 1726 did assure the Public that he could calculate the Values of Lives if he would, but that he would not, because the Observations then extant about the probable Duration of Lives were so uncertain and so few, that he could not rely upon them, but that as soon as he had procured from the Parish Clerks, the Bills of Mortality for a competent number of years, he would then shew what he is capable of; but as he was diffident he could not live so long, He, to make some compensation for the uncertainty of his Life, did communicate

to



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to the Public a most valuable Secret, by declaring that the best Method he could propose to his Reader for the valuation of Lives, was to consider *with himself*, what number of years certain is equivalent to the Life or Lives in question, having always regard to the Age and Health of the Annuitant or Annuitants; which done, his Book will tell you the rest: What pity it is, that after so rare an Expedient for calculating a single Life, he has not been pleased to consider *with himself*, what must be the Method of calculating combined Lives; for it does not clearly follow, that because a good Physiognomist can guess at the Value of one Life, he can also guess at the Value of two or three joint Lives, or at the longest of them; it is to be hoped that some time or other, there will be found some ingenious Person who will improve so useful a Hint, and prosecute what he had so happily begun.

I have one word to add, which is, that the Tables I have printed in this Book concerning the Values of Lives, being upon a supposition of an Interest of 5 *per Cent.* may be thought not to be well fitted to the present Time: to which I answer, *first*, That in the Body of my Book I have given Rules for calculating the Values of Lives for any rate of Interest; *secondly*, That those Tables were originally composed when Interest was at five *per Cent.* *Thirdly*, that a Man by purchasing a Life at a certain price, may, without offending the Law, make 5 *per Cent.* of his Money or upwards: *Lastly*, that I intend shortly to publish a little Pocket-Book, containing Tables for readily estimating the Values of Single and Combined Lives, for an Interest both of 5 and 4 *per Cent.* wherein I shall endeavour to prove, that the rate of Interest which a Man ought to value his Money at, when he purchases a *Life*, is not to be regulated by the Interest he can make, by placing his Money in the Funds.



The most remarkable Errata, are as follows:

Pag. 83. Lin. 19. for 6 *Matadors*, read 3 *Matadors*.

Pag. 113. Lin. 2. for Cylinder, read Prism.

Pag. 152. Lin. 7. from the Bottom, fill up the Blank by writing  
xviii<sup>th</sup>.