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Edited for Trinity College by William Whewell
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The Mathematical Works of Isaac Barrow

EDITED FOR TRINITY COLLEGE
BY WILLIAM WHEWELL



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THE
MATHEMATICAL WORKS

OF

ISAAC BARROW, D.D.

MASTER OF TRINITY COLLEGE, CAMBRIDGE.

Edited for Trinity College

BY

W. WHEWELL, D.D.

MASTER OF THE COLLEGE.

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P R E F A C E.

It was thought right, by Barrow's College, that a new edition of his Mathematical Works should accompany the edition of his Theological Works lately published by the University, under the editorial care of Mr Napier; and I have willingly undertaken to superintend the printing of the edition thus agreed upon. I have already, in the Preface to Vol. ix. of Mr Napier's Edition, given an account of Barrow's mathematical writings; but for the sake of convenience I will here resume the subject.

Barrow's first mathematical publication appears to have been his edition of Euclid. I have supposed in my former Notice that this was published in 1654, before he set out upon his travels. But the first edition is dated 1655. It is probable that he left the manuscript to be printed after his departure; for in the Preface he says that he has tried to reduce the book into a small space: "Id quod assecutus videor, si *absentem* Typographi cura non frustretur." This edition contains all the fifteen Books of Euclid's Elements¹. I shall not insert the bulk of this publication in the present edition; but at the end of this Prefatory Notice I will insert Barrow's Dedication and Preface to this book. The Dedication is addressed to three young men, who, from the terms in which they are spoken of, must have been, I conceive, Barrow's pupils. They were Edward Cecil, son of the Earl of

¹ Euclidis *Elementorum* Libri xv. breviter demonstrati, Opera Is. Barrow, Cantabrigiensis, Coll. Trin. Soc. MDCLV. There are many later editions in Latin and in English.

Salisbury, John Knatchbul, and Francis Willoughby¹. The latter was afterwards the celebrated reformer of Ichthyology, the friend of Ray, the botanist. He addresses these young men with expressions of great affection and esteem; and says that no one can know their good qualities better than himself, in virtue of the sweet habitual intercourse which he has had with them. In his Preface he speaks of the two main objects which he had in editing Euclid, to reduce the whole of the Elements into a portable volume, and to gratify those readers who prefer symbolical to verbal reasoning. He would have been satisfied, he says, with Tacquet's edition, if Tacquet had not confined it to eight books (the I. II. III. IV. V. VI. XI. and XII.); omitting the remaining seven. The symbols which he has used are mostly those of William Oughtreed, "to which most of us are accustomed." These are the usual algebraical symbols, of which the introduction was then recent.

But the principal contents of the present volume are the Lectures which Barrow delivered as Lucasian Professor of Mathematics; an office which he held from 1664 to 1670. And these consist of three series: the *Lectiones Mathematicæ*, the *Lectiones Opticæ*, and the *Lectiones Geometricæ*; the first being on the general Principles of Mathematics, the second containing propositions of Optics proved geometrically, and the third treating of properties of Curve Lines. I will make a few remarks on some of these Lectures.

The first or Inaugural Lecture², contains an account

¹ The entries of these names in the College Admission Book are, (p. 23.) Edvardus Cecil, filius H. D. Comitum Sar. admissus commensalis Nov. 30, 1652.

(p. 2.) Johannes Knatchbul, Cantianus, admissus commensalis May 6, 1652.

(p. 23.) Franciscus Willoughby, Warwicensis, admissus commensalis Sept. 9, 1652:

All the three being admitted as pupils of Mr Duport.

² Though this Lecture is printed in the ninth volume of Mr Napier's edition, I have reprinted it here, as the first of the Lucasian Lectures; and

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PREFACE.

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of Henry Lucas, the founder of the Professorship, given in Barrow's usual rhetorical manner. He begins by referring to the tranquillity which had been restored to the nation by the cessation of the civil wars. "The turmoil of public business being reduced to rest, and the restoration of tranquillity having given you some heart to tell and to hear news, listen to me, Academics, while I haste to tell you something strange and almost a prodigy. There has shone forth of late—What? you will say. Some dire comet, the presage of calamity, such as are seen in numbers every day (in spite of the heavens themselves) by the distorted vision of fanatics? No: but a new and benignant star, shining with a ray both true and propitious, such as has not for many years risen above the academical horizon. And it is that I may measure its magnitude, explain its motions, and interpret its presages, that I now come forwards, no vain astrologer I." It is by this image, and others in the same strain, that he describes Lucas, and his good deeds towards the University. The Lucasian Professorship which he founded he endowed with the rents of an estate in Bedfordshire, amounting at present to £155. Barrow mentions the principal circumstances of Lucas's life: stating that he had studied at St John's College, and was then taken into the family of the Earl of Holland, (who was Chancellor of the University from 1625 to 1648,) as his Secretary. Here, by attention to business and economy, he accumulated a considerable fortune; and continuing unmarried, resolved to make posterity his heir. He represented the University in Parliament from 1640; and as his end approached, considered and consulted with his friends how he could best promote its interests. The result was that he determined to give encouragement to mathematical studies, which, though admired always, and

have also ventured to repeat in this Preface some of the remarks which I made in the Preface to that volume.

especially in recent times, had hitherto enjoyed no patronage in the University. Barrow goes on to extend his praises to the two trustees of Lucas's will: Robert Raworth, a lawyer, and Thomas Buck, a resident in the University, whom he refers to as well-known to his hearers; "the same whose stately presence and dignified countenance is every day before your eyes;" and to him he ascribes the suggestion of Lucas's benefaction. These two, in conjunction with the Heads of Colleges, and especially the Vice-chancellors of 1663 and 1664, carefully and diligently took the proper steps for carrying the bequest into effect.

He then proceeds to speak of his own tastes, and of his purposes with regard to Lectures; as I have stated in the Notice of him already referred to; and ends his praise of Mathematics in language which, as I have said, may remind us of the expressions of Francis Bacon.

The Lectures which in the present volume next follow this Prefatory Lecture are those which were the earliest delivered, namely in 1664, 5 and 6, but the latest published, namely, not till 1685. They treat, as I have already said, of the general principles and arrangements of Mathematics, with a notice of some of the leading controversies and criticisms which had appeared on the subject; extending through twenty-three lectures, and ending with a vindication of Euclid's Doctrine of Proportion. I have annexed to these, at the foot of the page, a brief summary of their contents, which may enable the English reader to trace the general course of the argument. They display great metaphysical subtlety, logical precision, and a large acquaintance with the literature of Mathematics, both ancient and modern. As I have stated in the former account of Barrow, an English translation of these Lectures was published in 1734; but so badly executed that it cannot be of use to any one. The Lectures themselves are of interest to those who love to dwell on the meta-

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physical grounds of mathematical truths; but do not belong to that progressive line of mathematical speculation to which Barrow had referred in his Prefatory Lecture when he spoke of Galileo, Gassendi, Gilbert, Mersenne, Cartesius, and others. Barrow's contributions to this kind of mathematics appear in his *Lectiones Opticæ*, and *Lectiones Geometricæ*, as we shall see.

But next after the *Lectiones Mathematicæ*, I have printed four Lectures, in which he proposes to himself, as he says, to expound the method by which Archimedes invented his beautiful theorems: (those, namely, concerning Cones and Spheres, their Solid Contents and Surfaces). This he says he will do by reducing the steps to problems, such as Archimedes proposed to himself, and from the solution of which he deduced both his theorems, and the mode of demonstrating them: whence, he says, it will appear what was the analysis which he used, and how like our modern analysis it was. It is a thought which has often suggested itself to the readers of the ancient Greek mathematicians, and especially of Archimedes, that those writers must have been led to their geometrical theorems and proofs by some methodical analytical process. Their constructions and propositions are so complex, recondite, and abstruse, that it seemed impossible that any one should be led to them by mere direct exercise of ingenuity or felicity of conjecture. We know that some modern mathematicians, particularly Newton, have followed this practice, of discovering a proposition by analysis, and then proving it by a geometrical synthesis. Barrow has, in the four Lectures here given, proved, partly by the use of algebraical analysis, some of the most difficult of the Propositions of Archimedes concerning Spheres and Cones: for instance, that which forms the last proposition in these Lectures, that of all Segments of Spheres, of equal extent of surface, the Hemisphere has the largest content: a proposition which might be found difficult to demonstrate by a good mathematician of the

present day. And as a previous step, he has to solve this Problem (Lect. xxvii. Prob. iv.); To cut a given Sphere into two Segments which have a given Ratio. He obtains, by algebraical processes, a certain proportion which gives the point of section of the axis; and which is, he says, the very proportion to which Archimedes reduces the problem: and this, he says, shows what kind of analysis that author must have employed. For that he got at it by employing the various compositions, divisions, permutations and inversions of proportion, in the same order in which he presents them in his text, is beyond belief. If he had done this, his lighting upon the right solution would have been rather a matter of chance than of reason or skill; and that this should have happened so constantly, is inconceivable and impossible.

Barrow's love of the ancient Greek geometers, and his desire to abridge and simplify their demonstrations by introducing into them analysis, led him at a later period to publish an edition of the works of Archimedes, Apollonius, and Theodosius¹.

To include this work among Barrow's Mathematical Works would have made this publication too bulky: I have printed, at the end of these prefatory remarks, the brief preface which he prefixes to it. His edition contains all the extant works of Archimedes, namely the Two Books on the Sphere and Cylinder, agreeing in substance with the four Lectures here given: The Treatises on the Measurement of the Circle: On Spirals: On Conoids and Spheroids: On the Centres of Gravity in Plane Figures: On the Quadrature of the Parabola: On Floating Bodies: On numbering the Sand. Also his Lemmas, which include, among other propositions concerning tangencies of circles,

¹ *Archimedis Opera; Apollonii Pergæi Conicorum Libri quatuor; Theodosii Sphærica Methodo novo illustrata et succinetè demonstrata.* Per Is. Barrow, exprofessorem Lucasianum Cantab. et Societatis Regiæ Soc. 1675.

the proposition concerning the figure which he calls the *Arbelon*, a figure bounded by three semicircles, and named from its resemblance to a leather-cutter's knife.

Of Apollonius, this edition by Barrow contains only four Books, the only ones which have come down to us in Greek. Books v. vi. and vii. were afterwards found to exist in an Arabic Translation; were brought from the East by Golius, translated by Abraham Ecchellesius, and published by Borelli in 1661. The Eighth Book has never been found, but has been restored conjecturally by Halley and by Vieta.

The Three Books of the Spherics of Theodosius contain various propositions concerning the circles of the sphere, Small Circles as well as Great. The propositions here contained are the basis of Spherical Trigonometry; but of course the ancient Greek Geometer does not attempt the solution of spherical triangles.

The *Opticæ Lectiones*, as I have mentioned in my former account of Barrow, are noticed by historians of mathematics as an important work. In his *Optical Speculations*, says Montucla, Barrow quitted the beaten track, and discussed questions hitherto imperfectly treated; as the theory of the foci of spherical surfaces and lenses, the apparent places of images, and the like. He also explains the Rainbow, simplifying Cartesius's calculations; and seeing that the Cartesian explanation of the colours is not satisfactory, proposes one of his own (Lect. xii. Art. xvi.). It remained for Newton to give the true explanation. In the Epistle to the Reader, he states that Isaac Newton had revised and corrected the copy and added matter of his own; and that Collins (who, he says, may be called the *Mersenne* of England, for his merits in promoting mathematical science both by his own labours and those of others) had superintended the edition with great attention. The mathematical reader will recollect that Mersenne was

a correspondent of most of the scientific men of his time, and a centre of correspondence among them.

The Geometricæ Lectiones are full of curious methods of determining the areas and tangents of curves, many of which are very close anticipations of Newton's methods. The most noted of these is the method of drawing tangents to curves, given in Lect. x. Art. xiv. This method is justly held to be an anticipation of the Differential Calculus, and to approach very near to it. It will be best explained by taking Barrow's first example, which is this.

In Fig. 116, ABH is a right angle, K any point in BH ; A being a fixed point, AK is joined; and in it, AM is taken equal to BK : it is required to draw a tangent to the curve AM .

If, as in modern notation, we draw an ordinate MP perpendicular to the line of abscissas AB , and call AP and PM , x and y respectively, AB being $= r$, it is evident that $DK = \frac{ry}{x}$, and $AM = \sqrt{x^2 + y^2}$; whence the equation of the curve is $x^2 + y^2 = \frac{r^2 y^2}{x^2}$; or $x^4 + x^2 y^2 = r^2 y^2$. And if we differentiate this we obtain, for the subtangent PT ,

$$\frac{ydx}{dy} = \frac{r^2 y^2 - x^2 y^2}{2x^3 + xy^2}.$$

Barrow's mode of proceeding is this: (he uses p and m for x and y , which I shall alter so as to fall in with modern notation:)

Take an ordinate and abscissa near to x and y , and let these be $x - e$ and $y - a$, e and a being small. Therefore

$$\begin{aligned} (x - e)^2 + (y - a)^2 &= AQ^2 + QN^2 = AN^2 \\ &= BL^2 = x^2 - 2xe + e^2 + y^2 - 2ay + a^2. \end{aligned}$$

But $AQ : QN :: AB : BL$;

that is, $x - e : y - a :: r : BL$,

whence $BL^2 = \frac{r^2 y^2 + r^2 a^2 - 2r^2 ya}{x^2 + e^2 - 2xe}$;

and, *rejecting superfluous terms* (1), in these values of BL^2 , they are

$$= \frac{r^2y^2 - 2r^2ya}{x^2 - 2xe} \text{ and } x^2 - 2xe + y^2 - 2ay.$$

And, equating these, and multiplying up,

$$r^2y^2 - 2r^2ya = x^4 - 2x^3e + x^2y^2 - 2x^2ya - 2x^3e + 4x^2e^2 - 2xy^2e + 4xyae.$$

That is, *rejecting the terms which our rule rejects*, (2),

$$- 2r^2ya = - 4x^3e - 2x^2ya - 2xy^2e,$$

or $r^2ya - x^2ya = 2x^3e + xy^2e;$

or, putting the ordinate y and the subtangent t for a and e , (since they are in the same proportion as those lines,)

$$r^2y^2 - x^2y^2 = 2x^3t + xy^2t,$$

whence $\frac{r^2y^2 - x^2y^2}{2x^3 + xy^2} = t = PT.$

The peculiarity of the method consists in the steps which I have marked (1) and (2); that is, the rejection of superfluous terms. And the Rule given by Barrow is this:

After constituting the equation to the curve, put $x - a$ and $y - e$ for the ordinates x and y : expand, and reject all the terms in which there is no a or e ; (for they destroy each other by the nature of the curve;) reject all the terms in which a or e are above the first power, or are multiplied together: (for they are of no value compared with the rest, as being infinitely small:) then put y for a , and t the subtangent for e ; and PT is found.

It is plain that the terms thus retained are the terms involving the first powers of a and e , when, in the equation to the curve, $x - a$ and $y - e$ are put for x and y and the equation is expanded. But the ratio of the coefficients of these terms is the ratio of dx to dy in the differential calculus: and hence the substantial identity of the two methods is evident. What remained was, to devise a notation, and to assign general rules of obtaining those coefficients.

Barrow applies this method to the following curves :
 (I use the modern notation for the co-ordinates :)

Ex. II. The curve $x^3 + y^3 = r^3$.

Ex. III. The curve $x^3 + y^3 = rxy$, which it appears
 was called *La Galande*.

Ex. IV. *The Quadratrix*, of which the equation is

$$y = \overline{r - x} \tan \frac{\pi x}{2r}.$$

Ex. V. The curve in which the abscissa being equal
 to an arc of a circle, the ordinate is equal to its trigonome-
 trical tangent :

$$y = r \tan \frac{\pi x}{2r}.$$

Also we may regard Barrow's mode of finding the areas
 of curves by comparing them with the sum of the inscribed
 and circumscribed parallelograms (Lect. XII. Append. 2,
 fig. 175, 176), as leading the way to Newton's method of
 doing the same, given in the First Section of the Principia.

I have, for the most part, retained Barrow's notation,
 with slight alterations where it would have been likely to
 mislead a modern reader. Thus where we write $A : B$ he
 writes $A . B$, and where we write $A > B$ he writes $A \square B$.
 I have retained Aq for A^2 , $A \text{ cub}$ for A^3 , Aqq for A^4 and
 the like.

It is a matter of labour and difficulty for a reader in
 these days to follow out the complex constructions and
 reasonings of a mathematician of Barrow's time ; and I do
 not pretend that I have in all cases gone through them to
 my satisfaction. I have however, in several cases, en-
 deavoured to assist my reader to follow these demonstrations
 with moderate trouble.

TRINITY COLLEGE,
 November 9, 1860.

Dedication of Barrow's Euclid,
 1655.

NOBILISSIMIS ET GENEROSISSIMIS

ADOLESCENTIBUS

D^{no} EDVARDO CECILIO,

Illustriss. Comitiss Sarisburiensis Filio;

D^{no} JOHANNI KNATCHBUL,

ET

D. FRANCIS WILLOUGHBY,

ARMIGERIS.

UNICUIQUE vestrum (Optimi Adolescentes) tantum me debere repute, quantum homo homini debere potest. Meâ enim sententiâ, ultra sincerum amorem non est quod quispiam de alio bene mereri possit. Hunc autem jamdiu est quo ex singulari vestrâ bonitate mihi indultum experior, ejusque sensus intimis animi medullis inhærens, ipsi ardens studium impressit, quovis honesto modo reciprocos affectus prodendi. Quandoquidem vero ea fortunarum mearum tenuitas, ea vestrarum amplitudo existit, ut nec ego aliâ quâ gratæ alicujus agnitionis significatione uti queam, nec vos aliam admittere velitis, eapropter haud illibenter hanc occasionem arripio, honoris et benevolentiæ, quibus vos prosequor, publicum hoc et durable *μνημόσυνον* edendi. Etsi cum oblatis anathematis exilitatem, et libellum vestris nominibus consecratum, quam is longe infra vestrorum meritorum dignitatem subsidat, attentius considero, timor subinde aliquis et dubitatio animum incessant, ne hoc studium erga vos meum vobis dehonestamento sit potius, quam ornamento; scilicet memor cum sim, ut malæ causæ, sic et mali libri patrocinium in patroni contumeliam magis quam in gloriam

cedere. Sed quum vestrarum virtutum id robur, eam fore soliditatem recognoscerem, quæ vestrum decus, meo quantumvis labefactato, inconcussum sustinere possint, idcirco non dubitavi vos in aliquatenus commune mecum periculum induere. Virtutes illas intelligo, quibus nemo unquam in vestrâ ætate, aut in vestro ordine, saltem me iudice, majores deprehendit, quæ vos insigniter gratos omnibus et amabiles reddunt, eximiam modestiam, sobrietatem, benignitatem animi, morum comitatem, prudentiam, magnanimitatem, fidem; præclaram insuper ingenii indolem, quæ vos ad omnem ingenuam scientiam non tantum excellenti captu, sed et appetitu forti ac sincero instruxit. Quas vestras præclarissimas dotes prout nemo est fortassis, qui me melius novit, aut pro consuetudine, quam jamdudum vobiscum dulcissimam coluisse ex vestro favore mihi contigit, penitius introspexerit, ita nemo est, qui impensius miratur, et suspicit; aut qui ipsas libentius prædicare, ac celebrare vellet, si non cum eloquii mei vires supergrederentur, tum etiam quæ in singulis vobis elucent, prolixius alicujus commentarii, aut panegyricæ orationis libertatem, potius quam præstitutas hujusmodi salutationibus angustias, exposcerent. Quin potius divinam clementiam imploro, ut vos earundem virtutum sancto tramiti insistere, atque hos egregios fructus vernæ vestræ ætatis felicibus incrementis maturescere concedat; vitamque vobis in hoc seculo ingenuam, innocentem, jucundam, et in futuro beatam ac sempiternam transigere largiatur. Minime autem dubito, ne pro consueto vestro in me candore, hoc ultimum fortassis, quod vobis præstare postero, benevolentiae erga vos et observantiae testimonium, alacriter accepturi sitis, quod vobis propensissimo affectu offert.

*Vestri in æternum amantissimus,
et observantissimus,*

I. B.

Preface of Barrow's Euclid,
 1655.

BENEVOLO LECTORI.

SI quid in hac elementorum editione præstitum sit scire desideras, amice Lector, accipe, pro genio operis, breviter. Ad duos præcipue fines conatus meos direxi. Primum ut cum requisitâ perspicuitate summam demonstrationum brevitatem conjungerem, quo eam libello molem compararem, quæ commode absque molestiâ circumferri posset. Id quod assecutus videor, si absentem Typographi cura non frustretur. Concinnius enim quispiam meliori ingenio, aut majori peritiâ excellens, at nemo forsâ brevius plerasque propositiones demonstraverit, præsertim cum in numero et ordine propositionum ipse nihil immutârim, nec licentiam mihi assumpserim quamcunque propositionem Euclideam procul ablegandi tanquam minus necessariam, aut quasdam faciliores in axiomatum censum referendi, quod nonnulli fecerunt; inter quos peritissimus Geometra A. Tacquetus *C* quem ideo etiam nomino, (quod quædam ex eo desumpta agnoscere honestum duco,) post cujus elegantissimam editionem, ipse nihil attentare voluissem, si non visum fuisset doctissimo viro non nisi octo *Euclidis* libros suâ curâ adornatos publico communicare, reliquis septem, tanquam ad elementa Geometriæ minus spectantibus, omnino quasi spreto atque posthabitis. Mihi autem jam ab initio alia provincia demandata fuit, non elementa Geometriæ utcunque pro arbitrio conscribendi, verum *Euclidem* ipsum, eumque totum, quam possem brevissime, demonstrandi. Quod enim quatuor libros spectat, septimum, octavum, nonum, decimum, quamvis illi ad Geometriæ planæ et solidæ elementa, ut sex præcedentes, et duo subsequentes, non tam prope pertineant,

quod tamen ad res Geometricas admodum utiles sint, tam propter Arithmeticæ et Geometricæ valde propinquam cognationem, quam ob notitiam commensurabilium et incommensurabilium magnitudinum ad figurarum tam planarum, quam solidarum apprime necessariam, nemo est e peritioribus Geometris qui ignorat. Quæ vero in tribus ultimis libris continetur, 5 corporum regularium nobilis contemplatio, illa non nisi injuriâ prætermitti potuit, quando nempe illius gratiâ noster *στοιχειωτής*, Platonicæ familiæ philosophus, hoc elementorum systema universum condidisse perhibetur, uti testis est *Proclus*¹, iis verbis, "Ὅθεν δὴ καὶ τῆς συμπασῆς στοιχειώσεως τέλος προεστῆσατο τὴν τῶν καλουμένων πλατωνικῶν σχημάτων σύστασιν. Præterea facile in animum induxi ut opinarer, nemini harum scientiarum amanti non futurum esse cordi, penes se habere integrum Euclideum opus, quale passim ab omnibus citatur, et celebratur. Quare nullum librum, nullamque propositionem negligere volui earum, quæ apud *P. Herigonium* habentur, cujus vestigiis presse insistere necesse habui, quoniam ejusce libri schematismis maximâ ex parte uti statutum erat, quod præviderem mihi ad novas describendas tempus non suppetere, etsi nonnunquam id facere præoptâssem. Eâdem de causâ nec alias plerasque quam Euclidean demonstrationes adhibere volui, succinctiori formâ expressas, nisi forte in 2, et 13, et parce in 7, 8, 9 libris, ubi ab eo nonnihil deflectere operæ pretium videbatur. Bona igitur spes est saltem in hac parte cum nostris consiliis, tum studiosorum votis aliquo modo satisfactum iri. Nam quæ adjecta sunt in Scholiis problemata quædam et theoremata, sive ob suum frequentem usum ad naturam elementarem accedentiâ, sive ad eorum, quæ sequuntur, expeditam demonstrationem conducentiâ, seu quæ regularum practicæ Geometriæ quarundam præcipuarum rationes innuunt ad suos fontes relatas, per ea, ut spero, libellus ultra destinatam molem magnopere non intumescet.

¹ Lib. II.

Alter scopus, ad quem collineatum est, eorum desideriis consuluit, qui demonstrationibus symbolicis potius quam verbalibus delectantur. In quo genere cum plerique apud nos *Gulielmi Oughtredi* symbolis assueti sint, ea plerumque usurpare consultius duximus. Nam qui *Euclidem* hâc viâ tradere et interpretari aggressus sit, hactenus, quod ego sciam, præter unum *P. Herigonium*, repertus est nemo. Cujus viri longe doctissimi methodus, sane in multis egregia, ac ejus peculiari proposito admodum accommodata, duplici tamen defectu laborare mihi visa est. Primo, quod cum Propositionum ad unius alicujus theorematis aut problematis probationem adductarum, posterior a priori non semper dependeat, quando tamen illæ inter se cohærent, quando non, nec ex ordine singularum, nec ullo alio modo satis prompte innotescere potest; unde ob defectum conjunctionum, et adjectivorum *ergo, rursus, &c.* non raro difficultas et dubitandi occasio, præsertim minus exercitatis, inter legendum oboriri solent. Deinde sæpe evenit, ut prædicta methodus nimis frequenter supervacaneas repetitiones effugere nequeat, a quibus demonstrationes est quando prolixæ, aliquando et magis intricatæ evadunt. Quibus vitiis noster modus facile per verborum signorumque arbitrariam mixturam medetur. Atque hæc de opellæ hujus intentione et methodo dicta sufficiant. Cæterum quæ in laudem Matheseos in genere, aut Geometriæ ipsius; et quæ de historiâ harum scientiarum, ideoque de *Euclide* horum elementorum digestore dici possent, et reliqua hujusmodi *ἔξωτερικὰ*, cui hæc placent, apud alios interpretes consulere potest. Neque nos angustias temporis, quod huic operi impendi potuit, nec interpellationes negotiorum, nec adjumentorum ad hæc studia apud nos egestatem, et quædam alia, ut liceret non immerito, in excusationem obtendemus, metu scilicet inducti, ne hæc nostra omnibus minus satisfaciant. Verum quæ ingenui Lectoris usibus elaboravimus, eadem in solidum ipsius censuræ ac judicio submittimus, probanda si utilia sibi comperit, sin omnino secus, rejicienda.

I. B.

Preface of Barrow's Archimedes, &c.

1674.

LECTORI.

VETUSTOS autores, scientiarum parentes, ab interitu salvos præstari, posterorum interesse videtur, ne ingrati audiant. Neque tametsi quæ continent pleraque novis artificiis vel promptius elici, vel concisius astrui possint, fructu penitus destituitur illorum lectio. Nam amcenum imprimis videtur quibus a fundamentis tantum in fastigium evectæ sunt scientiæ dispicere; tum haud inutile fuerit degustare fontes, e quibus cuncta ferme recentiorum inventa dimanârunt; istorum quippe perquam ingeniosas atque subtiles persequendo vel æmulando methodos horum emicuit industria. Porro sincerum demonstrandi gustum ac peritiam non aliunde quis opinor feliciter hauserit, quam ex iis, quorum in theorematis deducendis præcipuæ relucet solertia ac elegantia; quas ut nemo transgredi possit, ita vix assequi quisquam valeat ab illorum Scriptis peregrinus: Ut taceam, cum a posteris hæc scripta suis firmandis præsternantur, allegenturque passim, illorum referre qui hæc studia tractant, ea præsto ad manum, ne dicam ad unguem, habere: Id ut prompte tibi succedat, et quam exiguo impendio, præstitura videtur hæc editio; saltem præ illis, quæ enormi juxta mole spissæ ac pretio caræ hactenus prostant; sin hæc nihilominus displiceat, det ille quæso tibi pœnas, qui amicitiam præpotenter abusus, me nequicquam reclamante, protrusit hæc crepundia, luci publicæ minime nata vel debita. *Vale.*